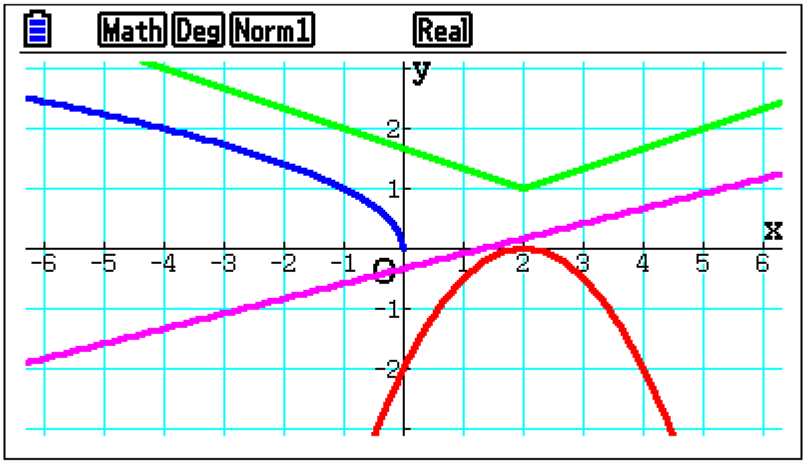
A FUNdamental Approach to Connecting Families of FUNctions Using Transformations



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Common Core State Standards Addressed

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

Content Standards

Interpret functions that arise in applications in terms of the context F-IF
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

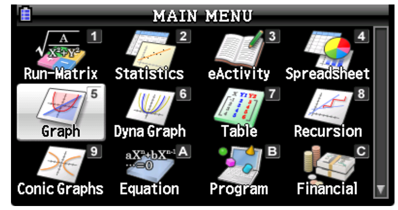
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Analyze functions using different representations F-IF
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Build new functions from existing functions F-BF
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

I. The Linear Family: Y=Ax+B (Slope-Intercept Form)

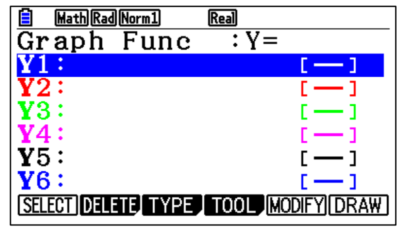
1. From the Main Menu ([MENU]), enter the Graph mode ([5])



If there are any equations stored in the Equation Editor, move the cursor to the equation.

Delete it using the key sequence:

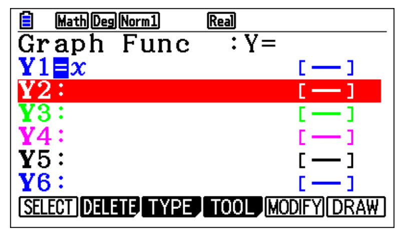
[F2] (DELETE) [F1] (Yes)



2. Use the key sequence below to store the equation Y = x in Y1.

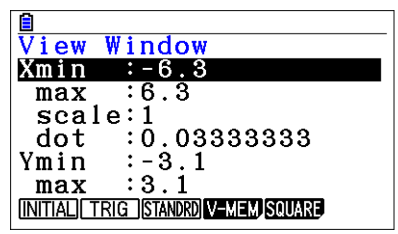
This is the “Parent Function.”

[X,θ,T] [EXE]

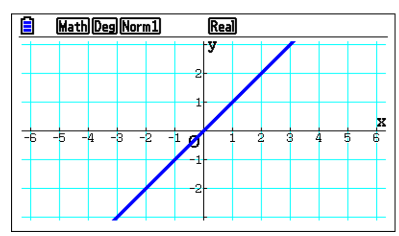


3. Use the key sequence below to set the Viewing Window to the [INITIAL WINDOW]

[SHIFT] [F3] [F1] [EXIT]

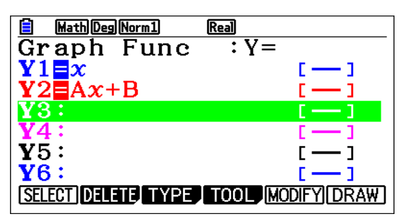


4. Press [F6] (DRAW). This graph can be traced. Press the [F1] (Trace) key followed by the ◀ and ▶ keys.

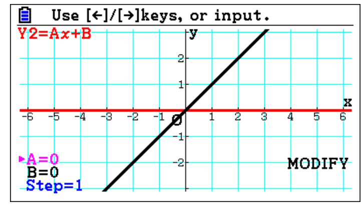


5. Press [EXIT] to return to the Graph Func (aka Y=) screen. Use the key sequence below to store the equation Y=Ax+B in Y2.

