Explorations, Investigations, and Applications

(It's why we study mathematics!)

Area Versus Perimeter

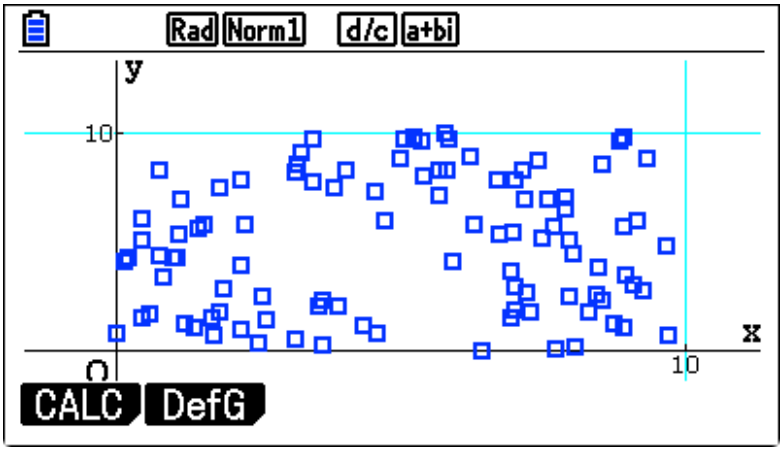
Of Random Rectangles

Projectile Motion

A look at physics

Recursion

The environment and medicine



seq(10 rnd#,x,1,100,1) -> L1

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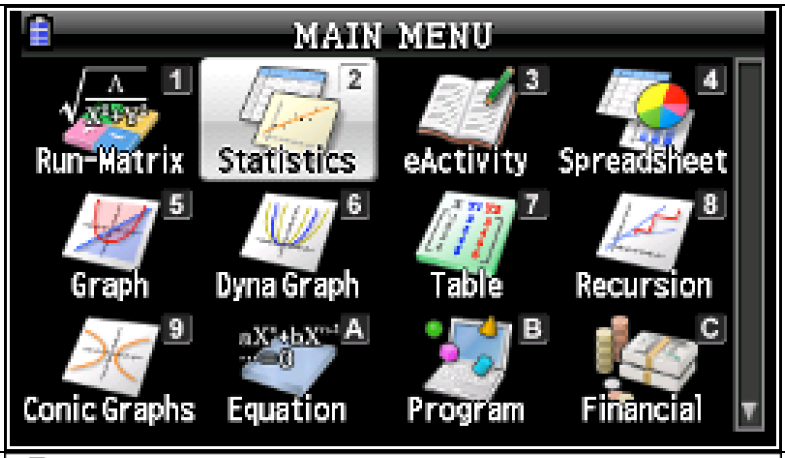
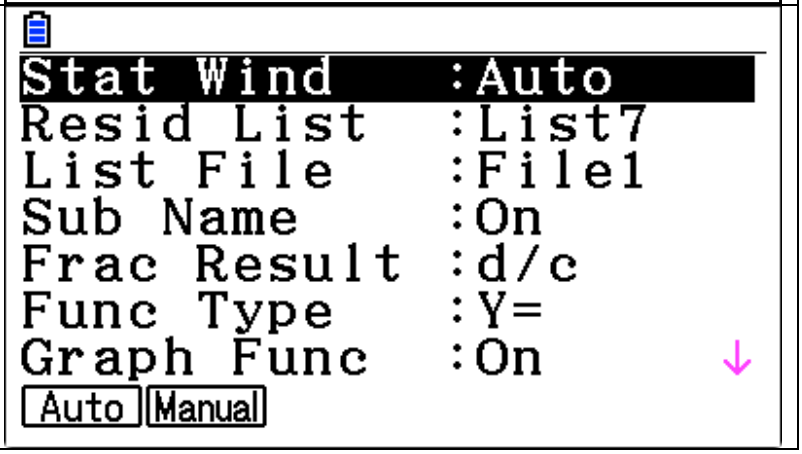
How are the Area and Perimeter of a rectangle related? Everyone knows the formulas:

Area = Length * Width
Perimeter = 2(Length + Width)

