Mathematical Discourse

from Question Asking to Question Answering

General guidelines and specific ideas for promoting and implementing effective mathematical discourse in the classroom.

CORE MATH

Meaningful Mathematical Discourse

Classroom discourse develops students' understanding of key ideas. Student dialogue provides additional information and engages students in deeper understanding and reflection, and ultimately promotes greater conceptual development.

Adapted from *Adding It Up* 2001 and Nathan & Kim 2007

Discourse and Writing in the CCSS Mathematical Practices

Students . . . understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements, They justify their conclusions, communicate them to others, and respond to the arguments of others . . . making plausible arguments that take into account the context from which [they] arose.

Students . . . communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. . . . By the time they reach high school they have learned to examine claims and make explicit use of definitions.

—From CCSSM Mathematical Practice Standards 3 and 6

Discourse in Math

Math discussions differ from discourse/discussions in other content areas.

- In many content areas the point of discussion is often to express and support opinions and ideas.
- In math the point of discussion is usually to determine and prove a correct answer, or to recognize and describe mathematical relationships.

Initiate, Manage, and Connect & Conclude Discourse

- **Initiate –**

  Getting discourse started

- **Manage –**

  Getting students to engage and persevere

- **Connect & Conclude –**

  Getting the mathematical point across

Initiate, Manage, and Connect & Conclude Discourse

Initiate

- **Formation** – questioning and posing problems
- **Foundation** – students ready to engage in dialogue (knowledge base, attitude, setting, expectations)

Initiate, **Manage**, and Connect & Conclude **Discourse**

Manage

- **Know when to hold them** – leave it alone and let them work
- **Know when to scaffold them** – managing the room, scaffolding, extensions, etc.. Combination of guide on the side and sage on the stage as you work and walk the room.

Initiate, Manage, and Connect & Conclude Discourse

Connect & Conclude

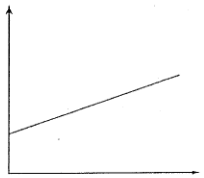
- **Selection** – decided which student work and ideas to share with the whole class
- **Connection** – connect student work/ideas together, connect to the mathematics, and connect to the lesson objective

Initiating Discourse – Formation
Rich Tasks and Deeper Level Questions

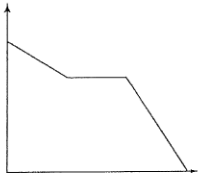
Graph Stories

What story might each graph be telling?

A.



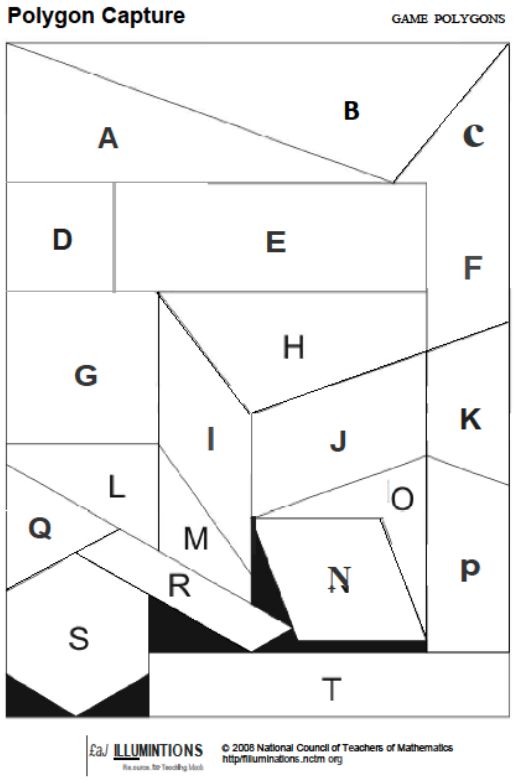
B.



A.	B.
A tree growing planted a tree and going up steadily.	Going down a water slide. Going down the water slide steady then not changing at all then go down a drop really fast.

Polygon Capture Game
(from NCTM Illuminations)

Especially good for discourse when you have two students work as a team competing against another team of two students.



File Cabinet Problem

(from Dan Meyer as shared on NCSM under Three-Act Math)

How many sticky notes will it take to cover this entire cabinet? What is your guess? Share your guess with your neighbor and say why you think it is so.

http://www.mathedleadership.org/resources/threeacts/filecabinet.html

http://blog.mrmeyer.com/category/3acts/

Mr. Torres Sports Cards
(from SMARTER Balance -SBAC)

Mr. Torres sold a total of 30 boxes of sports cards at his store on Monday. These boxes contained only baseball cards and football cards.