[ALPHA] [X,θ,T] (A) [X,θ,T] [+] [ALPHA] [log] (B) [EXE]

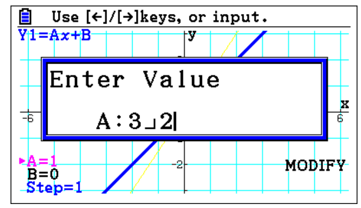


6. Press for [F5] (MODIFY).



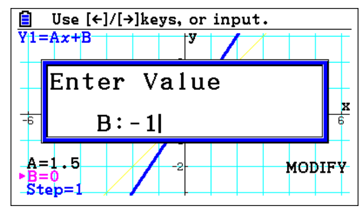
The values of A, B and the Step can be adjusted by pressing the arrow keys on the wheel, ◀, ▶, or by typing in a value followed by [EXE]. The ▲ and ▼ arrow keys can be used to change the values of A, B, and Step as one desires. Set the values for A and B as shown on the screen.

7. Use the ◀ and ▶ keys, to explore the changes to the graph of the "Parent Function" as you modify the value of A. Alternatively, press [EXE] and enter a value for A such as $\frac{3}{2}$.



Use the key sequence [3] [a b/c] [2] [EXE].

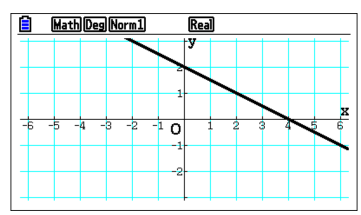
8. Use the ▼ to move to the cursor to the value of B. Then use the ◀ and ▶ to explore what happens to the "Parent Function" as you modify the values of B. Alternatively, press [EXE] and enter a value for B such as -1.



9. To leave the (MODIFY) Feature:

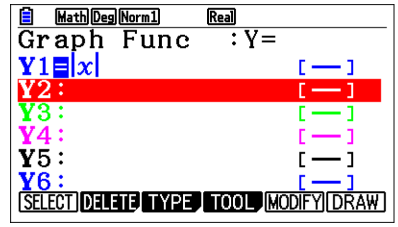
- Press [EXIT] **once** to return to the graph of the equation with the stored values for the variables. *This graph can be traced.*
- Press [EXIT] **twice** to return to the (Y=) Screen.

10. What is the equation of the function graphed on the screen to the right?



II. The Absolute Value Family: Y = A | x – H | + K

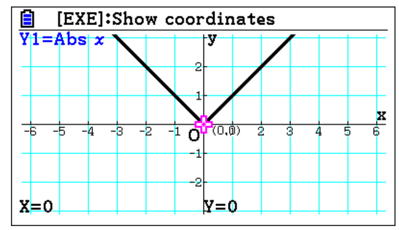
1. From the Main Menu ([MENU]), enter the GraphMode ([5]).



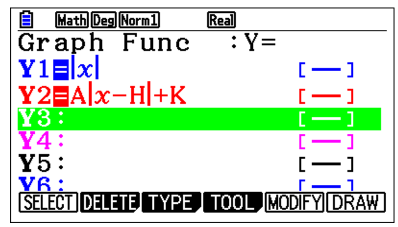
Use the key sequence below to store the equation Y=|x| in Y1. This is the "Parent Function."

[OPTN] [F5] (NUMERIC) [F1] (Abs) [X,θ,T] [EXE]

2. Press [F6] (DRAW). This is the graph of the "Parent Function." Press [F1] to trace along the graph. Pay careful attention to the values of y when x<0 and when x>0.