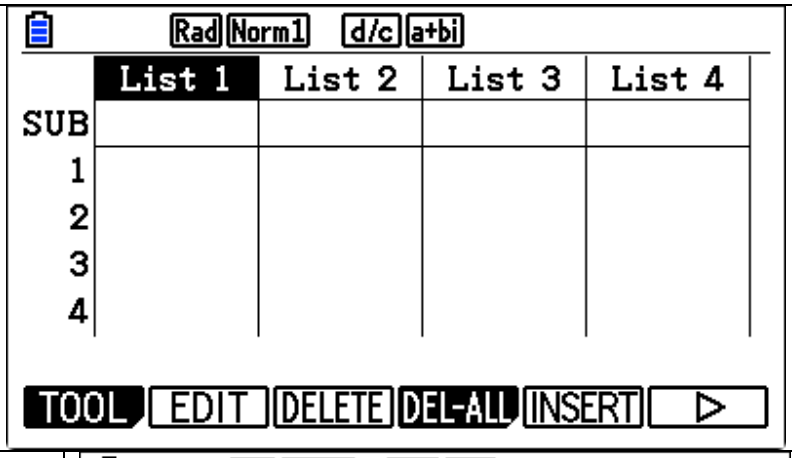
But if you look at the data for many different rectangles, what patterns would you notice? Are there any relationships between Area and Perimeter?

Random Rectangles – Area versus Perimeter

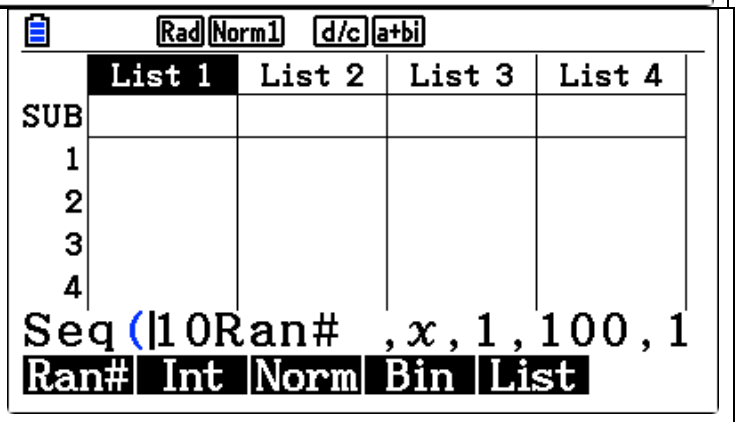
Using the Casio PRIZM

From the Main Menu (MENU), enter the Statistics mode (2)	
Set the Stat Window to Auto: Shift – Setup – Stat Wind – F1 - EXE	

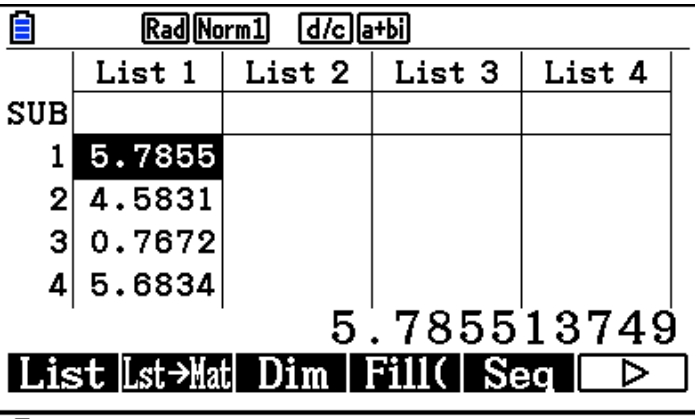
Go to the top of List 1 and press DEL-All (F4). Repeat until all lists are cleared.



While at the top of List 1, fill in random data, press: OPTN -List (F1)-Seq(F5)-10- OPTN -PROB(F5)-RAND(F4)-RAND#(F1)- , - X,θ,T - , -1- , -100- , -1-) - EXE

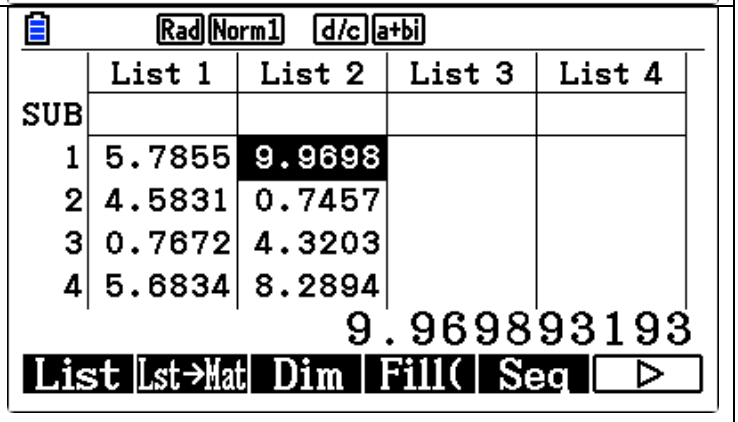


After pressing EXE the screen should look like this:



