Flips, Slides, Turns, and Tessellations, Oh My!
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NCTM Standards

- In grades 3 – 5 students can investigate the effects of transformations and begin to describe them in mathematical terms. In the middle grades, students should learn to understand what it means for a transformation to preserve distance, as translations, rotations, and reflections do.

NCTM Focal Points

In grades 3 – 5 students should:

- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.
- Describe a motion or series of motions that will show that two shapes are congruent.
- In grades 6 – 8 students should:
- Describe sizes, positions, and orientations of shapes under informal transformations.
- Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.

Common Core Standards

In grade 8, students are expected to:

- Verify experimentally the properties of rotations, reflections, and translations.
- Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that exhibits the congruence between them.

Definitions

- A translation is a transformation of a plane that moves every point of the plane a specified distance in a specified direction along a straight line (vector). (Billstein, Libeskind & Lott, 2013)
- A rotation is a transformation of the plane determined by holding one point (the center) fixed and rotating the plane about this point by a certain amount in a certain direction. (Billstein, Libeskind & Lott, 2013)
- A reflection in a line l is a transformation from the plane to the plane that pairs each point P with a point P' in such a way that l is the perpendicular bisector of PP', as long as P is not on l. If P is on l, then P = P' (Billstein, Libeskind & Lott, 2013)
- A tessellation of a plane is the filling of the plane with repetitions of figures in such a way that no figures overlap and there are no gaps. (Billstein, Libeskind & Lott, 2013)
- A **Penrose tiling** is a non-periodic tiling generated by anaperiodic set of prototiles. Penrose tilings are named after mathematician and physicist Roger Penrose who investigated these sets in the 1970s. The aperiodicity of the Penrose prototiles implies that a shifted copy of a Penrose tiling will never match the original. A Penrose tiling may be constructed so as to exhibit both reflection symmetry and fivefold rotational symmetry. (Tessellations.org, 2014)