3. Press [EXIT] twice to go back to the Equation Editor. Change Y2 to the equation as shown. Use the key sequence below.

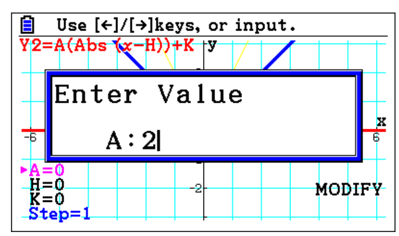


[ALPHA] [X,θ,T] (A) [OPTN] [F5] (NUMERIC) [F1] (Abs)

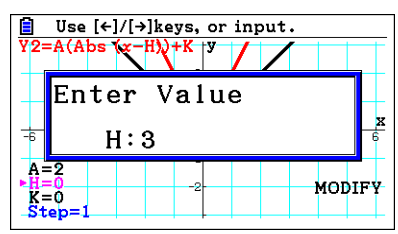
[X,θ,T] [−] [ALPHA] [F↔D] (H) [▶] [+] [ALPHA] [,] (K) [EXE]

4. Press for [F5] (MODIFY).

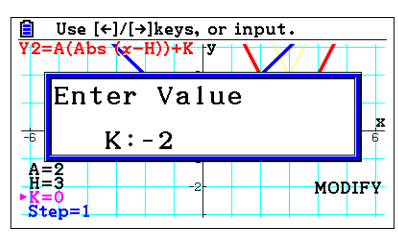
Use the arrow keys to explore what happens to the "Parent Function" as you modify the value of A. Use the arrow keysto explore what happens to the "Parent Function" as you modify the value of A. Alternatively you can press [EXE] and enter a value for A such as 2.



5. Move to the cursor to the value of H. Use the arrow keys to explore what happens to the "Parent Function" as you modify the value of H. Alternatively you can press [EXE] and enter a value for H such as 3



6. Move to the cursor to the value of K. Press [EXE] and enter a value for K such as -2. Explore what happens to the line as you modify the values of K. Is the ordered pair



(H, K) on your graph?

Exploration Questions for the Absolute Value Family

A) Some students think that the absolute function could be called the “rebound” function. What do you think this means? Do you agree?

B) How can you predict whether the graph of the equation will open upward or downward?

C) If A= -1, describe the geometric transformation of the Parent Function.

D) Summarize how changing the value of A affects the graph of the function.

E) What happens to the graph of the function as the value of |A| increases?

F) Summarize how changing the value of H affects the graph of the function.

G) Summarize how changing the value of K affects the graph of the function.

H) What are the coordinates of the point where the function “rebounds?”

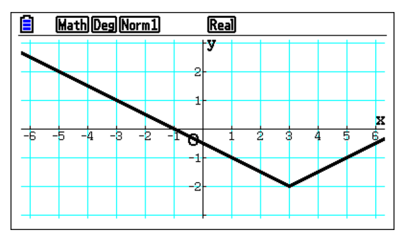
I) How can you determine the slope of each “branch” of the function?

J) What is the domain?

K) What is the range?

L) Can the range be determined from the equation?

M) What is the equation of the function that is graphed?



N) Write the equation of an Absolute Value Function whose domain all real numbers, whose range is $y \leq 2$, with a vertex in 1st Quadrant and is **narrower than** $Y = |x|$.

III. The Quadratic Family: $Y = A(x – H)^2 + K$

Enter the equations shown below in your calculator.

$Y1 = x^2$
$Y2 = A(x – H)^2 + K$

For Y2, use the key sequence:

[ALPHA] [X,θ,T] (A) [(] [X,θ,T] [−] [ALPHA] [F↔D] (H) [)] [x^2] [+] [ALPHA] [,] (K) [EXE]

Begin with A = 1, H = 0, and K = 0. Investigate what happens to the graph as you modify the values of A, H, and K. (Hint: Change the variables individually!)

A) Summarize how changing the value of A affects the graph of the function.

B) What happens to the graph of the function as the value of |A| increases? In other words, what happens to the graph of the function if A is positive and you increase its value?

C) If A= -1, describe the geometric transformation of the Parent Function.

D) Summarize how changing the value of H affects the graph of the function.

E) Summarize how changing the value of K affects the graph of the function.

F) What is the domain?

G) What is the range?

H) The axis of symmetry is a vertical line that passes through the vertex of the function. If the vertex is **not** on the x-axis, what could you conclude about the distance from the x-intercepts to the axis of symmetry?