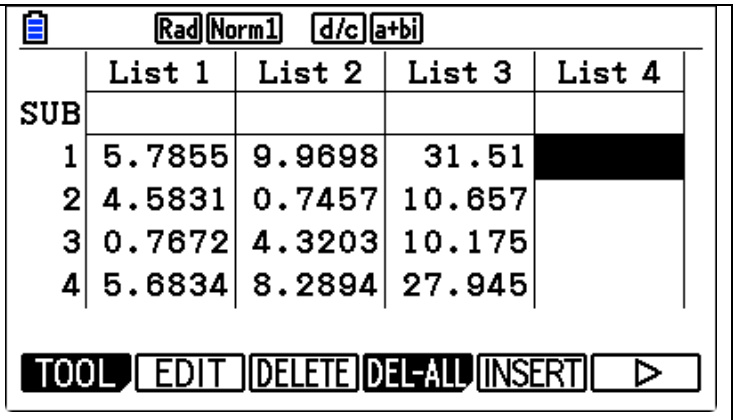
Repeat this process for List 2:

OPTN -List (F1)-Seq(F5)-10- EXIT -PROB(F5)-RAND(F4)-RAND#(F1)- , - X,θ,T - , -1- , -100- , -1-) - EXE



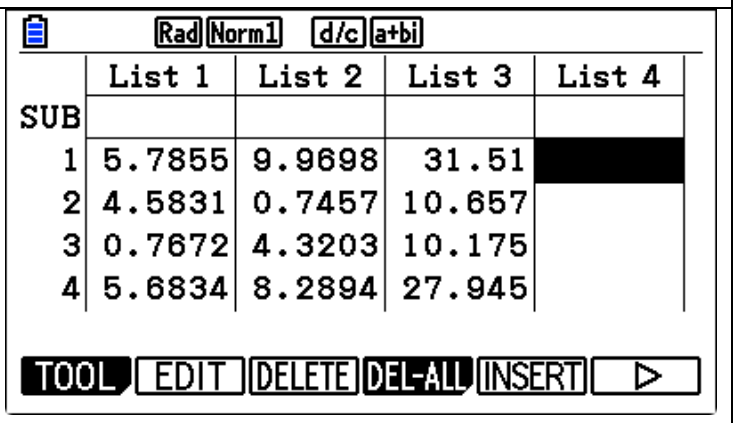
Next, scroll to the top of List 3 multiply 2(List 1 ⊞ List 2) to generate a list of Perimeter.