References

- Billstein, R., Libeskind, S., & Lott, J. (2013). *A Problem Solving Approach to Mathematics for Elementary School Teachers* (11th ed.). Boston, MA: Pearson.
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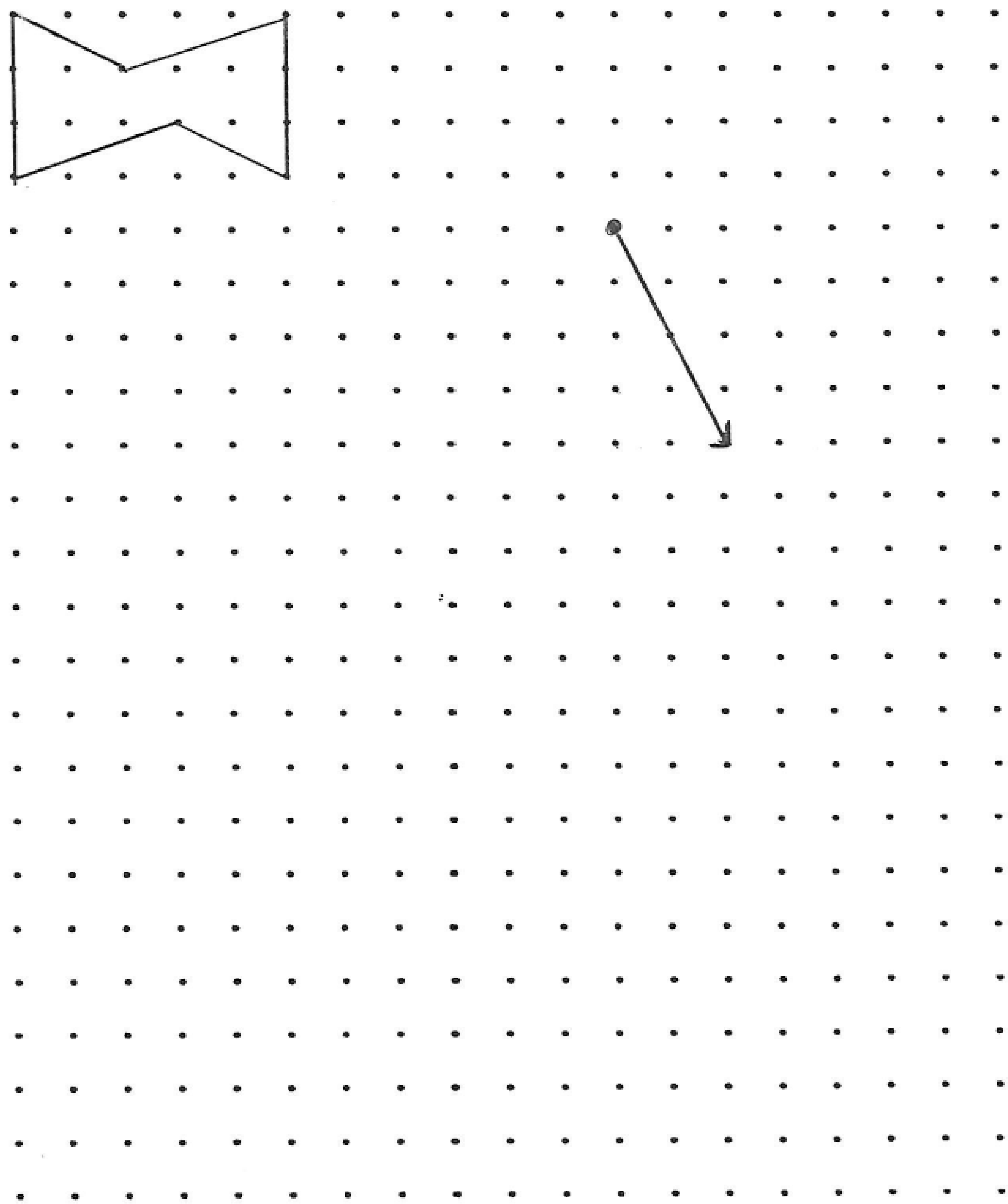
Activity 1 (Translations)

- Place your patty paper over the shape on the Activity 1dot paper and trace the shape.
- Trace the vector on the paper paper as well.
- Move the patty paper so that the origin of the vector matches up to the endpoint of the vector.
- Trace the new shape onto the dot paper.
- Measure the distance between several points and their images.
- On the dot paper, draw another shape and a vector.
- Translate your shape along your vector.
- Measure the distance between several points and their images.

Translation Conjectures

- In a translation transformation the distance between any point and its image is _______________________________.
- The image of a segment is another ___________ that is ____________ to the pre-image.
- The image of a shape through a translation is _________________ to the pre-image.

Comment on any other results you found interesting.



Activity 2 (Rotations)

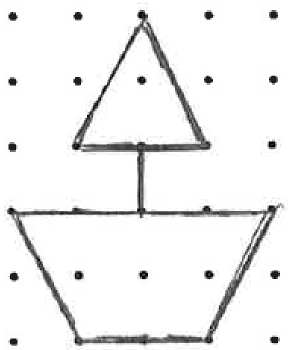
- Place a piece of patty paper on the shape on the Activity 2 dot paper and trace the shape.
- Trace the rotation angle and the direction of the rotation.
- Put your pencil on the vertex of the rotation angle and turn your paper in the direction of the turn arrow until the first side of the angle matches with the second side.
- Trace your shape in this location.
- Measure the distance between some of the points on the original shape and the corresponding image on the rotated shape.
- Draw another shape and repeat the process.

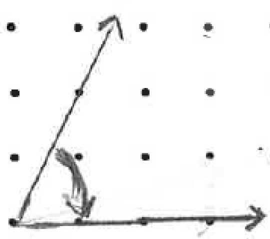
Rotation Conjectures

- In a rotation transformation the distance between any point and its image is ___________________________.
- In a rotation transformation the image and its pre-image are _______________

Comment on any other results you found interesting. Compare the results of a translation to those of a rotation. How are they similar? How are they different?

