CARE

Conceptual Algebra Readiness for Everyone

Conceptual Algebra Readiness for Everyone, CARE is a curriculum development project funded through a grant from the Indiana Mathematics and Science Partnership to the Michigan City Area Schools in collaboration with Purdue University North Central for grades 3-7. The goal of the project is to help children develop conceptual algebra readiness through weekly classroom activities and by providing extensive professional development for teachers.

Algebra Readiness

Conceptual algebra readiness is sometimes referred to as ‘early algebra’ or ‘algebraic reasoning’. One of the key points we want to make is that conceptual algebra readiness is not (formal) algebra early (Carraher, Schliemann & Schwartz, 2008). Our intent is not to teach children how to solve algebraic equations with x’s and y’s; rather, our intent is to help children understand the underlying concepts of algebra so that when they do solve equations in algebra; they will have a conceptual basis, stemming from their work with whole numbers, fractions, decimals, and percent, for interpreting and operating meaningfully on algebraic equations and symbols. In other words, conceptual algebra readiness lays a foundation for students to make sense of algebra rather than to manipulate symbols mindlessly!

Activities

The activities in this project are designed to be done once per week and take two forms: 1) **Problem Sets** and 2) **Whole-class Activities**. A blend of the two activities is recommended. A sequence of activities is provided but it can be adjusted to fit teachers’ schedules and students’ needs. It is essential that students be engaged with these activities continually, approximately once per week. Students need continued experiences to develop conceptual algebra readiness.

Problem Sets typically consist of 10-15 problems in 4^{th} and 5^{th} grade and 5 problems in 6^{th} and 7^{th} grade for students to solve. These problems are a mixture of problem types and topics. Research has shown a mixed review approach to be an effective way to help children learn mathematics. This approach provides students with multiple experiences with the same concepts.

Whole-class Activities vary in form. Some are game-like activities and others are one or two problems that the class may work on in groups and then discuss.

CARE is designed to challenge students at a variety of developmental levels. Our primary purpose is to help children develop conceptual understandings to prepare them for algebra.

Teacher Notes

The teacher notes in CARE are extensive. They include the sequence of activities, an explanation of algebra readiness, a general description of the activities, a discussion with specific suggestions for realizing class discussion and small-group work in the classroom, and a rationale for the activities with practical suggestions to help teachers make sense of the curriculum, so that students develop conceptual algebra readiness. The CARE curriculum guide provides specific notes and suggestions for <u>each problem</u> in the Problem Sets and for each Whole-class Activity

Professional Development

Teachers participating in this project receive two weeks of professional development and daylong follow-up workshops. In the workshops teachers experience the activities, learn about the theories guiding the project, model the classroom group work and whole-class discussion, and engage in rich mathematical thinking.

CCSS and NCTM Standards

CARE incorporates the NCTM National Standards (2000) and is aligned with the Common Core State Standards in Mathematics (2010). The appendices include a table where each problem and whole-class activity is aligned with the Common Core Stand Standards and *Everyday Math*. A second table aligns the CCSS with CARE.

Effectiveness

Preliminary results of a statistical analysis examining the effects of the CARE program on students' mathematical achievement indicate that the use of the program had a significant impact on student achievement (Pratt, Schroer, Feikes, submitted paper). Specifically, teachers' use of CARE was positively correlated with their students' mathematics ISTEP scores. Specifically, students showed significant score increases in the topics of Algebraic Functions and Measurement. These findings were also consistent for students from low SES backgrounds and students of color. The results suggest the program may be effective in decreasing the achievement gap in mathematics for at risk students.

Teachers' Perspective of CARE

> This year I watched my kids move on their own from arithmetic thinkers to algebraic thinkers. They make generalizations every time now and they want to use variables. They love using variables and the best part is they understand them, not everyone, but most do. They also want to justify everything now.

> The use of the CARE activities has dramatically increased my student's confidence in their problem solving abilities. Problems my students wouldn't have even attempted in the past are now met with enthusiasm. My students see the CARE activities as a challenge to take on, and not something that is over their heads. More importantly, I have seen my students start to believe in themselves and their ability to tackle complex challenges.

> CARE Math has made a world of difference in my classroom. I was able to see each child grow algebraically on a weekly basis and ultimately in their standardized tests. My students soared to all-time highs in all math standards, specifically algebra and functions. These children's ability to generalize, reason, and find an easier way to solve problems was nothing short of amazing!