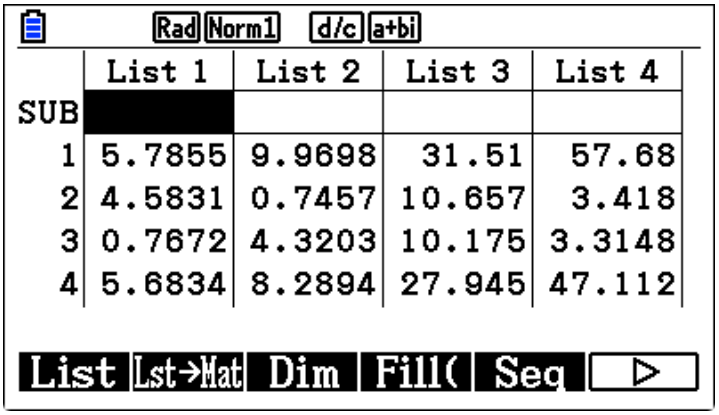
2 (OPTN F1 F1 1 ⊞ F1 2) EXE

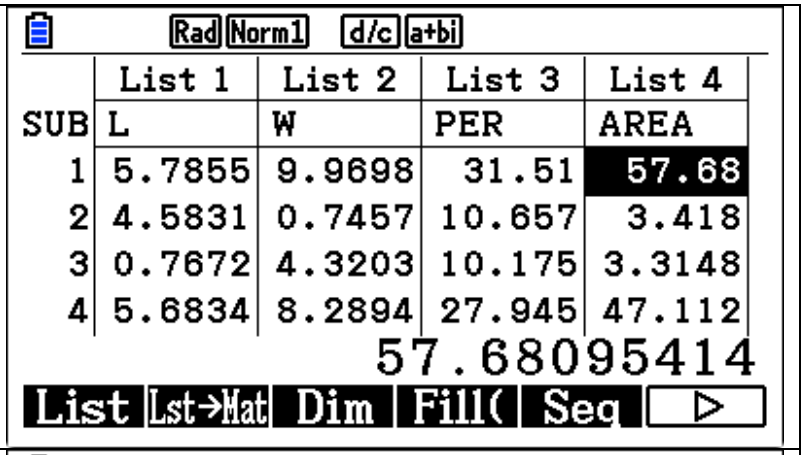
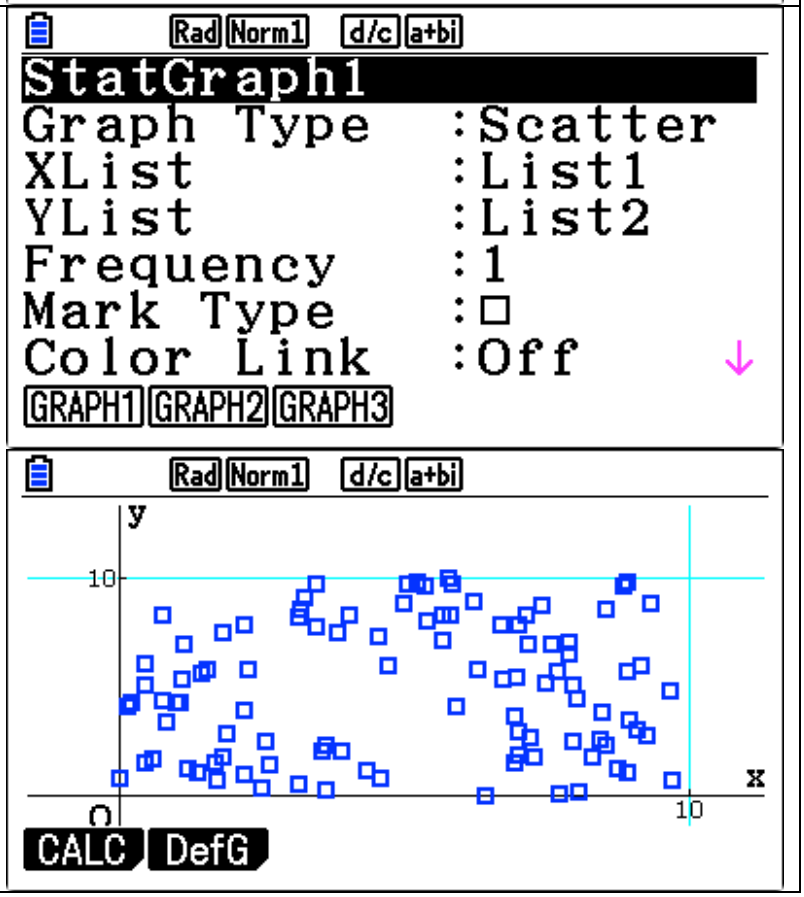