- Each box contained 25 sports cards.
- He earned $3 for each sports card he sold.
- He earned a total of $1,134 from the football cards he sold.

What amount of money did Mr. Torres earn from the baseball cards?

- Smarter Balanced Assessment Consortium (SBAC). Downloaded from http://www.smarterbalanced.org/sample-items-andperformance-tasks/.

Always, Sometimes, or Never

A. When is the following statement always true, when is it sometimes true, and when is it never true?

Subtraction results in a lesser value.

In other words, If $a - b = c$*, then* $c < a$*.*

B. When is the following statement always true, never true, and sometimes true?

If $a + b + c = d$*, then* d *is a multiple of 5.*

C. When is the following statement always true, never true, and sometimes true?

$$\frac{a}{b} < \frac{b}{a}$$

T-shirt Sale *(from map.mathshell.org)*

T-shirt Sale: Any 3 T-shirts for $14.50

1. Tom bought these three T-shirts at the sale price of $14.50.
   How much money did he save compared to the original total price of the T-shirts?
   Show your calculations. $ _____

2. What percentage of the original total price did Tom save? _____%
   Show your work.

3. Harry also paid $14.50 for three T-shirts at the sale. The sale price saved Harry 30% of the original price of the three T-shirts.
   What is the original total price of his three T-shirts? $_____
   Show your calculations.

Hess' Cognitive Rigor Matrix

Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions

Revised Bloom's Taxonomy	Webb's Depth of Knowledge Levels			
	1 Recall & Reproduction	2 Skills & Concepts	3 Strategic Thinking/Reasoning	4 Extended Thinking
Remember				
Understand				
Apply				
Analyze				
Evaluate				
Create				

Initiating Discourse – Processes or Input for Student Thinking

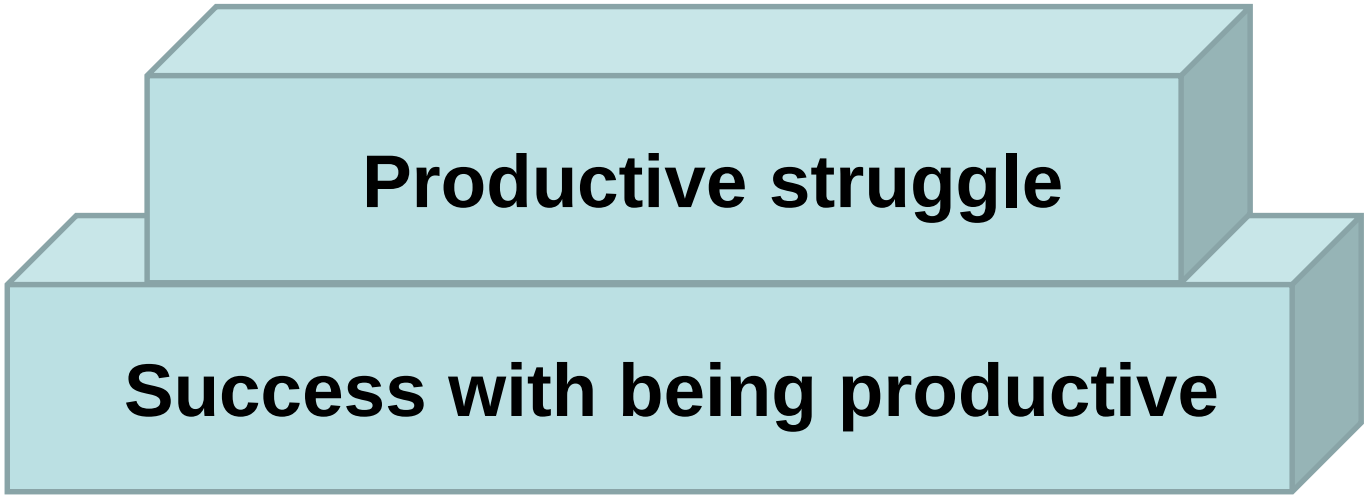
Process Focus	Type of Question Associated with Discourse Focused on Process
Plan	*How will you solve this?*
Explore	*What have you discovered?*
Apply	*How did you solve this?*
Model	*Which model best represents this?*
Analyze	*How is this connected to . . . ?*
Compare	*What are similarities and differences?*
Conjecture/Predict	*What will happen if . . . ?*
Translate/Interpret	*What does this graph tell us?*

