Do The Math:
Like Your Life Depends On it

NCTM Regional Conference
Indianapolis, IN
October 30 & 31, 2014

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Investigation 7.5:
How Old Is That Artifact?

Did You Know?

In 2007, a reindeer herder in northwest Siberia, found the frozen carcass of a 6-month-old baby mammoth. Scientists believe the mammoth died at the end of the last Ice Age about 10,000 years ago. The baby's eyes and trunk were still intact, and the baby mammoth even still had fur on its body!

Reference: http://news.bbc.co.uk/2/hi/science/nature/6284214.stm

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You have probably watched or read news stories where they talk about fascinating ancient artifacts found by archaeologists. At an archaeological dig, objects such as stone tools, pottery, metal objects, buttons, jewelry, clothing, and bone fragments are often found. For example, you might hear that a piece of wooden tool has been unearthed and the archaeologist finds it to be 5,000 years old. You might learn that a child mummy was found high in the Andes and the archaeologist says the child lived more than 2,000 years ago. You might learn of the largest Tyrannosaurus Rex ever found that lived in what is now South Dakota, a dinosaur named Sue (named for the paleontologist who found it) and is believed to have lived between 60 and 70 million years ago.

How do scientists know how old an object or a bone fragment is? What methods do they use and how do these methods work?

Let's investigate how scientists use radioactivity to determine the age of objects by focusing on Carbon-14 dating. To determine the age of certain archaeological artifacts up to 50,000 years old, scientists often use Carbon-14 dating. However, scientists only use Carbon-14 dating to determine the age of objects that once were living, such as bone, cloth, wood, and plant remains. Carbon-12 and Carbon-14 are atoms found in all living organisms. As soon as a living organism dies, it stops taking in new Carbon. The ratio of Carbon-12 to Carbon-14 at the time of death is the same as the ratio of these atoms in all living things, but while the Carbon-12 remains, the Carbon-14 begins to decay and is not replaced.

HOW OLD IS AN ARTIFACT? (CONTINUED)

The rate at which Carbon-14 decays is predictable. Carbon-14 is an isotope that has a half-life of about 5700 years. This means that 5700 years after a living organism dies, it will have only half of the Carbon-14 it had at the time of its death. The rest will have decayed, and actually will have become Nitrogen-14 atoms. In the next 5700 years, it will lose half of what was remaining, so that after a living organism dies, the amount of Carbon-14 remaining 11,400 years later will be approximately one-fourth of the original amount. It continues to lose half of the remaining Carbon-14 every 5700 years. Meanwhile, the amount of Carbon-12 remains constant. By determining the ratio of Carbon-12 to Carbon-14 in a sample and comparing it to the ratio of a living organism, it is possible to determine the age of a formerly living organism to a reasonable degree of accuracy.

In this investigation, we will model the radioactive decay of an isotope such as Carbon-14, the process by which an unstable Carbon-14 atom loses energy by emitting radiation and decaying into a different type of atom.

Each group will need a paper plate and a cup of M&M's. The more M&M's you have, the better (for more than one reason!). Be careful! All of these M&M's are radioactive.

- Count the number of M&M's in the cup, and record this number as the amount of Carbon-14 atoms the organism has at the time of its death.
- Pour the M&M's on the paper plate. Remove all of the M&M's that have their M showing, and count them. Record this number in the table provided. This represents the number of Carbon-14 atoms that emitted radiation and became Nitrogen-14 atoms. These M&M's are no longer radioactive, and are now safe to eat if you choose.
- Next, put all of the radioactive M&M's back in the cup so you may again pour them onto the paper plate. Again, remove and count, all of the M&M's that have their M showing, and record this number. Continue this process until you have fewer than 5 radioactive M&M's left on the paper plate.

A Create a scatterplot of the number of radioactive M&M's as a function of the number of trials. Describe the graph.

B Can the number of radioactive M&M's with respect to the number of trials be modeled using an exponential function? Explain your reasoning.

C Write an algebraic equation to represent the number of radioactive M&M's that you would expect there to be as a function of the number of trials. What is the b-value of this exponential function? What is the value of a? What is the real-world meaning of both of these constants?