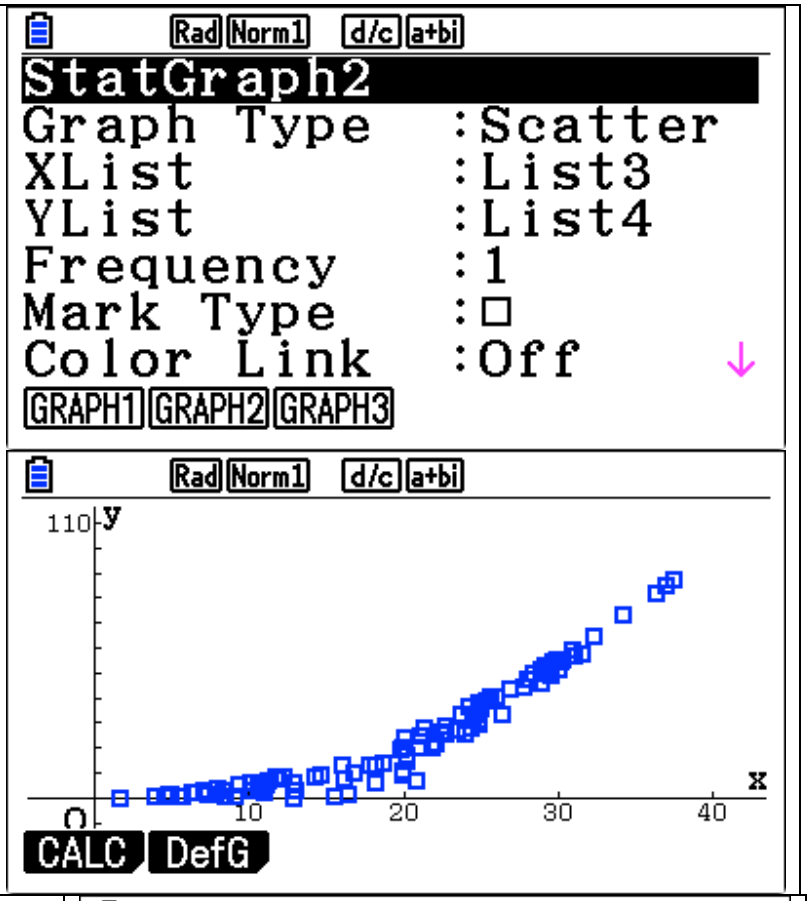
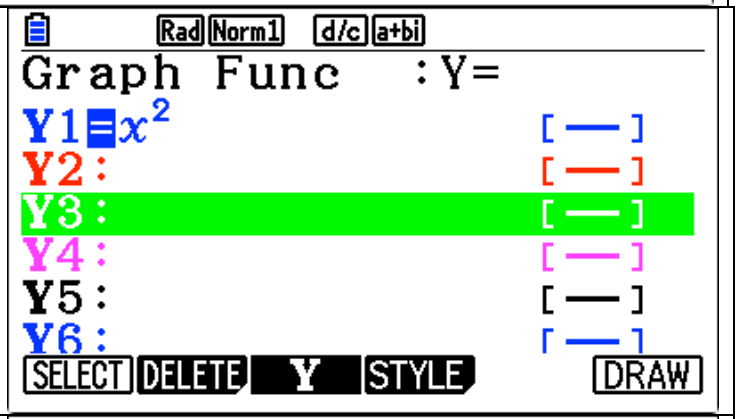
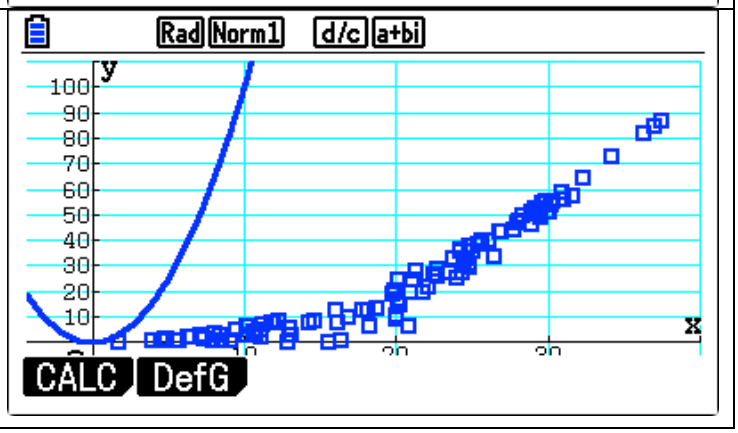
Scroll to the top of List 4 multiply List 1 ⊠ List 2 to generate a list of Area.

