

Common Core State Standards

3.OA.B.5 Apply properties of operations as strategies to multiply and divide. *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

Objective

Apply the distributive property as a strategy to multiply.

How does the arrangement of clothespins depict the distributive property?

Answer: $2 \times (5 + 5 + 5) = 10 + 10 + 10$.

Distributive Property

Students have worked with multiplication concepts for some time now and have learned some strategies, such as using the commutative and associative properties. Another useful strategy is to apply the distributive property. This strategy draws on the student's experience with array models and decomposing numbers, and it will be useful to students as they develop their mental math skills and prepare for algebra.

Vocabulary

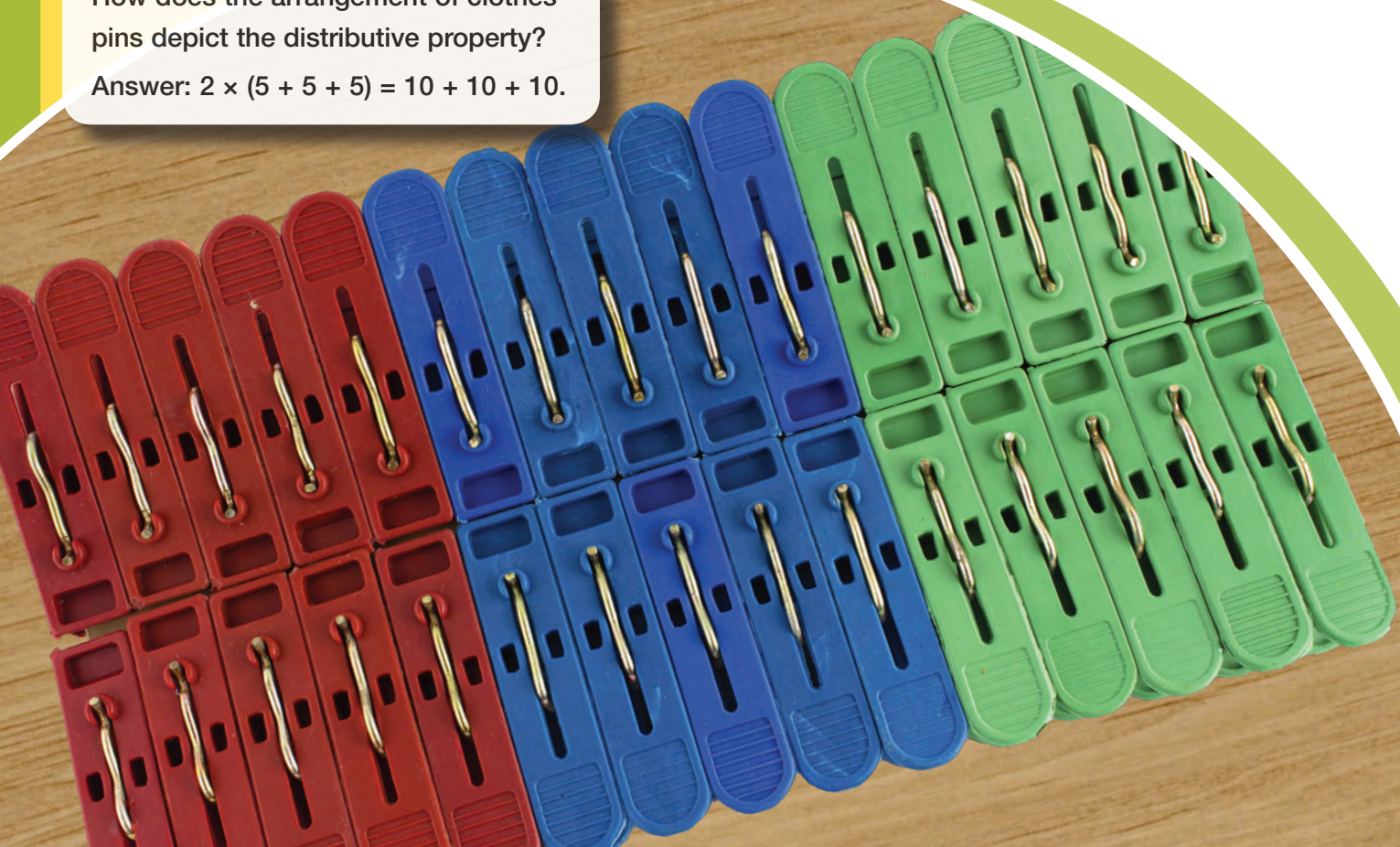
■ **Ask:** *If I distribute a pencil and 2 pieces of paper to 5 of you, how many pencils do I need? How many pieces of paper?*

Demonstrate by giving a pencil and 2 pieces of paper to each of 5 students, one student at a time.

