

Mathematical Discourse

From Question Asking to Question Answering




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Discourse and Writing in the Common Core Standards

Mathematical Practice Standard 3:

Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematical Practice Standard 6:

Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, expressing numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Strategies to Initiate, Manage, and Connect & Conclude Discourse

I. Initiate Discourse: Formation and Foundation

1. Ask students engaging and challenging questions. For example,
 - Simply ask “why?”
 - Ask “compare-and-contrast” questions.
 - Ask “agree/disagree and why” questions.
 - Ask, “How does this connect to previous work?” questions.
 - Ask questions that have students build mathematical concepts one upon another. For example, after reviewing subtraction of whole numbers on a number line, ask, “What do you think it means if we subtract 5 from 3?”
2. Provide think and discussion time. Use *turn-and-talk*, *think-pair-share*, and *think-write-pair-share*. Keep in mind that not all discourse is student-to-student in pairs or small groups. Discourse may also be whole-class and student-to-teacher.
3. Let students know what is expected—to think about (possibly write about) and then discuss a math idea. Let them know this may be different from prior experiences and expectations in math classes.

II. Manage Discourse: Know when to hold them and Know when to scaffold them

1. Begin with small expectations in terms of discussion time and provide time frames for students. For example, “*Class, you have 30 seconds to decide if you agree or disagree with Jerry’s answer and why. Then you will have two minutes to discuss this with your partner and be ready to explain your answer to the class.*”
2. Circulate and listen in on conversations while students talk.
3. While circulating during student-to-student discussions, be prepared to guide, refocus, and move past blockages in discussions as needed by asking questions to help scaffold ideas, or making suggestions to discussions. Just be sure to leave room for students to think. Don’t do all the thinking for them.

III. Connect & Conclude Discourse: Selection and Connection

1. Choose students to share using both selective methods to have specific ideas shared that you overheard, or using “cold call” to encourage all students to be ready to share and to prevent bias in selecting who shares.
2. Have students share various strategies to a solution.
3. Correct misconceptions as needed. Invite students into this process.
4. Build on student ideas whenever possible. Connect student ideas or ask students to connect ideas that are shared. (*Repeat, revise, restate, and add-on strategies.*)
5. Clarify and validate correct mathematical thinking. Connect ideas, methods, thinking, and/or conclusions to the day’s learning objective(s).

Hess' Cognitive Rigor Matrix

- **Bloom:** What type of thinking (verbs) is needed to complete a task?
- **Webb:** How deeply do you have to understand the content to successfully interact with it? How complex is the content?

DOK is about depth & complexity—not difficulty!

Hess' Cognitive Rigor Matrices

Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions

REVISED BLOOM'S TAXONOMY	WEBB'S DEPTH OF KNOWLEDGE LEVELS			
	1 RECALL & REPRODUCTION	2 SKILLS & CONCEPTS	3 STRATEGIC THINKING/ REASONING	4 EXTENDED THINKING
REMEMBER Retrieve knowledge from long-term memory, recognize, recall, locate, identify	Recall a fact, term, principle, or concept; perform a routine procedure			
UNDERSTAND Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion, predict, compare/ contrast, match similar ideas, explain, construct models		Use information, conceptual knowledge; select appropriate procedures for a task; perform two or more steps with decision points along the way; solve routine problems; organize or display data; interpret or use simple graphs.	Reason or develop a plan to approach a problem; employ some decision making and justification; solve abstract, complex, or nonroutine problems (DOK 3 problems often allow more than one possible answer.)	Perform investigations or apply concepts and skills to the real world that require time to research, problem solve, and process multiple conditions of the problem or task; perform nonroutine manipulations across disciplines, content areas, or multiple sources
APPLY Carry out or use a procedure in a given situation; carry out or use/apply to an unfamiliar task				
ANALYZE Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct (e.g., for bias or point of view)				
EVALUATE Make judgments based on criteria, check, detect inconsistencies/fallacies, judge, critique				
CREATE Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, produce				

Hess, Carlock, Jones, and Walkup (2009)