OPTN F1 F1 1 ⊠ OPTN F1 F1 2 EXE

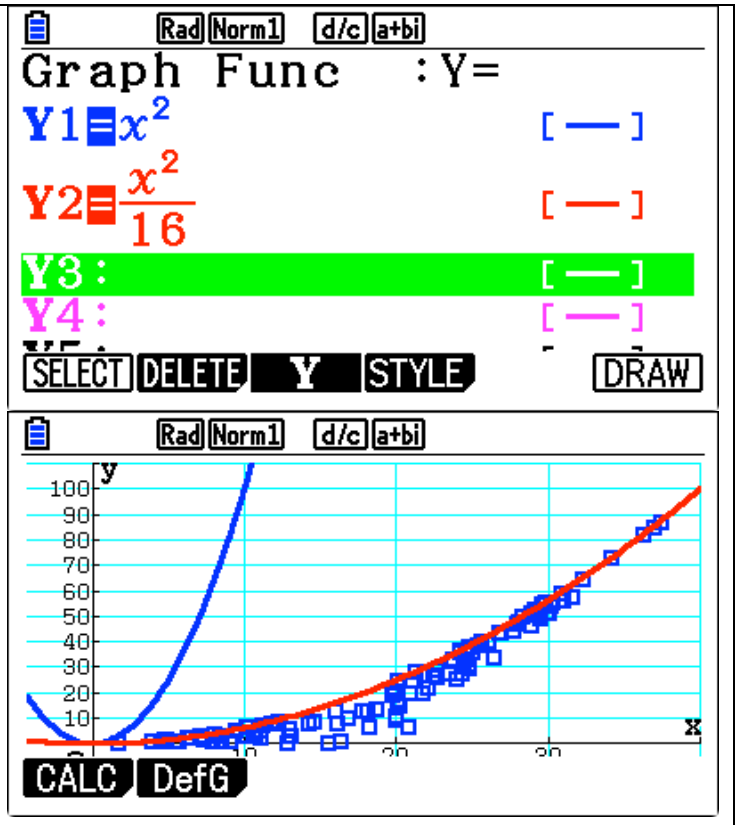




To designate what the data in each column means, type in the row labeled SUB; Use the Alpha key: L, W, PER, AREA in L1, L2, L3, L4.	
To demonstrate that the data is truly random, plot List 1 versus List 2. Exit-Graph(F1)-SET(F6)	

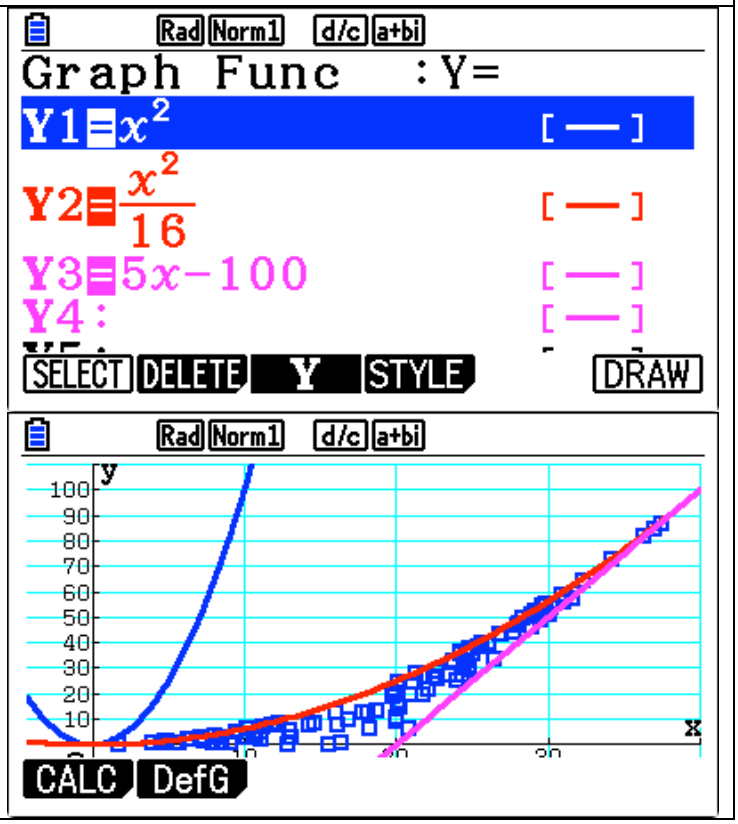
Plot Perimeter versus Area, i.e. List 3 versus List 4 using Graph 2.	
Let's examine the upper bound of this set of data. Notice that it is curved. Start with y = X^2. Press DefG(F2) to set the graph equation.	
Press DRAW(F6) to draw the function. Notice that the function needs to be lowered to approach the upper bound. To do so, the coefficient of X needs to be 0<A<1.	

The upper bound of the data set are squares. Therefore, the area of a square Area = Length^2. The perimeter is Perimeter = 4 x Length or Length = Perimeter/4. Substituting for Length gives Area = (Perimeter/4)^2 or Area = Perimeter^2/16. Plot that as Y2.