■ **Ask:** *How many pencils did I distribute? [5] How many pieces of paper? [10]*

Elicit that $5 \times (1 \text{ pencil} + 2 \text{ pieces of paper}) = ?$ describes the situation and that the result of the distribution is $(5 \times 1 \text{ pencil}) + (5 \times 2 \text{ pieces of paper}) = 5 \text{ pencils} + 10 \text{ pieces of paper}$. Tell students this demonstrates the **distributive property**.

■ $5 \times (1 + 2) = (5 \times 1) + (5 \times 2) = 5 + 10$ is an illustration of the **distributive property**.



Set the Stage



Engage WHOLE CLASS

Distribute Color Tiles (at least 20 per student).

- **Ask:** *What if you had 2 bags with 3 tiles in each bag? How would this represent multiplication? [2 groups of 3] How could you show the same multiplication as an array? [2 rows of 3]*

Have students build the array of tiles, all the same color, and write the multiplication. [$2 \times 3 = 6$]

Next have students add a tile of a different color to each row.

- **Ask:** *If you look only at the new tiles, what multiplication do you see? [$2 \times 1 = 2$] If you look at all the tiles, what multiplication do you see? [$2 \times 4 = 8$, or $2 \times (3 + 1) = 8$] What other number sentences can you write to describe the model? [$6 + 2 = 8$, or $(2 \times 3) + (2 \times 1) = 8$]*



Warm-Up

Use this short thinking exercise to jump-start the instructional session.

Name Answer Key

3
Complete each addition table.

a.	$\begin{array}{r} + \square \square \\ 3 \mid 11 \ 13 \\ 4 \mid \square \square \end{array}$
----	--

b.	$\begin{array}{r} + 2 \square \\ \square \mid \square \ 5 \\ 5 \mid 7 \ 9 \end{array}$
----	--

c.	$\begin{array}{r} + 3 \ 9 \\ \square \mid 5 \ \square \\ \square \mid 8 \ \square \end{array}$
----	--

ANSWER:

a.	$\begin{array}{r} + 8 \ 10 \\ 3 \mid 11 \ 13 \\ 4 \mid 12 \ 14 \end{array}$
----	---

b.	$\begin{array}{r} + 2 \ 4 \\ 1 \mid 3 \ 5 \\ 5 \mid 7 \ 9 \end{array}$
----	--

c.	$\begin{array}{r} + 3 \ 9 \\ 2 \mid 5 \ 11 \\ 5 \mid 8 \ 14 \end{array}$
----	--

COMMENTS & EXTENSIONS: Make up more grid problems like this for students to complete. Start with a completed grid (no blanks) and blank out some of the numbers. Be sure to solve the grid problems yourself.



Foundation Skill Practice

Use this VersaTiles® activity to help students activate their prior knowledge.

What's the Total?

Find the addition or multiplication sentence that describes the total.

1	2	3
4	5	6
7	8	9
10	11	12

Answer Box

A $2 + 4 + 3 = 9$	B $5 \times 3 = 15$	C $2 + 2 + 2 = 6$	D $4 + 2 + 4 = 10$	E $6 + 6 = 12$	F $3 \times 4 = 12$
G $3 \times 3 = 9$	H $4 \times 2 = 8$	I $2 + 3 + 2 = 7$	J $2 \times 2 = 4$	K $5 + 5 = 10$	L $4 + 5 + 4 = 13$

22 Objective: Name an addition or multiplication sentence that is used to find a total.



Introduce the Concept

Explore WHOLE CLASS

Present the problem—

Jaylon's teacher made 4 rows of 7 chairs in the back of the room. How many chairs are there in all?

- **Ask:** *What do you need to do to solve this problem? [multiply] How do you know? [It asks how many there are in 4 groups of 7.] What multiplication sentence do you need to solve? [$4 \times 7 = ?$]*

Ask students to recall what they did during the Engage session and think about how they might build up an array of tiles to model the problem.