Activity 2





Activity 3 (Reflections)

- Using the dot paper on Activity 3, place your MIRA on the reflecting line, so that the beveled edge faces you and the shape.
- Look through the MIRA and trace the image of the shape.
- Draw your own shape and reflecting line and create a reflection.
- Draw several segments connecting points on the original shape to their images.
- What do you notice about the reflecting line and the segments?
- Label the points on your original shape and then label each of the reflections using prime notation.
- Has something has changed that did not change in a translation and a rotation?

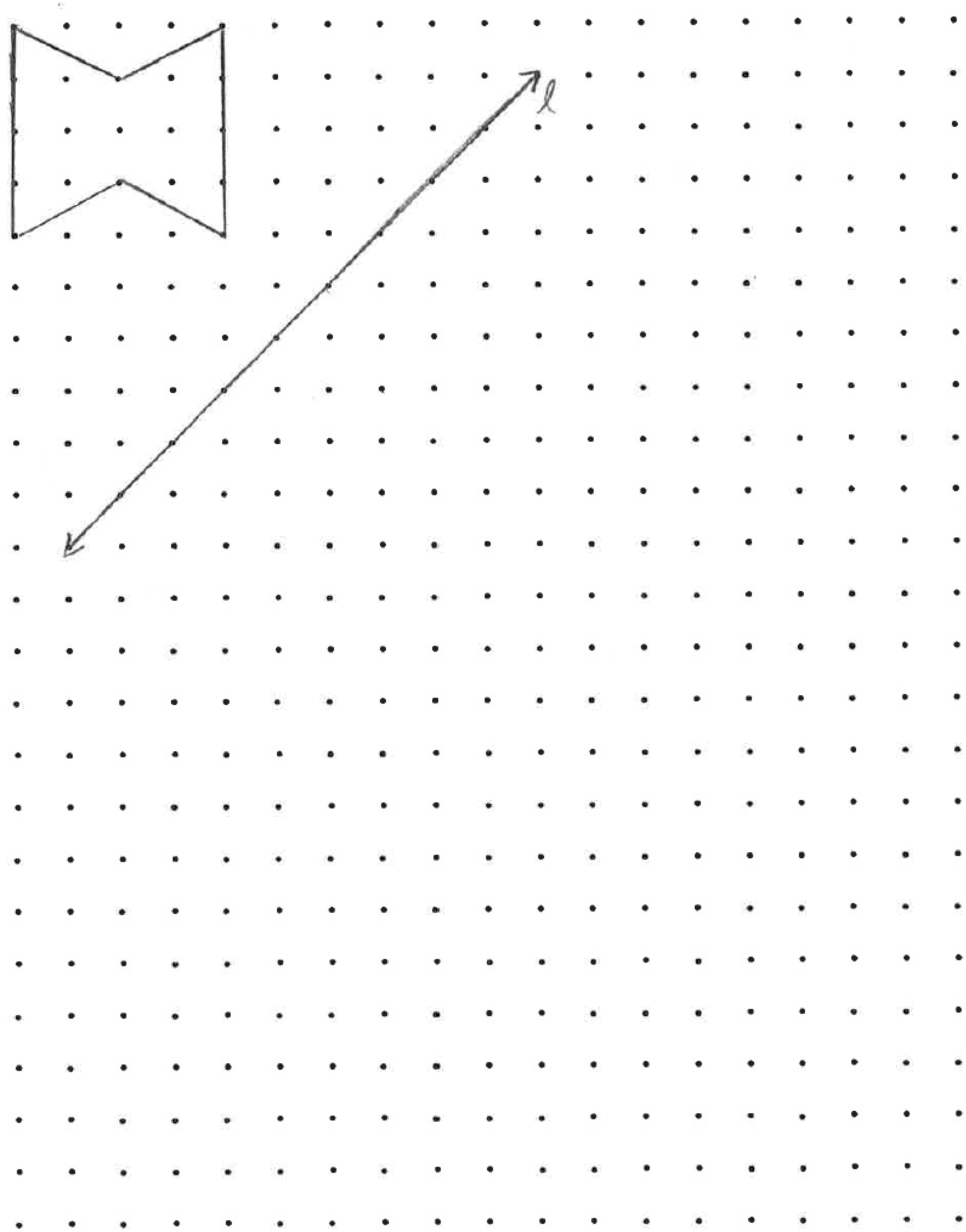
Rotation Conjectures

The line of reflection is the ______________________ of each segment connecting a point with its corresponding point in the reflected image.