Levels of Discourse – Outcomes or Output from Student Thinking

Output Level	Type of Question Associated with Discourse Focused on Output
Confirm	*Is it true?*
Recall	*What is it?*
Explain	*How did you get the answer?*
Justify	*Why is it true?*
Generalize	*Is it always true?*
Prove	*What is the evidence that it is true?*

Initiating Discourse – Foundation

Build Productive Struggle on Productive Success



Initiating Discourse – "Scratcher" Strategy

1. What is ½ of 50?

   A. 5 B. 10 C. 100 D. 75 E. 25

2. Which of the following is true?

   A. $0^0 = 0$

   B. $0^0 = 1$

   C. 0^0 is undefined

   D. 0^0 does not exist

   E. 0^0 is infinite

Initiating Discourse – "Scratcher" Strategy

Work with a partner – choose the correct answer on the handout and scratch it off. If incorrect, discuss some more and choose again.

3. Arrange the fractions in order from least to greatest without making common denominators or using decimals. $\frac{7}{8}, \frac{7}{9}, \frac{13}{15}$

4. When is the following statement never true?

   If $a - b = c$, *then* $c < a$. *(Subtraction results in a lesser value.)*

5. How many times should you tickle an octopus?

Managing Discourse

Manage the classroom + Manage the math =

Opportunities for meaningful discourse

Managing Discourse –

Levels of Classroom Discourse from Hufford-Ackles, Fuson, and Sherin (2014)

	Teacher role	Questioning	Explaining mathematical thinking	Mathematical representations	Building student responsibility within the community
Level 0	Teacher is at the front of the room and demonstrates conversation.	Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.	Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.	Representations are missing, or teacher shows them to students.	Culture supports students looking to themselves or just providing answers when asked.
Level 1	Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.	Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.	Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in or explain. Students provide brief descriptions of their thinking in response to teacher probing.	Students learn to create math drawings to explain their mathematical thinking.	Students believe that their ideas are accepted by the classroom community. They begin to listen to others and listen supportively and to restate in their own words what another student has said.
Level 2	Teacher facilitates conversation between students, and encourages students to ask questions of one another.	Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.	Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.	Students label their math drawings so that others are able to follow their mathematical thinking.	Students believe that they are math learners and that their ideas and the ideas of the classmates are important. They listen actively so that they can contribute significantly.
Level 3	Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher allows for students to clarify thinking of others.	Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask "why" and call for justification. Teacher questions may still guide discourse.	Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.	Students follow and help shape the descriptions of others' math thinking through math drawings and may suggest edits in others' math drawings.	Students believe that they are math leaders and can help shape the thinking of others. They help shape others' math thinking, support views, value all ways and accept the same support from others.

Fig. 11. Levels of classroom discourse. From Hufford-Ackles, Fuson, and Sherin (2014), table 1.

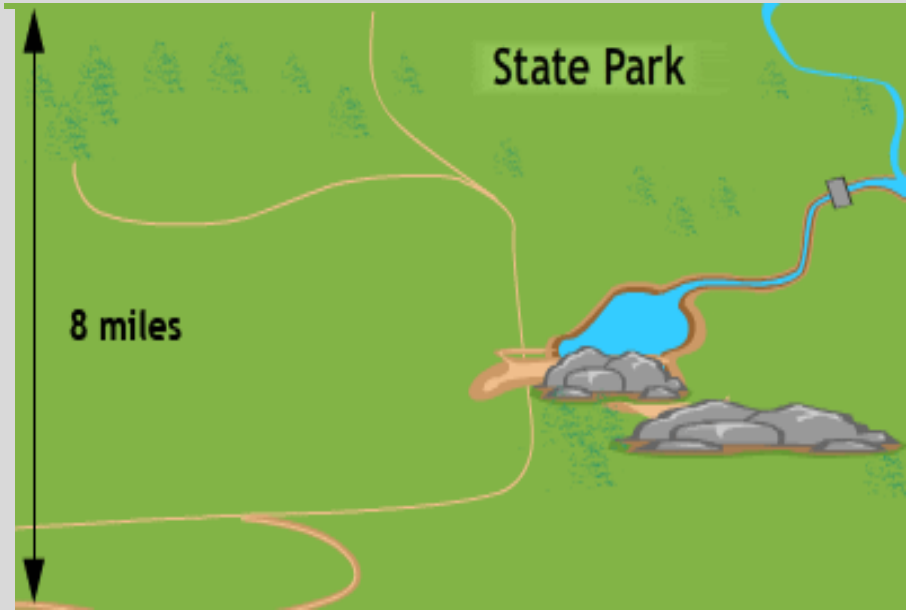
Principles to Actions, NCTM, 2014

Managing Discourse – Hold or Scaffold Prepare for both

The perimeter of the rectangular state park shown is 42 miles.