- **Ask:** *Do you know what 4×1 is? 4×2 ? 4×3 ?*

Have students explore, using the color of the tiles to help them see how the problem can be turned into a combination of simpler problems.

- **Ask:** *If you know $4 \times 5 = 20$ and $4 \times 2 = 8$, how can you use this to solve the problem?*

Acknowledge that together, (4×5) and (4×2) make $4 \times (5 + 2)$, or 4×7 . So 4×7 is $20 + 8$, or 28.

Have students draw their models and share their findings with partners.

Materials

- Color Tiles



Explore & Explain

LESSON

3

Distributive Property

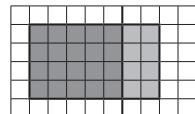
Name Answer Key

Try This

Andrea has 4 boxes of pencils. Each box contains 7 pencils. How many pencils are there in all? Show how to make an easier problem.

- Use Color Tiles to build an array for the problem.
- Draw your array on the grid.
- Show the parts by drawing a line.
- Complete the number sentences.

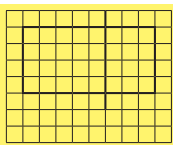
$$4 \times 7 = 4 \times (5 + 2) = 28$$



$$(4 \times 5) + (4 \times 2) = 20 + 8$$

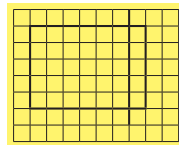
Answers will vary. Sample answers are given.

1. $4 \times 8 = 4 \times (5 + 3) = 32$



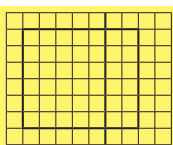
$$(4 \times 5) + (4 \times 3) = 20 + 12$$

2. $5 \times 7 = 5 \times (6 + 1) = 35$



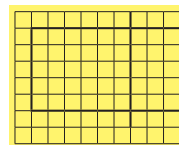
$$(5 \times 6) + (5 \times 1) = 30 + 5$$

3. $6 \times 7 = 6 \times (5 + 2) = 42$



$$(6 \times 5) + (6 \times 2) = 30 + 12$$

4. $5 \times 9 = 5 \times (6 + 3) = 45$



$$(5 \times 6) + (5 \times 3) = 30 + 15$$

Properties and Problem Solving ■ Lesson 3

continued on the next page
Hands-On Standards® Number & Operations

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OOO

Online resource available at hand2mind.com/hosnumber3

Reinforce the Concept

Explain & Elaborate WHOLE CLASS

Ask students to list some simple multiplication facts that they know from memory.

- **Say:** *If you don't know it really fast, then don't list it.*
- **Say:** *Now use the easy facts to find some facts you don't know well. Use the distributive property. Show your work using drawings and equations.*
[Example: $6 \times 9 = (6 \times 5) + (6 \times 4) = 30 + 24 = 54$]

Evaluate WHOLE CLASS

- **Ask:** *When you break apart a factor, can you break it into more than two parts? [Yes]*
- **Say:** *Show how you might do this to find 4×9 .*

Have students use drawings and/or equations to show their work. They can decompose either factor into as many addends as they like so long as the sum of the addends is equal to the factor and the product is 36.



Independent Practice

Use this VersaTiles® activity to give students more practice with the skills they learned in the lesson.

Distributive Property

Example

Complete the number sentence.

$2 \times (2 + 3) = \square + (2 \times 3)$
 $2 \times (2 + 3)$ is modeled as an array with 2 rows and $(2 + 3)$ columns. The shaded part models 2×2 . The unshaded part models 2×3 . The array is the sum of the shaded and unshaded parts. So the completed number sentence is $2 \times (2 + 3) = (2 \times 2) + (2 \times 3)$



Complete the number sentence.

- | | |
|---|--|
| 1 $4 \times (5 + 3) = (4 \times 5) + (\square)$ | 2 $3 \times (3 + 2) = (\square) + (3 \times 2)$ |
| 3 $4 \times (4 + 3) = (\square) + (4 \times 3)$ | 4 $6 \times (5 + 3) = (6 \times 5) + (\square)$ |
| 5 $6 \times (\square) = (6 \times 5) + (6 \times 3)$ | 6 $5 \times (\square) = (5 \times 6) + (5 \times 3)$ |
| 7 $5 \times (4 + 3) = (\square) + (5 \times 3)$ | 8 $5 \times (6 + 3) = (\square) + (5 \times 3)$ |
| 9 $3 \times (\square) = (3 \times 3) + (3 \times 2)$ | 10 $4 \times (5 + 2) = (4 \times 5) + (\square)$ |
| 11 $6 \times (\square) = (6 \times 5) + (6 \times 4)$ | 12 $3 \times (2 + 3) = (\square) + (3 \times 3)$ |

