# Reflection Challenges (long form)   Name:

In this activity you will reflect a point and then compare the motion of the point to the motion of its reflected image.

REFLECT A POINT

Begin by reflecting a point and describing how the reflection behaves.

1. In a new sketch, construct an independent point ⬚ and label it *x* Ⓐ.

2. Construct a vertical line ⬚ in the center of the sketch.
   To make it exactly vertical, hold the Shift key while constructing it.

3. Label Ⓐ the line *j*. (Don't label the points that determine the line.)

4. Drag the point ⬚ around in your sketch.

   Because you can drag this point anywhere in your sketch, and it doesn't depend on other objects, we say that point *x* is *independent.*

5. Mark line *j* as a mirror, by double-clicking it with the **Arrow** tool ⬚.

6. Select point *x* ⬚, and choose **Transform | Reflect.** Label the reflected point Ⓐ
   r[j](x). Because point $r_j(x)$ depends on point *x,* we say that $r_j(x)$ is *dependent.*

   You can read $r_j(x)$ as "the reflection of *x* across line *j.*"

7. Change the color of the dependent point $r_j(x)$ by selecting it ⬚ and choosing
   Display | Color.

Q1 Drag point *x* up. Which way does $r_j(x)$ go? Drag *x* left. Which way does $r_j(x)$ go?

8. Turn on tracing for points *x* and $r_j(x)$, by selecting them both ⬚ and choosing **Display | Trace Points.**
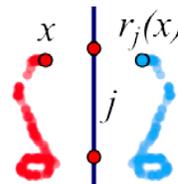
9. Click in empty space ⬚ to deselect both points. Then drag independent point *x* to trace out an interesting shape.

Q2 Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing showing your shapes.

10. Erase the traces by choosing **Display | Erase Traces,** and then trace a new shape. Make this shape go on both sides of the mirror.

Q3 What happens when you drag *x* across the mirror? Describe the traced shapes, and include a drawing on your paper.

Q4 Click the **Information** tool ⓘ on point *x,* and then on $r_j(x)$. How do the resulting balloons describe each point?
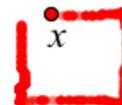
Q5 Experiment by dragging ⬚ your mirror points and your independent variable so that you can create traces that look like these. Explain how you did it, and draw your shapes and mirror on your paper.
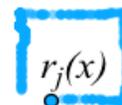
11. Adjust the mirror so it's neither horizontal nor vertical.

12. Erase the traces and drag x so that points x and $r_j(x)$ come together in the same location. (Recall that a location like this is called a *fixed point*.)

Q6 Drag x to make a shape that includes several fixed points. Draw the mirror and both shapes on your paper. Label the fixed points on your drawing.

Q7 How are these shapes different from the shapes you made when the mirror was vertical?

RESTRICT THE INDEPENDENT VARIABLE TO A POLYGON

Now you'll merge point x to a polygon and observe the effect on $r_j(x)$.

13. Construct an interesting polygon 🔷 with at least 5 vertices. To finish the polygon, click the first point again.

14. To hide the vertices of the polygon, select the vertices ➤ (but not the polygon) and choose **Display | Hide Points.**

15. Merge point x to the polygon by selecting both the point and the polygon and choosing **Edit | Merge Point to Polygon.**

16. Drag independent point x.

Q8 Describe the function's domain: where can you drag the independent variable?

A domain like this is called a *restricted domain.*
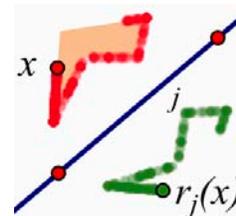
17. Select x, and choose **Edit | Action Button | Animation.** Then press OK.

18. Press the *Animate Point* button that appears.

Q9 While the points are being animated, describe their motion. Compare their speeds: which moves faster? Compare their directions: which way does $r_j(x)$ go when you drag x in various directions?

Q10 How does the range compare to the domain? What features are similar, and what features are different? How do these features relate to the relative motions?

REFLECTION CHALLENGES

Now you'll look at some reflections and find the location of the invisible mirror.

19. Open **Reflection Challenges.gsp** and try Challenges 1 through 6.

Q11 Describe in step-by-step detail how you can create a new reflection to exactly match any given reflection. How can you find the location of the invisible mirror? Describe any shortcuts you can invent to make it easy to find the location of the mirror.

Reflection Challenges (short form)   Name:

In this activity you will reflect a point and compare the motion of the point to the motion of its reflected image.