I) Write the equation of a quadratic function that is narrower than the parent function, opens down, and has a vertex at (2, -2)?

IV. The Polynomial Family: $Y = Ax^N$

$$Y1 = Ax^N$$

Use the key sequence [ALPHA] [X,θ,T] (A) [X,θ,T] [^] [ALPHA] [8] (N) [EXE]

Begin with A = 1, N = 2. Investigate what happens to the graph as you modify the values of A, and N. (Hint: Change the variables individually!)

A) What do the equations of graphs that **do not** cross the x-axis have in common?

B) What do the equations of graphs that cross the x-axis have in common?

C) What is a common feature of all graphs that have an even value of N?

D) What is a common feature of all graphs that have an odd value of N?

E) If N is even:

1. What is the difference between the graphs for positive values of A (A > 0) and those with negative values of A (A < 0)?

2. If A > 0, what is the domain?

3. If A > 0, what is the range?

4. If A > 0, and the value of x increases and gets very large, that is as x approaches positive infinity (symbolically written as $x \to \infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

5. If A < 0, what is the domain?

6. If A < 0, what is the range?

7. If A > 0, and the value of x decreases and gets very small, that is as x approaches negative infinity (symbolically written as $x \to -\infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

8. If A < 0, and the value of x increases and gets very large, that is as x approaches positive infinity (symbolically written as $x \to \infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

9. If A < 0, and the value of x decreases and gets very small, that is as x approaches negative infinity (symbolically written as $x \to -\infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

10. Write the equation of a function that has the end behavior described below.

    As $x \to -\infty$, $y \to \infty$ **and** as $x \to \infty$, $y \to -\infty$

F) If N is odd:

1. What is the difference between the graphs for positive values of A (A > 0) and those with negative values of A (A < 0)?

2. If A > 0, what is the domain?

3. If A > 0, what is the range?

4. If A > 0, and the value of x increases and gets very large, that is as x approaches positive infinity (symbolically written as $x \to \infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

5. If A > 0, and the value of x decreases and gets very small, that is as x approaches negative infinity (symbolically written as $x \to -\infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

6. If A < 0, what is the domain?

7. If A < 0, what is the range?

8. If A < 0, and the value of x increases and gets very large, that is as x approaches positive infinity (symbolically written as $x \to \infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

9. If A < 0, and the value of x decreases and gets very small, that is as x approaches negative infinity (symbolically written as $x \to -\infty$), what happens to the value of y? This is generally referred to as the **end behavior** of a polynomial function.

10. Write the equation of a function that has the end behavior described below.

As $x \to -\infty$, $y \to \infty$ **and** as $x \to \infty$, $y \to -\infty$

G) Is there a relationship between the sign of A, the coefficient, and the value of N, the exponent? Consider the different situations you have explored!

V. The Square Root Family: $\boldsymbol{Y = A\sqrt{x - H} + K}$

Enter the equations shown below in your calculator.

$\boldsymbol{Y1 = \sqrt{x}}$
$\boldsymbol{Y2 = A\sqrt{x - H} + K}$

For Y2, use the key sequence:

[ALPHA] [X,θ,T] (A) [SHIFT] [x^2] [X,θ,T] [−] [ALPHA] [F↔D] (H) [▶] [+] [ALPHA] [,] (K) [EXE]

Begin with A = 1, H = 0, and K = 0. Investigate what happens to the graph as you modify the values of A, H, and K. (Hint: Change the variables individually!)

A) Summarize how changing the value of A affects the graph of the function.

B) What is the domain?

C) What is the range?

D) How can the value of A be changed so that the Parent Function is reflected over the x-axis?

E) Summarize how changing the value of H affects the graph of the function.

F) Summarize how changing the value of K affects the graph of the function.

G) For the square root family, how can the domain be determined from the equation?

H) For the square root family, how can the range be determined from the equation?

I) Write the equation of a square root function whose domain is $x \geq -5$, whose domain is $y \leq 2$, and has an x-intercept between -3 and 0?

J) In the function $\boldsymbol{Y = A\sqrt{-x}}$, explain why the domain is all real numbers less than or equal to zero.