In a rotation, the _______________________ of the points is_______________.

Comment on any other results you found interesting. Compare the results of a translation to those of a rotation or a translation. How are they similar? How are they different?

Activity 3



Activity 4 (Composition of Transformations)

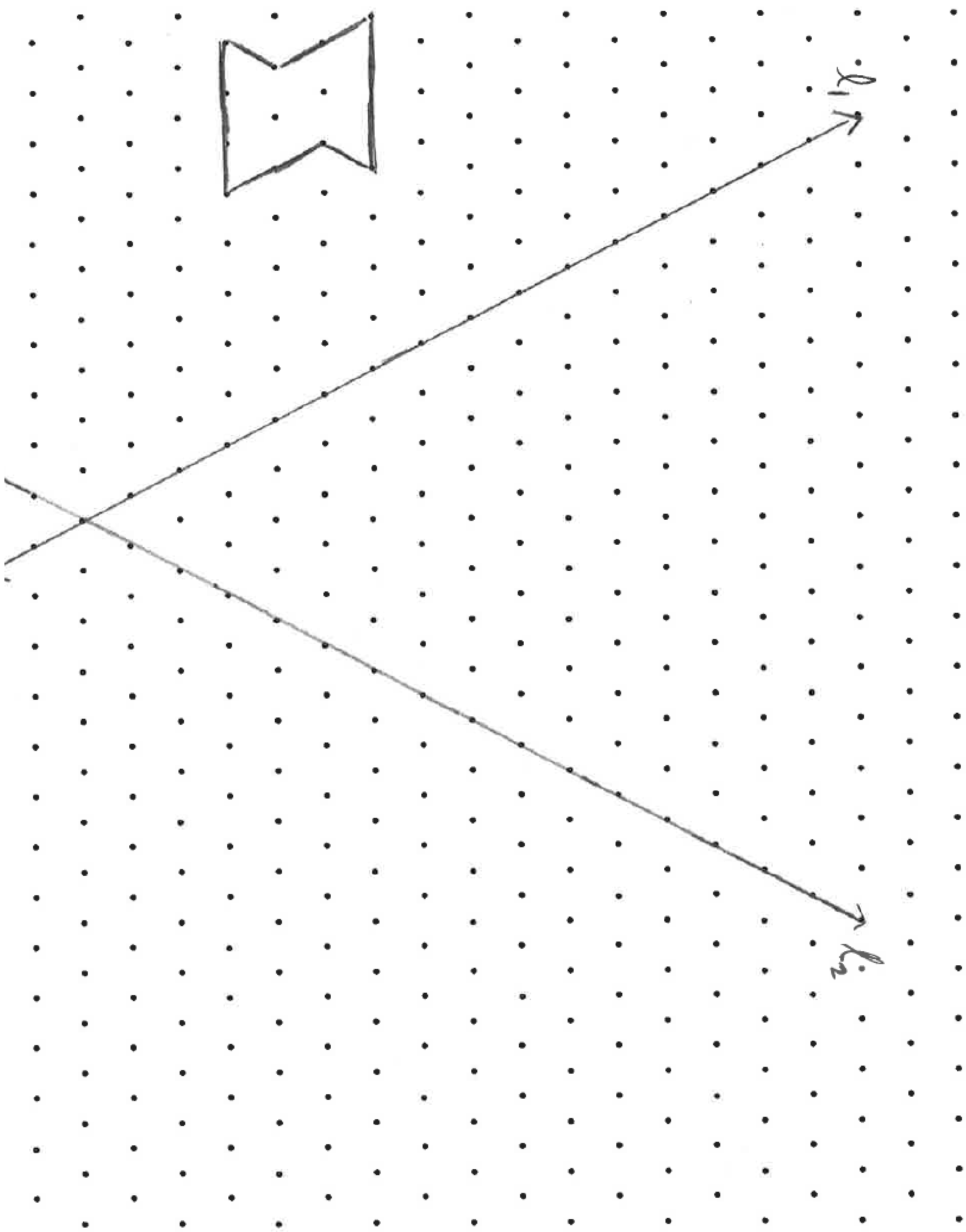
- A transformation is a function, and so it is possible to create new transformations through a composition of transformations.
- Perform a reflection of the shape across line *l*1 and then across line *l*2.
- Is there another single transformation that would have achieved the same results?
- Measure the angle of the two intersecting lines. Measure the angle of rotation.