REFLECT A POINT

Begin by reflecting a point and describing how the reflection behaves.

1. In a new sketch, construct an independent point and label it *x*.

2. Construct a vertical line and label it *j*.

3. Reflect point *x* across the mirror, label the reflected point r[j](x), and change its color. This point is called the *dependent* variable because it depends on point *x*.

   You can read $r_j(x)$ as "the reflection of *x* across line *j*."

Q1 Drag point *x* up. Which way does $r_j(x)$ go? Drag *x* left. Which way does $r_j(x)$ go?

4. Turn on tracing for points *x* and $r_j(x)$. Deselect both points and then drag independent point *x* to trace out an interesting shape.

Q2 Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing showing your shapes.
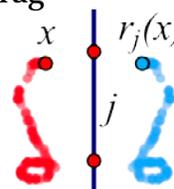
5. Erase the traces and trace a new shape that goes across the mirror.

Q3 What happens when you drag *x* across the mirror? Describe the traced shapes, and include a drawing on your paper.

Q4 Click the **Information** tool on point *x,* and then on $r_j(x)$. How do the resulting balloons describe each point?
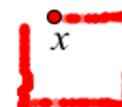
Q5 Drag your mirror points and your independent variable so that you can create traces that look like these. Explain how you did it, and draw your shapes and mirror on your paper.
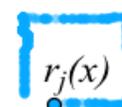
6. Adjust the mirror so it's neither horizontal nor vertical.

7. Erase the traces and drag *x* so that points *x* and $r_j(x)$ come together in the same location. (A location like this is called a *fixed point.*)
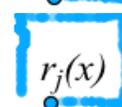
Q6 Continue dragging *x* to make a shape that includes several fixed points. Draw the mirror and both shapes on your paper. Label the fixed points on your drawing.

Q7   How are these shapes different from the shapes you made when the mirror was vertical?

RESTRICT THE INDEPENDENT VARIABLE TO A POLYGON

8. Construct an interesting polygon interior with at least 5 vertices. Hide the vertices, and merge independent variable *x* to the polygon.

Q8 Describe the function's *domain*: where can you drag the independent variable?

A domain like this is called a *restricted domain.*

9. Create an Animation action button for *x,* and press the button.

Q9 While the points are being animated, describe their motion. Compare their speeds: which moves faster? Compare their directions: which way does $r_j(x)$ go when you drag *x* in various directions?

Q10 How does the range compare to the domain? What features are similar, and what features are different? How do these features relate to the relative motions?

REFLECTION CHALLENGES

Now you'll look at some reflections and find the location of the invisible mirror.

10. Open **Reflection Challenges.gsp** and try Challenges 1 through 6.

Q11 Describe in step-by-step detail how you can create a new reflection to exactly match any given reflection. How can you find the location of the invisible mirror? Describe any shortcuts you can invent to make it easy to find the location of the mirror.

Reflection Challenges Answers   Name:

Q1   Drag point x up. Which way does $r_j(x)$ go? Drag x left. Which way does $r_j(x)$ go?

Q2   Describe the traced shapes. How are they similar, and how are they different? Consider position, size, angle, and anything else you think of. On your paper include a drawing showing your shapes.

Q3   What happens when you drag x across the mirror? Describe the traced shapes, and include a drawing on your paper.

Q4   Click the **Information** tool on point x, and then on $r_j(x)$. How do the resulting balloons describe each point?

Q5   Drag your mirror points and your independent variable to create traces that look like the picture. Explain how you did it, and draw your shapes and mirror on your paper.

Q6   Drag x to make a shape that includes several fixed points. Draw the mirror and both shapes on your paper. Label the fixed points on your drawing.

Q7   How are these shapes different from the shapes you made when the mirror was vertical?

Q8   Describe the function's *domain*: where can you drag the independent variable?

Q9   While the points are being animated, describe their motion. Compare their speeds: which moves faster? Compare their directions: which way does $r_j(x)$ go when you drag x in various directions?

Q10   How does the range compare to the domain? What features are similar, and what features are different?

Q11   Describe in step-by-step detail how you can create a new reflection to exactly match any given reflection. How can you find the location of the invisible mirror? Describe any shortcuts you can invent to make it easy to find the location of the mirror.

Reflection Challenges Exit Ticket   Name:

1. Describe one important thing you learned today about the reflection function family.

2. Describe one thing about the reflection function family that seems confusing to you.