Answer Box

A	B	C	D	E	F
6 + 3	3 × 3	5 × 4	4 × 2	4 × 4	3 + 2
G	H	I	J	K	L
3 × 2	5 × 6	6 × 3	4 × 3	5 + 3	5 + 4



Objective: Apply the Distributive Property. 23



Re-Engage

Use this page to give students additional concrete-to-representational-to-abstract practice.

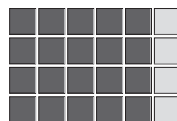
LESSON 3

Distributive Property

Name Answer Key

Use Color Tiles to build the model. Write the missing numbers.

1. You had 4 rows of 5 chairs. Then you added a chair to each row. How many chairs do you have now?
 $4 \times (5 + 1) = ?$



$$(4 \times 5) + (4 \times 1) = \frac{20}{= 24} + \frac{4}{= 24}$$

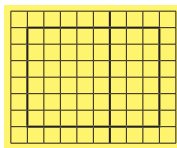
2. You had 3 groups of 3 books. Then you added 2 books to each group. How many books do you have now?
 $3 \times (3 + 2) = ?$



$$\left(\frac{3 \times 3}{= 9} \right) + \left(\frac{3 \times 2}{= 6} \right) = \frac{15}{= 15}$$

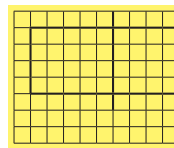
Use Color Tiles to model the problem. Draw your model on the grid. Write the missing numbers. [Answers may vary.]

3. $6 \times 8 = 6 \times (5 + \square)$



$$\left(\frac{6 \times 5}{= 30} \right) + \left(\frac{6 \times 3}{= 18} \right) = \frac{48}{= 48}$$

4. $4 \times 9 = 4 \times (\square + \square)$



$$\left(\frac{4 \times 5}{= 20} \right) + \left(\frac{4 \times 4}{= 16} \right) = \frac{36}{= 36}$$

Complete the number sentence.

- | | |
|---|---|
| 5. $3 \times (5 + 1) = (\square) + (3 \times 1)$ | 6. $8 \times (5 + 3) = (\square) + (\square)$ |
| 7. $\square \times (4 + 1) = (5 \times 4) + (5 \times 1)$ | 8. $\square \times (5 + 4) = (6 \times 5) + (6 \times 4)$ |

Online resource available at hand2mind.com/hosnumbergr3



Daily Routine

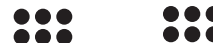
Desk Arrays

To show how a factor can be distributed, build an array by rearranging the student desks in your room before class.

For example, if you have 24 desks in your room, one side of the classroom might show 2×6 as:



The other side might show $(2 \times 3) + (2 \times 3)$ as:



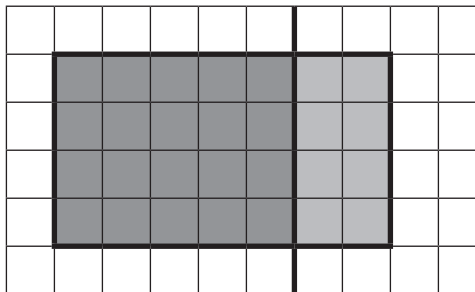
Discuss with students how both models represent the same number of desks. Have students think of other arrays that model 24.

Try This

Andrea has 4 boxes of pencils. Each box contains 7 pencils. How many pencils are there in all? Show how to make an easier problem.