Composition Conjecture

- Given two consecutive reflections over intersecting lines, there is a single ____________ that has the same result. The measure of the ______________ is ___________ the measure of the angle between the pair of intersecting lines.

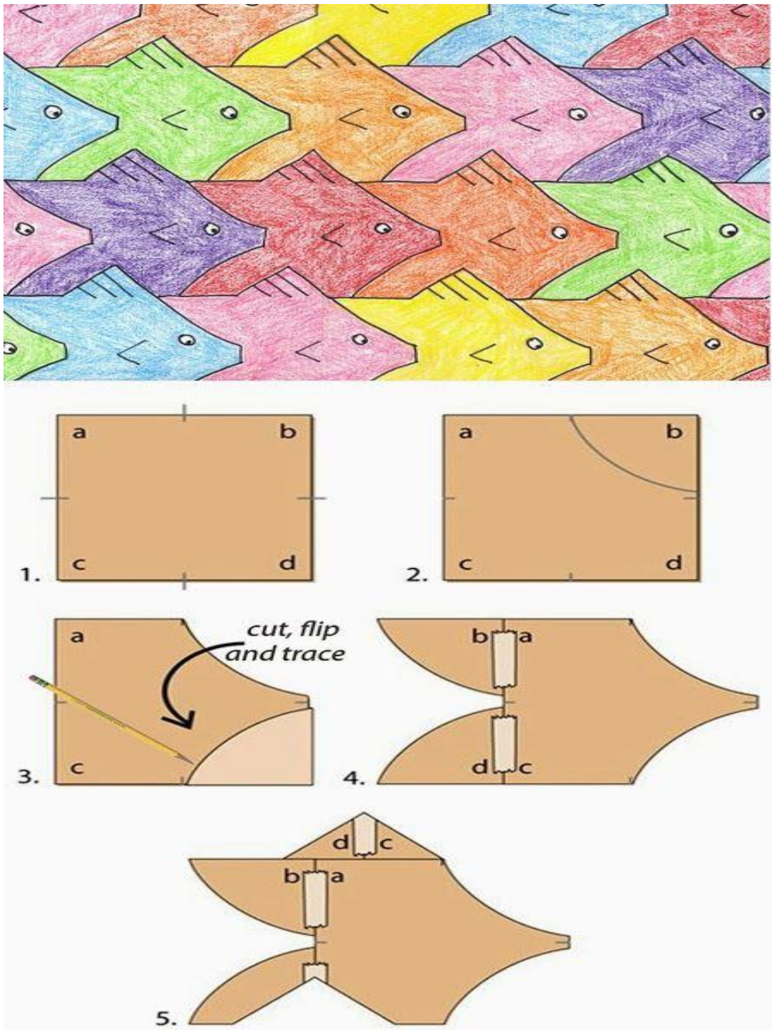
Comment on any other properties you may have noticed. Are there any other transformations that could be accomplished by more than one method?