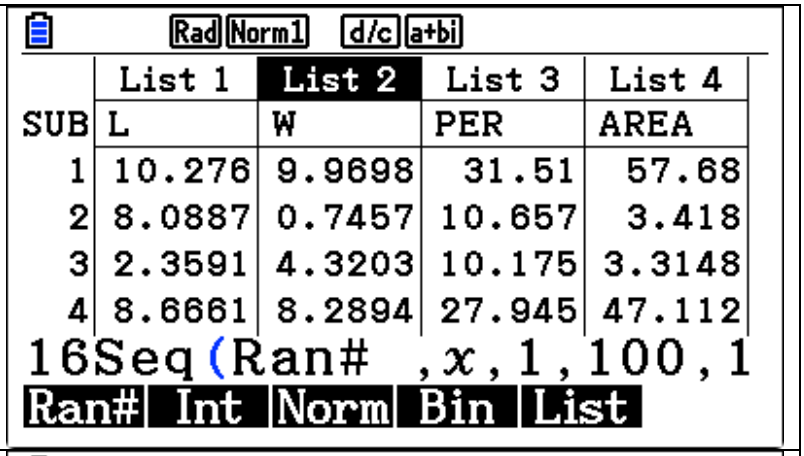
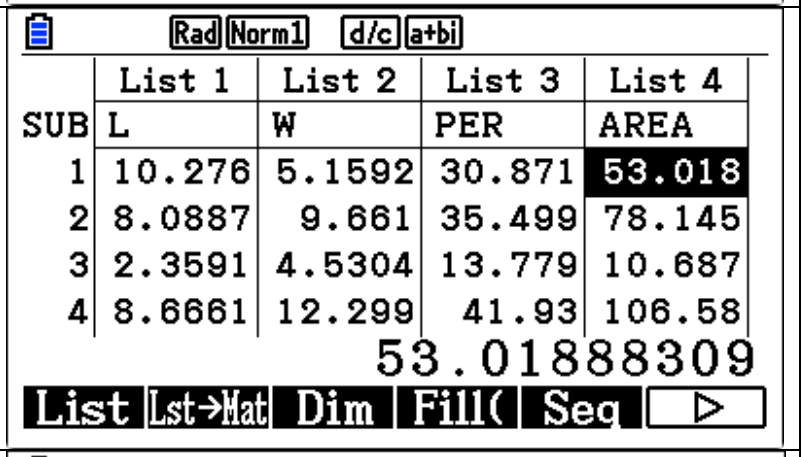
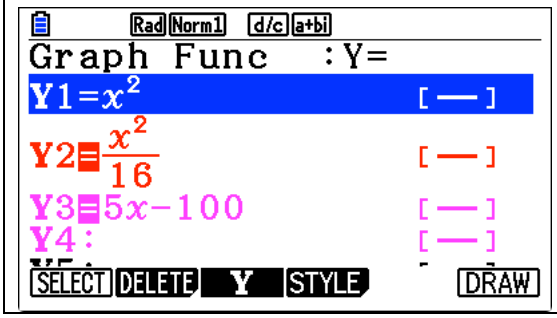
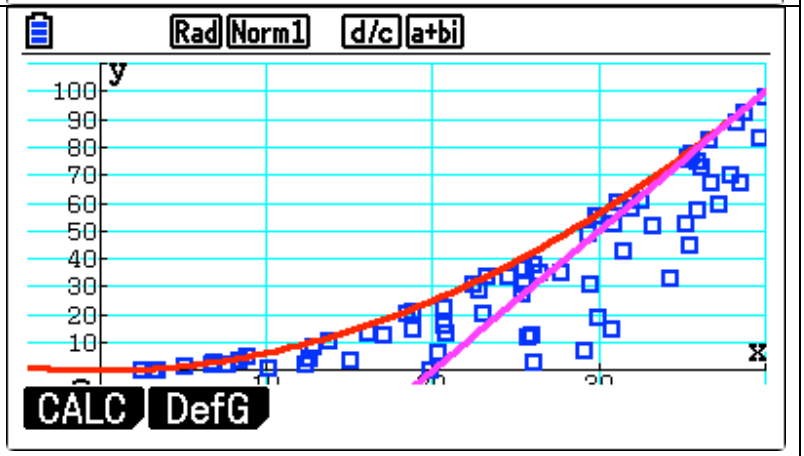


