Two Sides of a Coin: Transformations, Functions, and the Common Core

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Common Core Standard G-CO2 says that students should "describe transformations as functions that take points in the plane as inputs and give other points as outputs." This statement only hints at the potential impact that this connection has for students' understanding of functions.

Recent findings in brain science and in cognitive science—the science that studies how we think, that analyzes the nature of our knowledge and understanding—support the age-old wisdom that we learn better by doing than by being told. Our bodies are not separate from our brains, and we learn best by exploiting that connection.

For functions, this means that students should experience, in as physical a way as possible, how variables vary, the shape and size of a domain and a range, the actual motion that underlies relative rate of change. These experiences enable students to form *conceptual metaphors* on which to base their abstract mathematical ideas:

> The mechanism by which the abstract is comprehended in terms of the concrete is called *conceptual metaphor*. Mathematical thought ... makes use of conceptual metaphor, as when we conceptualize numbers as points on a line. [Lakoff & Nuñez, *Where Mathematics Comes From,* p. 5]

Hans Freudenthal expressed the importance of learning based on our sensory-motor systems this way:

> Geometry is one of the best opportunities that exists to learn how to mathematize reality….[N]umbers are also a realm open to investigation…but discoveries made by one's own eyes and hands are more convincing and surprising. [Freudenthal, *Mathematics as an Educational Task,* p. 407]

In this session we'll make students' learning of function "more convincing and surprising" by connecting their eyes and hands more strongly to function concepts.

We'll do that by having students create, manipulate, and explore geometric transformations as functions, and by connecting students' understanding of functions in geometry (transformations) to their understanding of functions in algebra.

By treating geometric transformations as functions, students *vary* the variables by dragging them, they observe *rate of change* by observing the motion of variables, and even *function notation* becomes meaningful as students use it to describe their constructions. We'll do all this on our way to connecting *geometric* functions to *algebraic* functions via some surprising relationships.

Agenda

- Introduce Geometric Functions and their function families
- Use function notation to describe our constructions
- Compare relative rates of change to distinguish families from each other
- Dance the dependent variable
- Compose geometric functions
- Restrict the domain, and compare the corresponding range
- Relate domain and range to behavior
- Turn a restricted domain into a number line
- Translate the output to get a Dynagraph
- Rotate instead for a Cartesian graph
- Graph both linear and quadratic functions

For more information, email me, visit geometricfunctions.org, or check out blog.keycurriculum.com.