A ranger estimates that there are 9 deer in each square mile of the park.

If this estimate is correct, how many total deer are in the park? Explain your answer using numbers, symbols, and words.



PARCC Grade 4 Sample
(http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html)

Deer In Park – Scoffold Questions

Sample of teacher work:

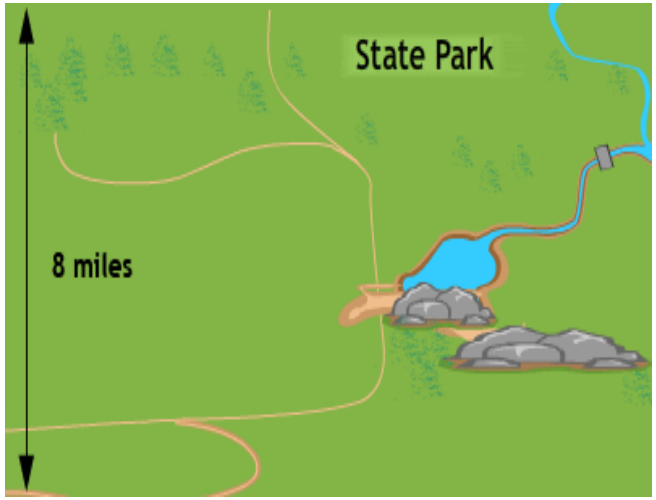
42-8 = 34

34-8=26

26/2=13

8x13=104

104x9=936 deer.



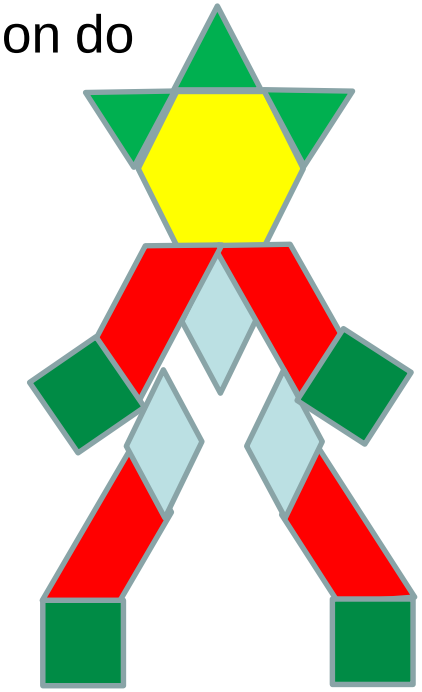
- *Why did you start by subtracting 8?*
- *What does the 34 represent?*
- *What does the 13 represent?*
- *What does the 104 represent?*
- *Why multiply by 9 rather than divide or add?*

Managing Discourse – Hold or Scaffold

1. If the hexagon represents one whole, what fraction do
   a) All the triangles represent together?
   b) All the trapezoids represent together?
   c) All the rhombi represent together?

   Explain your reasoning.

2. What total value do all four shapes represent together?



What would be a good extension for added discourse that would likely challenge students reasoning and allow you to see how they are understanding fractions with these shapes?

Connect and Conclude Discourse – Anticipating Responses

Mental math:

Subtract 385 from 529, that is 529 minus 385.

Do this in your head – no writing and no calculators.

- Standard subtraction algorithm?
- Counting on strategy, that is, started at 385 and counted up to 529?
  - Counted by 100s, 385, 485, 585, then counted from their to 629?
  - 385+15 is 400, then 400 to 629 is 229, and 229+15 is 244 (may have done in steps, such as 200 + 15 +29 is 244?"
- Counted down from 629 to 385?
- Counting down using 385 from 629 or 600 (with convenient numbers?)?

Steps for Meaningful Discourse

1. Initiate with a question or prompt that is focused on processes and/or outcomes that promote DOK 2–3.
2. Focus on the why behind the what.
3. Provide time to think.
4. Provide time to discuss.
5. Manage process for sharing and connecting ideas.
6. Make mathematical connections explicit.
7. Always ask, *“Why does this make sense?”*

Thank you!

Dean Ballard

dballard@corelearn.com

www.corelearn.com