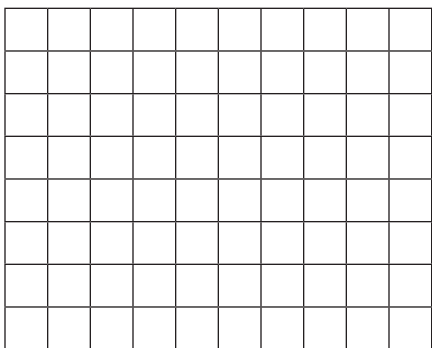
- Use Color Tiles to build an array for the problem.
- Draw your array on the grid.
- Show the parts by drawing a line.
- Complete the number sentences.

$$4 \times 7 = 4 \times (5 + 2) = 28$$



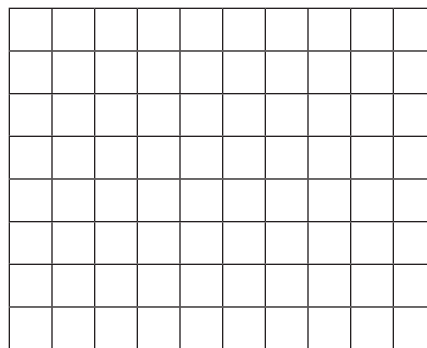
$$(4 \times 5) + (4 \times 2) = 20 + 8$$

1. $4 \times 8 = 4 \times (\underline{\quad} + \underline{\quad}) = \underline{\quad}$



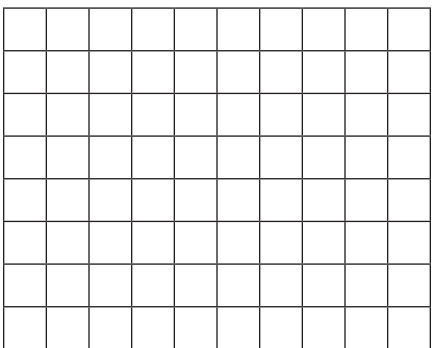
$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad}) = \underline{\quad} + \underline{\quad}$

2. $5 \times 7 = 5 \times (\underline{\quad} + \underline{\quad}) = \underline{\quad}$



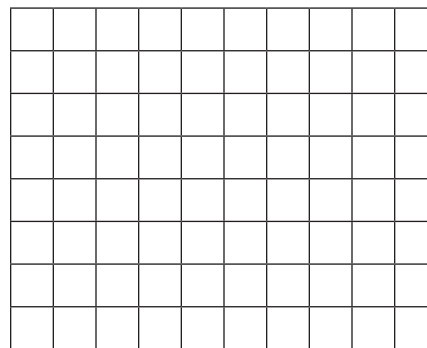
$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad}) = \underline{\quad} + \underline{\quad}$

3. $6 \times 7 = 6 \times (\underline{\quad} + \underline{\quad}) = \underline{\quad}$



$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad}) = \underline{\quad} + \underline{\quad}$

4. $5 \times 9 = 5 \times (\underline{\quad} + \underline{\quad}) = \underline{\quad}$



$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad}) = \underline{\quad} + \underline{\quad}$

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Distributive Property

Example

Complete the number sentence.

$$2 \times (2 + 3) = \square + (2 \times 3)$$

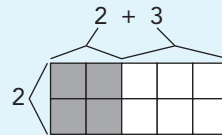
$2 \times (2 + 3)$ is modeled as an array with 2 rows and $(2 + 3)$ columns.

The shaded part models 2×2 .

The unshaded part models 2×3 .

The array is the sum of the shaded and unshaded parts.

So the completed number sentence is $2 \times (2 + 3) = (2 \times 2) + (2 \times 3)$



Complete the number sentence.

1 $4 \times (5 + 3) = (4 \times 5) + (\square)$

2 $3 \times (3 + 2) = (\square) + (3 \times 2)$

3 $4 \times (4 + 3) = (\square) + (4 \times 3)$

4 $6 \times (5 + 3) = (6 \times 5) + (\square)$

5 $6 \times (\square) = (6 \times 5) + (6 \times 3)$

6 $5 \times (\square) = (5 \times 6) + (5 \times 3)$

7 $5 \times (4 + 3) = (\square) + (5 \times 3)$

8 $5 \times (6 + 3) = (\square) + (5 \times 3)$

9 $3 \times (\square) = (3 \times 3) + (3 \times 2)$

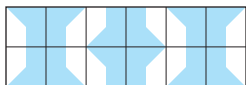
10 $4 \times (5 + 2) = (4 \times 5) + (\square)$

11 $6 \times (\square) = (6 \times 5) + (6 \times 4)$

12 $3 \times (2 + 3) = (\square) + (3 \times 3)$

Answer Box

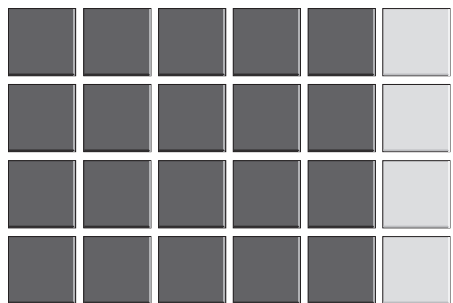
A $6 + 3$	B 3×3	C 5×4	D 4×2	E 4×4	F $3 + 2$
G 3×2	H 5×6	I 6×3	J 4×3	K $5 + 3$	L $5 + 4$



Use Color Tiles to build the model. Write the missing numbers.