Taking a Closer Look: Level 1, Recall & Reproduction

BLOOM'S	DOK LEVEL 1 RECALL & REPRODUCTION	
	ELA/SOCIAL STUDIES	MATH/SCIENCE
REMEMBER	<ul style="list-style-type: none"> Recall or locate basic facts, definitions, details, events Read words orally in connected text with fluency & accuracy 	<ul style="list-style-type: none"> Recall, observe, and recognize facts, principles, properties Recall/identify conversions among representations of numbers (e.g., customary and metric measures)
UNDERSTAND	<ul style="list-style-type: none"> Identify or describe literary elements (characters, setting, sequence, etc.) Select appropriate words when intended meaning/definition is evident Describe/explain who, what, where, when, or how Write simple sentences 	<ul style="list-style-type: none"> Evaluate an expression Locate points on a grid or on a number line Solve a one-step problem Represent math relationships in words, pictures, or symbols Read, write, compare decimals in scientific notation
APPLY	<ul style="list-style-type: none"> Use language structure (pre/suffix) or word relationships (synonym/antonym) to determine meaning Apply rules or resources to edit spelling, grammar, punctuation, conventions, word use Apply basic formats for documenting sources 	<ul style="list-style-type: none"> Follow simple procedures (e.g., recipe-type directions) Calculate, measure, apply a rule (e.g., rounding) Apply an algorithm or formula (e.g., area, perimeter) Solve linear equations Make conversions among representations of numbers, or within and between customary and metric measures
ANALYZE	<ul style="list-style-type: none"> Identify whether specific information is contained in graphic representations (e.g., map, chart, table, T-chart, diagram) or text features (e.g., headings, subheadings, captions) Decide which text structure is appropriate to audience and purpose 	<ul style="list-style-type: none"> Retrieve information from a table or graph to answer a question Identify whether specific information is contained in graphic representations (e.g., table, graph, T-chart, diagram) Identify a pattern/trend
EVALUATE		
CREATE	<ul style="list-style-type: none"> Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept 	<ul style="list-style-type: none"> Brainstorm ideas, concepts, or terms related to a topic

Verbs/Key Words

Locate, calculate, define, identify, list, label, match, measure, copy, memorize, repeat, report, recall, recite, recognize, state, tell, tabulate, use rules, answer who, what, when, where, why, how

Teacher Role

Questions to direct or focus attention, shows, tells, demonstrates, provides examples, examines, leads, breaks down, defines

Student Role

Recognizes, responds, remembers, memorizes, restates, absorbs, describes, demonstrates, follows directions, applies routine processes, definitions, and procedures

Question Stems

- Can you recall _____?
- When did ____ happen?
- Who was ____?
- How can you recognize ____?
- What is ____?
- How can you find the meaning of ____?
- Can you recall ____?
- Can you select ____?
- How would you write ____?
- What might you include on a list about ____?
- Who discovered ____?
- What is the formula for ____?
- Can you identify ____?
- How would you describe ____?

Taking a Closer Look: Level 2, Skill/Concept

BLOOM'S	DOK LEVEL 2 SKILLS & CONCEPTS	
	ELA/HISTORY SOCIAL STUDIES	MATH/SCIENCE
REMEMBER		
UNDERSTAND	<ul style="list-style-type: none"> Specify, explain, show relationships, explain why, cause-effect Give examples/nonexamples Summarize results, concepts, ideas in one text or one data set Identify main ideas or accurate generalizations of texts or issues Locate information to support explicit-implicit central ideas 	<ul style="list-style-type: none"> Specify and explain relationships (e.g., nonexamples/examples, cause-effect) Make and record observations Explain steps followed Summarize results or concepts Make basic inferences or logical predictions from data/observations Use models (e.g., diagrams to represent or explain mathematical concepts) Make or explain estimates
APPLY	<ul style="list-style-type: none"> Use context to identify the meaning of words/phrases Obtain and interpret information using text features Develop a text that may be limited to one paragraph 	<ul style="list-style-type: none"> Select a procedure according to criteria and perform it Solve routine problems, applying multiple concepts or decision points Retrieve information from a table, graph, or figure and use it to solve a problem Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) Construct models given criteria
ANALYZE	<ul style="list-style-type: none"> Categorize/compare literary elements, terms, facts/details, events Identify use of literary devices Analyze format, organization, and internal text structures (e.g., signal words, transitions, semantic clues) of different text Distinguish relevant-irrelevant information, fact/opinion Identify characteristic text features; distinguish between texts, genres 	<ul style="list-style-type: none"> Categorize, classify materials, data, figures based on characteristics Organize or order data Compare/contrast figures or data Select appropriate graph and organize & display data Interpret data from a simple graph Extend a pattern
EVALUATE		
CREATE	<ul style="list-style-type: none"> Generate conjectures or hypotheses based on observations or prior knowledge and experience 	Generate conjectures or hypotheses based on observations or prior knowledge and experience