Activity 4



Activity 5 (Tessellations)

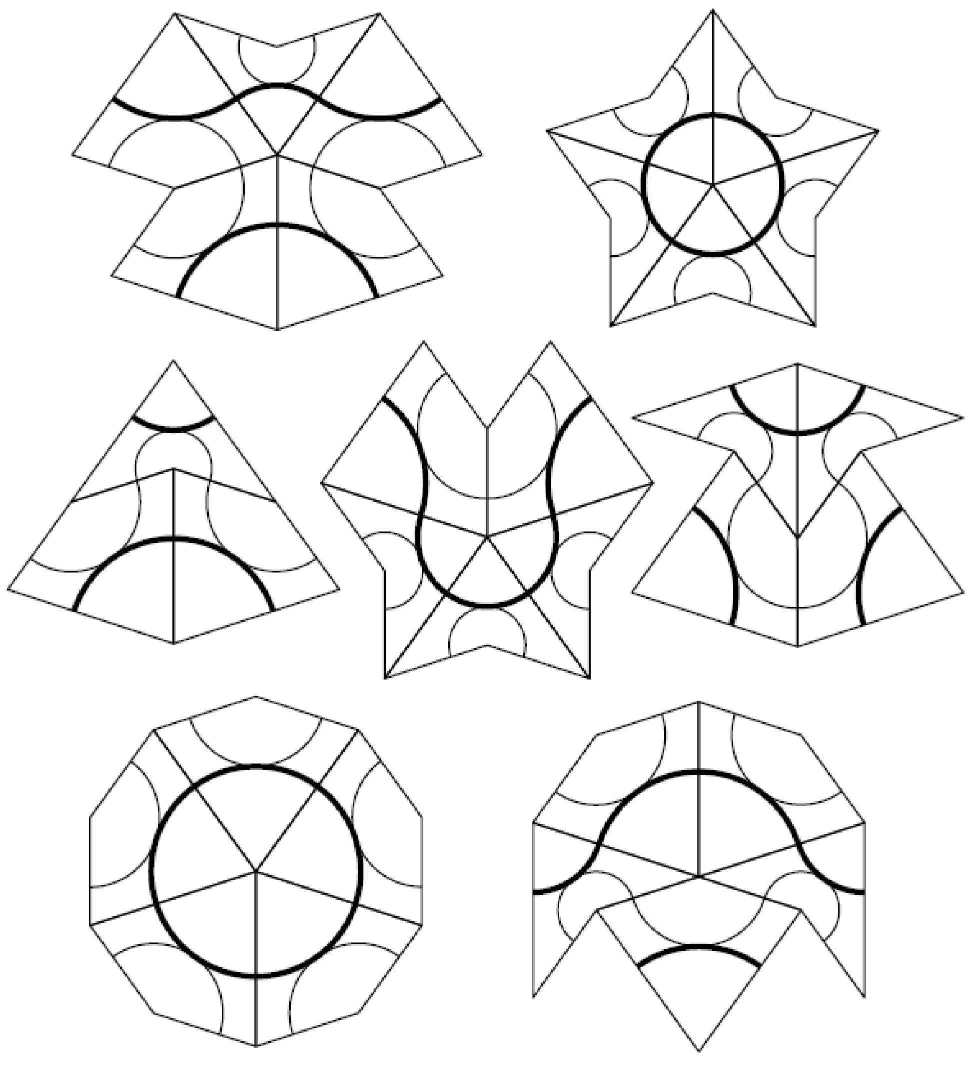
We will use a combination of patty paper, tracing paper, and dot paper to make the translation.



Activity 6 (Examining an Escher tessellation)

Activity 7 (Penrose Tilings)

Use the kites and darts to recreate the basic shapes below.



Overhead Transparency 1: Seven Arrangements

From the January 1999 *Mathematics Teacher*