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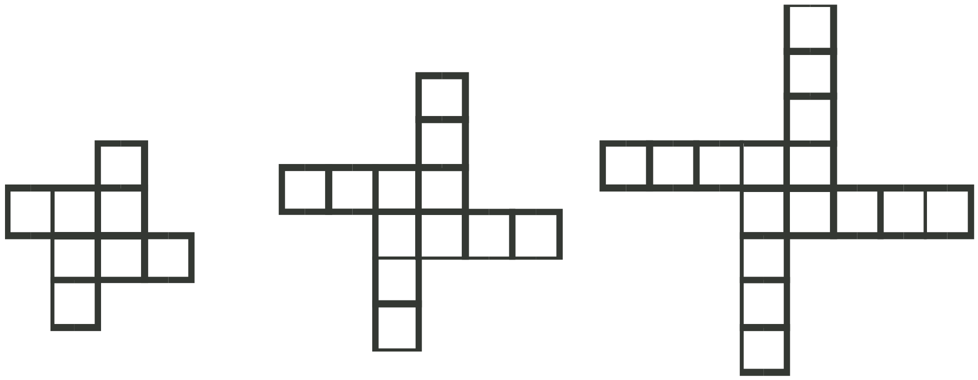
Problem Set B12

Solve each problem. On some problems you will be asked to explain your thinking! 1a. Draw the next shape in the pattern.



Without drawing the shapes how many squares will be in the:

1b. 10^{th} shape 1c. n^{th} shape

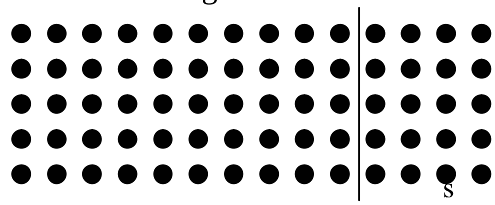
1d. Using the figure explain why the rule for the n^{th} shape works.

1e. Fill in the table.

Figure #	Number of Squares in the Shape
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
25	
100	
n	

Problem Set B12-2

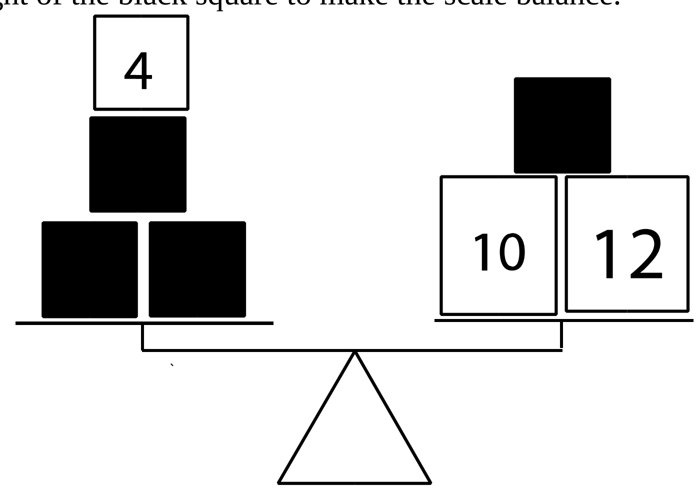
2. How many dots are in this figure?



Cindy multiplied 5 x 14 to find the number of dots in the picture.
Charles multiplied 5 x 10 and then added it to 5 x 4 to find the number of dots in the picture.

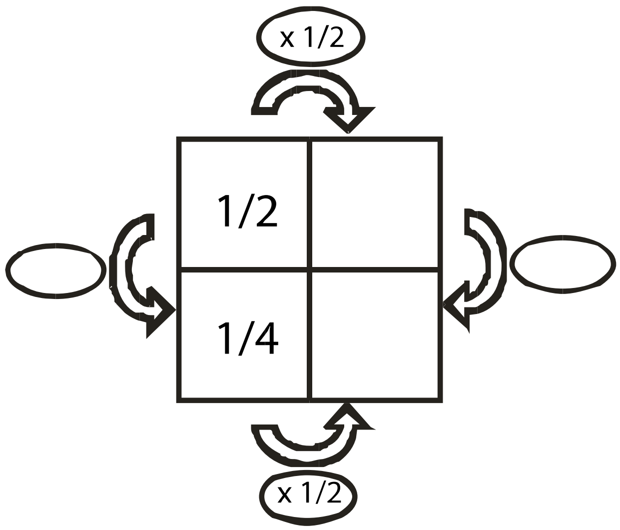
b. Why do both methods give the same answer?

3. Find the weight of the black square to make the scale balance.



Problem Set B12-3

4. Find the missing numbers and the operations in the ratio box.



5. A loaded trailer truck weighs 26,643 kilograms. When the trailer is empty it weighs 10,547 kilograms. About how much does the load weigh?

A. 14,000 kilograms
B. 16,000 kilograms
C. 18,000 kilograms
D. 36,000 kilograms

6. Fill in the missing numbers in the pattern and give the rule.

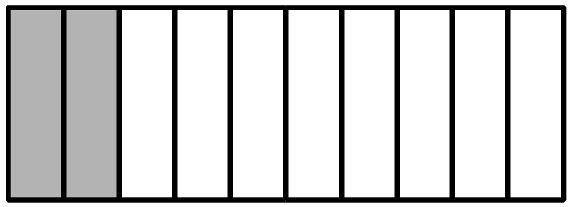
___ ___ 8 ___ ___ 26 ___ ___ ___

Rule ____________

7. I have exactly $10.00 in nickels, dimes, and quarters. If I have the same number nickels, dimes, and quarters, how many of each kind of coin do I have?