1. You had 4 rows of 5 chairs. Then you added a chair to each row. How many chairs do you have now?

$$4 \times (5 + 1) = ?$$

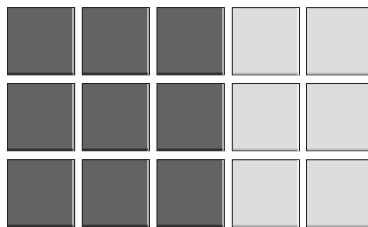


$$(4 \times 5) + (4 \times 1) = \underline{\quad} + \underline{\quad}$$

$$= \underline{\quad}$$

2. You had 3 groups of 3 books. Then you added 2 books to each group. How many books do you have now?

$$3 \times (3 + 2) = ?$$

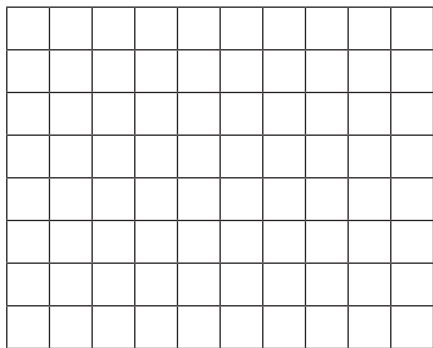


$$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$$

$$= \underline{\quad} + \underline{\quad} = \underline{\quad}$$

Use Color Tiles to model the problem. Draw your model on the grid. Write the missing numbers.

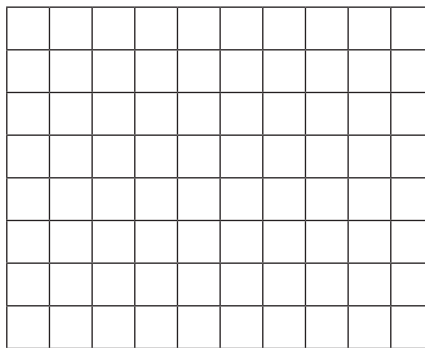
3. $6 \times 8 = 6 \times (5 + \underline{\quad})$



$$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$$

$$= \underline{\quad} + \underline{\quad} = \underline{\quad}$$

4. $4 \times 9 = 4 \times (\underline{\quad} + \underline{\quad})$



$$(\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$$

$$= \underline{\quad} + \underline{\quad} = \underline{\quad}$$

Complete the number sentence.

5. $3 \times (5 + 1)$
 $= (\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$

6. $8 \times (5 + 3)$
 $= (\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$

7. $\underline{\quad} \times (4 + 1) = (5 \times 4) + (5 \times 1)$

8. $\underline{\quad} \times (5 + 4) = (6 \times 5) + (6 \times 4)$

Complete the number sentence.

5. $6 \times (5 + 4) = (\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$

6. $8 \times (5 + 1) = (\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad})$

7. $\underline{\quad} \times (5 + 5) = (5 \times 5) + (5 \times 5)$

8. $\underline{\quad} \times (5 + 3) = (6 \times 5) + (6 \times 3)$

Solve the problem.

9. Roberta solved 8 math problems every day for 7 days. How many problems did she solve in all?

$$\begin{aligned}
 7 \times 8 &= 7 \times (\underline{\quad} + \underline{\quad}) \\
 &= (\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad}) \\
 &= \underline{\quad} + \underline{\quad} \\
 &= \underline{\quad}
 \end{aligned}$$

10. The library has 4 rows of tables. There are 9 tables in each row. How many tables are there?

$$\begin{aligned}
 4 \times 9 &= 4 \times (\underline{\quad} + \underline{\quad}) \\
 &= (\underline{\quad} \times \underline{\quad}) + (\underline{\quad} \times \underline{\quad}) \\
 &= \underline{\quad} + \underline{\quad} \\
 &= \underline{\quad}
 \end{aligned}$$