Key Words

Infer, categorize, organize and display, compare-contrast, modify, predict, interpret, distinguish, estimate, extend patterns, interpret, use context clues, make observations, summarize, translate from table to graph, classify, show cause/effect, relate, edit for clarity

Teacher Role

Questions to differentiate, infer, or check conceptual understanding, models, organizes/reorganizes, explores possible options or connections, provides examples and nonexamples

Student Role

Solves routine problems/tasks involving multiple decision points and concepts, constructs models to show relationships, demonstrates use of conceptual knowledge, compiles and organizes, illustrates/explains with examples or models, examines

Question Stems

- How or why would you use ____?
- Can you explain how ____ affected ____?
- What was the cause of ____?
- How would you compare ____? Contrast ____?
- How would you classify ____? What would you use to classify ____?
- How are ____ alike? Different?
- How would you classify the type of ____?
- What can you say about ____?
- How would you summarize ____?
- What steps are needed to edit ____?
- When would you use an outline to ____?
- How would you estimate ____?
- How would you organize to show ____?
- What examples/nonexamples can you find to ____?
- What do you notice about ____?

Taking a Closer Look: Level 3, Strategic Thinking & Reasoning

BLOOM'S	DOK LEVEL 3 STRATEGIC THINKING/ REASONING	
	ELA/HISTORY SOCIAL STUDIES	MATH/SCIENCE
REMEMBER		
UNDERSTAND	<ul style="list-style-type: none"> Explain, generalize, or connect ideas using supporting evidence (e.g., quote, example, text reference) Identify/make inferences about explicit or implicit themes Describe how word choice, point of view, or bias may affect the readers' interpretation of a text Write multi-paragraph composition for specific purpose, focus, voice, tone, & audience 	<ul style="list-style-type: none"> Use concepts to solve nonroutine problems Explain, generalize, or connect ideas using supporting evidence Make and justify conjectures Explain thinking when more than one response/solution is possible Explain phenomena in terms of concept
APPLY	<ul style="list-style-type: none"> Apply concepts to a new concept Revise final draft for meaning or progression of ideas Apply internal consistency of text organization and structure to composing a full composition Apply word choice, point of view, style, to impact readers' /viewers' interpretation of a text 	<ul style="list-style-type: none"> Design an investigation for a specific purpose or research question Conduct a designed investigation Use concepts to solve nonroutine problems Use and show reasoning, planning, and evidence Translate between problem & symbol notation when not a direct translation
ANALYZE	<ul style="list-style-type: none"> Analyze information within a text or source Analyze interrelationships among concepts, issues, problems Analyze or interpret author's craft (literary devices, viewpoint, potential bias) to create or critique a text or support text interpretation Use reasoning and evidence to generate criteria for making and supporting an argument or judgment (e.g., Was FDR a great president? Who was the greatest ballplayer?) 	<ul style="list-style-type: none"> Compare information data sets or texts or across related data sets Analyze and draw conclusions from data, citing evidence Generalize a pattern Interpret data from complex graph Analyze similarities/differences between research procedures or solutions
EVALUATE	<ul style="list-style-type: none"> Cite evidence and develop a logical argument for conjectures Describe, compare, and contrast solution methods Verify reasonableness of results Justify or critique conclusions drawn 	<ul style="list-style-type: none"> Cite evidence and develop a logical argument for concepts or solutions Describe, compare, and contrast solution methods Verify reasonableness of results
CREATE	<ul style="list-style-type: none"> Synthesize information within one source or text Develop a complex model for a given situation Develop an alternative solution 	<ul style="list-style-type: none"> Synthesize information within one data set, source, or text Formulate an original problem given a solution Develop a scientific/mathematical model for a complex situation

Key Words

Critique, appraise, revise for meaning, assess, investigate, cite evidence, test hypothesis, develop a logical argument, use concepts to solve nonroutine problems, explain phenomena in terms of concepts, draw conclusions based on data