Problem Set B12-4

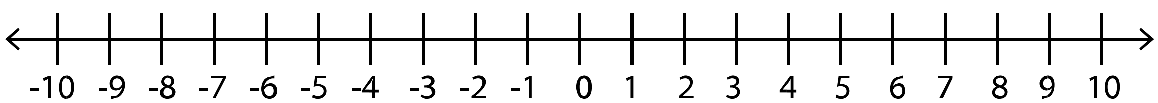
8. Which number represents the shaded part of the figure (TIMSS, 1995)?
   a. 2.8
   b. 0.5
   c. 0.2
   d. 0.02



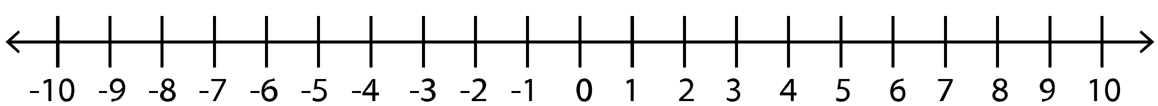
9. If you could not multiply the two numbers on each side of the equation to show they are equal, how could you convince someone that the following equation is equal? 20 x 16 = 40 x 8

10. List all the pairs of whole numbers that you can multiply together to get 72.

11. The Integer Express is a train that travels back and forth on the number line railroad. The engineer makes the train move to the right by entering a positive number and to the left by entering a negative number. If he started at 0 and pushed -8 and then +5, where would he end up?



b. The engineer sometimes forgets where he started. He knew he pushed -8 and +5. He ended up at -1. Where did he start?



CARE Notes

Problem Set B12

Overview: Here are some thoughts on introducing variables—letters to represent numbers. "*In early algebra formal notation is introduced only gradually*. Young student will not reinvent algebra on their own, and without a certain degree of guidance they are unlikely to express a need for a written notational for variables. Algebraic expressions need to be introduced, but judiciously, so to avoid *premature formalization* (Piaget, 1964)". These authors go on to say that in algebra, and mathematics, we often introduce unfamiliar notation and symbols to students, that they do not initially understand (Carraher, Schliemann, Schwartz, 2008, p. 236). However, we frequently must introduce new mathematical ideas with new, not quite fully understood, symbols and notations. Our intent is that many students will naturally begin to generalize the relationship in problems like #1. We cannot make or force them to understand 4n + 4! "Over time students will increasingly work conventional representations into their expressive repertoires—they will *make them their own*." (p. 237).

1. Purpose: Generalization, symbolic representation, using tables, and geometric patterning.
   Suggestions: The rule for this pattern is 4n + 4. Problem #1d asks students to explain how the algebraic rule, 4n + 4 is related to the figures. Our intent is that children will begin to connect the figures to the algebraic expressions.
2. Purpose: The Distributive Property
   Suggestions: Using dot arrays is one way of introducing and helping students understand the distributive property. There could be other way of distributing the array. For example, adding another line between the 5^{th} and 6^{th} column, the problem could be expressed as 5 x 14 = (5 x 5) + (5 x 5) + (5 x 4).
3. Purpose: Equality and Solving Equations.
   Suggestions: In this problem one black square cancels out similar to an equation like x + x + x + 4 = x + 10 + 12; subtracting x from both sides of the equation eliminates it just like one black square is eliminated without affecting the solution.
4. Purpose: Proportional Reasoning and Multiplication of Fractions.
   Suggestions: In this problem all the numbers and operations are fractional.
5. Purpose: Estimation and Number Sense.
   Suggestions: This problem was given on the 2009 NAEP Assessment and 53% of fourth grade students had a correct solution. This problem was also given to eighth graders and 84% had it correct.
6. Purpose: Number Sense.
   Suggestions: The process of taking the difference between two numbers that are three apart and dividing by 3, (26 – 8) ÷ 3 = 6 is a **generalization**. Students have generalized a process and this is algebraic reasoning.

Problem Set B12

7. Purpose: Problem Solving.
   Suggestions: A table or list is a good way to attempt this problem. Is the use of tables in generalization problems like #1 transferring to other problems like this one?
8. Purpose: Decimals.
   Suggestions: Internationally, 33% of third graders and 40% of fourth graders answered correctly on the 1995 TIMSS Assessment.
9. Purpose: Relational Understanding.
   Suggestions: When multiplying two numbers; doubling one and halving the other gives the same result. This is a generalization.
10. Purpose: Factors.
   Suggestions: Check to see if students have all the solutions. A good question to ask is: How do you know if you have all the solutions? Students should be confident they have them all, but how do they know?
11. Purpose: Integers
   Suggestions: We have expanded the number line from -10 to 10 on these problems.