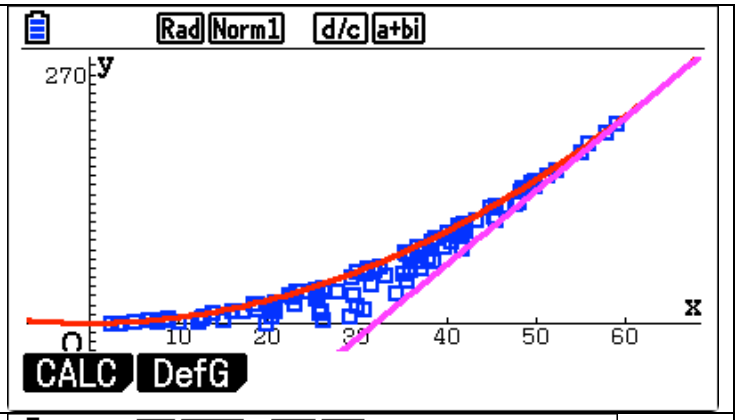
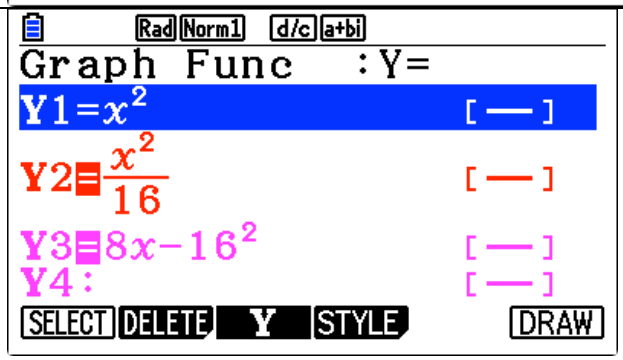
The empty region below the points exists because the maximum values of Length and Width limit the sizes of the rectangles. To minimize Area for a given Perimeter, two of the sides (say, Length) have to be as long as possible, 10 units each and 20 units combined, and the other two sides (say width) have to be as short as possible, or Width = (Perimeter – 20)/2.

Then the minimum area (lower bound) is Area = Length X Width = 10(Perimeter – 20)/2 = 5 x Perimeter – 100. Plot Y = 5x – 100 as the lower bound in Y3.



Extension: Let's consider what happens if we change the Length and Width. To keep things simple, we will allow for the same dimensional probability.

Change the Length and Width parameters to a max of 16. How might this affect our upper and lower bound functions?	
We must also redo the Perimeter and Area columns.	
Replot the data and the current function plots. Notice that the upper bound function in Y2 still fits, but the lower bound does not. Why? What should the lower bound function be? 	

Remember that the upperbound function is based on algebra that works for all such "square" data. Area = Perimeter^2/16 The lower bound depends on the data itself.	
The minimum area (lower bound) is Area = Length X Width = 16(Perimeter – 32)/2 =8(Perimeter – 32). Plot Y = 8x – 256 as the lower bound in Y3.	

Fathom Dynamic Data Software (Free)

In January, 2014, McGraw-Hill Education (MHE) ceased publishing Fathom Software. In March, 2014, MHE transferred the Fathom IP to William Finzer, Fathom's lead developer. This web site is his interim method of Fathom distribution.

What Doesn't Work in This Version

- Fathom Surveys—Because MHE is no longer maintaining the Fathom Surveys server, you will not be able to create surveys.
- Internet Downloads on Mavericks—A change in the way Mac OSX handles Internet access breaks Fathom's ability to download data from URLs and also census microdata.

Download Free, Expiring Fathom Software for Macintosh OS X and Windows

The copy you download is set to expire June 15, 2015. Be advised that we are not able to provide technical support for Fathom at this time.

http://concord.org/fathom-dynamic-data-software

Projectile Examples:

Bullet: If you fire a projectile horizontally in a vacuum and drop an object at the same time from the same height as the projectile, they will collide with each other every time, regardless of the initial velocity of the projectile. This can be demonstrated parametrically.

Xt1 = 700T Xt2 = 300
Yt1 = -4.9T^2 Yt2= -4.9t^2

Xmin : 0	Ymin : -2	Tmin : 0	Radian Mode
Xmax : 310	Ymax : 2	Tmax : .6	
Scale : 100	Scale : .5	Tpitch : .001	

Ball Toss: This example will model a ball tossed in the air vertically and also plot the associated height versus time graph. **(Format: Shift 5 to change to dot)**

Xt1 = T Xt2 = 14
Yt1 = -4.9T^2 + 50T Yt2= -4.9t^2 + 50T

Xmin : -3	Ymin : -100	Tmin : 0
Xmax : 16	Ymax : 200	Tmax : 12
Scale : 1	Scale : 50	Tpitch : .05

Moving Car: Suppose you have a car moving horizontally at a constant rate and a ball is tossed vertically (ignore air resistance). Then the ball will land in the car every time. Plot only the first and third graphs.

Xt1 = 3T
Yt1 = 2

Xt2 = 3T
Yt2 = -4.9(T-.2)^2 + 10(T-.2) + 2

Xt3 = 3T
Yt3 = Yt2/(Yt2≥2)

Xmin : 0	Ymin : 0	Tmin : 0
Xmax : 10	Ymax : 12	Tmax : 14
Scale : 1	Scale : 1	Tpitch : .005