Teacher Role

Questions to probe reasoning and underlying thinking, asks open-ended questions, acts as a resource and coach, provides criteria and examples for making judgments and supporting claims, encourages multiple approaches and solutions; determines when/where (text, concept) depth and exploration is most appropriate

Student Role

Uncovers and selects relevant and credible supporting evidence for analyses, critiques, debates, claims, and judgments; plans, initiates questions, disputes, argues, tests ideas/solutions, sustains inquiry into topics or deeper problems, applies to the real world

Question Stems

- How is ___ related to ___?
- What conclusions can you draw _____?
- How would you adapt ___ to create a different ___?
- How would you test _____?
- Can you predict the outcome if _____?
- What is the best answer? Why?
- What conclusion can be drawn from these three texts?
- What is your interpretation of this text?
- Support your rationale.
- How would you describe the sequence of _____?
- What facts would you select to support _____?
- What is the impact on the reader for use of this (rhetorical device, analogy, figurative language use, visual image, etc.)?
- What underlying bias is there _____?
- Can you elaborate on the reason _____?
- What would happen if _____?
- Can you formulate a theory for _____?
- How would you test _____?
- Can you elaborate on the reason?
- What is the author's chain of reasoning or point of view for _____?

Taking a Closer Look: Level 4, Extended Thinking

BLOOM'S	DOK LEVEL 4 EXTENDED THINKING	
	ELA/HISTORY SOCIAL STUDIES	MATH/SCIENCE
REMEMBER		
UNDERSTAND	<ul style="list-style-type: none"> Use multiple sources to elaborate on how concepts or ideas specifically draw from other content domains or differing concepts (e.g., research paper, arguments of policy: should this law be passed? What will be the impact of this change?) Develop generalizations about the results obtained or strategies used and apply them to a new problem or contextual scenario. 	<ul style="list-style-type: none"> Relate mathematical or scientific concepts to other content areas, other domains, or other concepts Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
APPLY	<ul style="list-style-type: none"> Select or devise an approach among many alternatives to research and present a novel problem or issue Illustrate how multiple themes (e.g., historical, geographic, social) may be interrelated within a text or topic 	<ul style="list-style-type: none"> Select or devise an approach among many alternatives to solve a problem Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
ANALYZE	<ul style="list-style-type: none"> Analyze multiple sources of evidence, or multiple works by the same author, or across genres or time periods Analyze complex/abstract themes, perspectives, concepts Gather, analyze, and organize multiple information sources Compare and contrast conflicting judgments or policies (e.g., Supreme Court decisions) 	<ul style="list-style-type: none"> Analyze multiple sources of evidence Analyze complex/abstract themes Gather, analyze, and evaluate information
EVALUATE	<ul style="list-style-type: none"> Evaluate relevancy, accuracy, and completeness of information across multiple sources Apply understanding in a novel way, provide argument or justification for the application Critique the historical impact (e.g., policy, writings, discoveries, etc.) 	<ul style="list-style-type: none"> Gather, analyze, and evaluate information in order to draw conclusions Apply understanding in a novel way, provide argument or justification for the application
CREATE	<ul style="list-style-type: none"> Synthesize information across multiple sources or texts in order to articulate a new voice, alternate theme, new knowledge, or nuanced perspective 	<ul style="list-style-type: none"> Synthesize information across multiple sources or text Design a mathematical model to inform and solve a practical or abstract situation

Key Words

Initiate, design and conduct, collaborate, research, synthesize, self-monitor, critique, produce/present

Teacher Role

Questions to extend thinking and broaden perspectives; facilitates teaming, collaboration, self-evaluation

Student Role

Designs, takes risks, researches synthesizing multiple resources, collaborates, plans, organizes, and modifies, creates concrete tangible products

Question Stems

- What changes would you make to solve or address this major problem or issue?
- How would you improve upon this invention or innovation?
- Can you propose an alternative solution to . . . ?
- What could be done to minimize/maximize ___ ?
- In what way would you design or redesign . . . and why?
- What evidence would you cite to defend the actions of ___?
- How would you evaluate ___?
- How would you prioritize criteria for making this decision . . . and why?
- How would you evaluate the works by this author over time?
- What information would you use to support a differing perspective ___?
- Can you formulate and test a conjecture for ___?
- Can you predict the potential benefits and drawbacks if this law does/does not pass?
- Can you construct a model that would change ___?
- Can you think of an original way to apply ___?
- What information can you gather to support your idea about ___?

Levels of Discourse

Discourse	Descriptions	Examples
Confirm	The teacher provides the answer and looks for students to agree or disagree.	<p><i>Is it true?</i></p> <ul style="list-style-type: none"> • Yes / No; Agree / Disagree • Thumbs up / Thumbs down
Recall	The teacher asks a question that requires a short response based on recall of a learned fact or procedure.	<p><i>What is it? What is the answer?</i></p> <ul style="list-style-type: none"> • A product • Knowledge from memory • Facts, calculations, definitions • Fill in the blank
Explain	The student is asked to explain how s/he got the answer. This can be as simple as “I did this, then I did that, then I did . . .” to an explanation that qualifies as a reasonable justification, or a proof.	<p><i>How did you get the answer?</i></p> <ul style="list-style-type: none"> • Explain or show your work. • What does the solution mean in the context of this problem? • Does the solution make sense? Why? • How does your way of solving the problem connect to another method shown?
Justify	The student explains why s/he solved the problem the way that s/he did. The student explains some or all of the mathematical reasoning behind the solution or method. A good justification may also qualify as a complete or incomplete proof.	<p><i>Why is it true?</i></p> <ul style="list-style-type: none"> • Why does that work? • Why would you do that? • Inductive and deductive reasoning. • See relationships and connections. • Identify patterns, and make and test conjectures.
Generalize	The student recognizes or hypothesizes that some aspect of a solution or characteristic of one or more individual situations applies to a wider group of situations, or to all situations.	<p><i>Is it always true?</i></p> <ul style="list-style-type: none"> • Move from the specific to the general—from one or many to all. • Identifying a rule to fit a pattern.
Prove	Explicit use of mathematical reasoning. A conclusion reached or confirmed through a deductive approach, building step by step on the use and application of previously established, agreed-upon, or proven mathematical properties and definitions.	<p><i>What is the evidence that it is true?</i></p> <ul style="list-style-type: none"> • Deductive reasoning. • Build or use proven or learned mathematical facts or properties. • Can take many forms—paragraph, two-column, verbal explanation, etc.

Adapted from the Sonoma County Office of Education, 2010

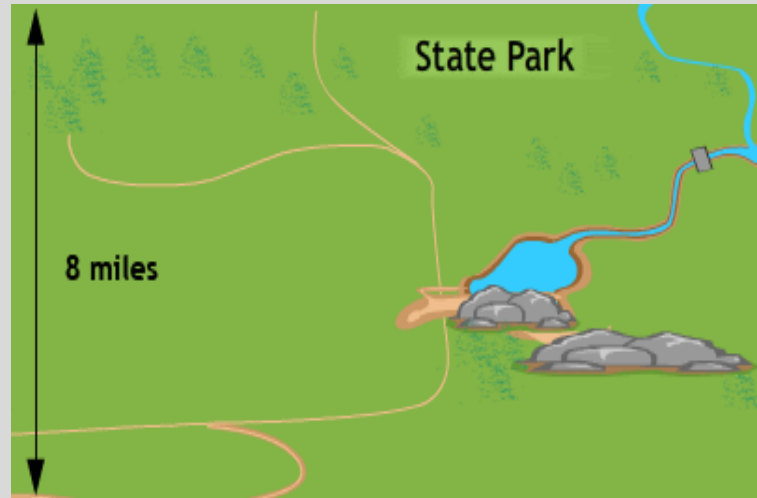
State Park

The perimeter of the rectangular state park shown is 42 miles.

A ranger estimates that there are 9 deer in each square mile of the park.

If this estimate is correct, how many total deer are in the park?

Explain your answer using numbers, symbols, and words.



PARCC Grade 4 Sample

(http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html)

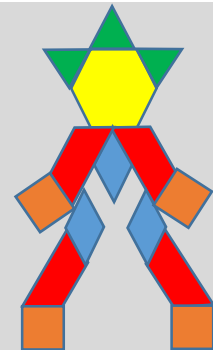
Pattern Block Man

1. If the hexagon represents one whole, what fraction do

- a) All the triangles represent together?
- b) All the trapezoids represent together?
- c) All the rhombi represent together?

Explain your reasoning.

2. What total value do all four shapes represent together?



What would be a good extension for added discourse that would likely challenge students reasoning and allow you to see how they are understanding fractions?