D Use regression to find an exponential function that models your real data.

E Compare your experimental equation with your theoretical equation.

F If the M&M's represent carbon-14 atoms, what does one trial represent? Explain.

Table for Investigation 7.5: **How Old Is That Artifact?**

Use this table to record the number of M&M's that remain and those that are no longer radioactive ("M" showing) for each trial.

TRIAL NUMBER	# OF RADIOACTIVE M&M'S REMAINING	# OF M&M'S NO LONGER RADIOACTIVE
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

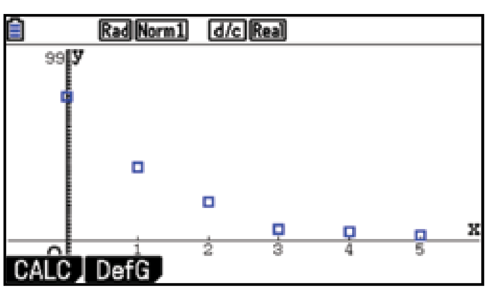
Sample Solution:
How Old Is That Artifact?

Results from one of our experiments are provided in the table below:

TRIAL NUMBER	# OF RADIOACTIVE M&M'S REMAINING	# OF M&M'S NO LONGER RADIOACTIVE
0	78	0
1	40	38
2	21	57
3	6	72
4	5	73
5	3	75
6	-	-
7	-	-
8	-	-
9	-	-
10	-	-

A Create a scatterplot of the number of radioactive M&M's as a function of the number of trials. Describe the graph.

Enter the Trial Number and the number of Radioactive M&M's remaining into two lists in the **Statistics** menu. Go to **GRAPH** and set up a scatterplot using the Trial Number as your **XList:** and the number of Radioactive M&M's remaining as your **YList:**. Note that we did not have that many trials before the number of M&M's was fewer than 5.



HOW OLD IS AN ARTIFACT? (CONTINUED)

We note that the graph is decreasing, that it is concave upward, meaning that it is decreasing at a slower and slower rate, and that it is solely in Quadrant 1.

B Can the number of radioactive M&M's with respect to the number of trials be modeled using an exponential function? Explain your reasoning.

Because we expect approximately half of the radioactive M&M's to remain after each trial, we have a geometric sequence. Whenever we have a constant multiplier (or a constant ratio between successive terms), we can model the situation with an exponential function. That is, a geometric sequence leads us to an exponential function.

C Write an algebraic equation to represent the number of radioactive M&M's that you would expect there to be as a function of the number of trials. What is the b-value of this exponential function? What is the value of a? What is the real-world meaning of both of these constants?

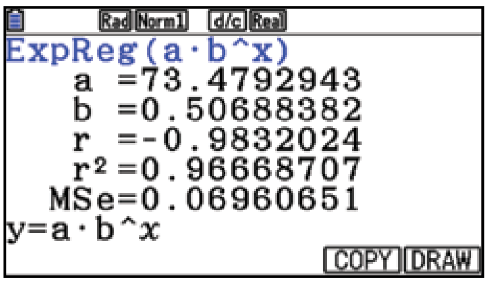
Let's assume we start with 78 M&M's. After 1 trial, we would expect $78 \cdot 0.5$ M&M's to still be radioactive. After 2 trials, we would expect $78 \cdot 0.5 \cdot 0.5$ M&M's to be radioactive. For each additional trial, we would multiply the previous result by 0.5, which leads to the equation $y = 78 \cdot 0.5^x$, where x represents the number of trials and y represents the number of radioactive M&M's. We note, however, that our model cannot be perfect. Suppose 39 are left after the first trial, then we would expect 19.5 after the second trial. We know we can't have a partial M&M; either an M&M is radioactive or it isn't.

In the general form, $y = ab^x$, for our specific equation, a is 78, the number of M&M's with which we start, and b is 0.5, our multiplier, the factor we expect to have remaining after each trial.

D Use regression to find an exponential function that models your real data.

From the scatterplot as shown above:

- Press F1 (CALC), F6, and F3 (EXP).
- Then press F2 (ab^x) to choose the exponential form mentioned above.



Our result using regression is $y \approx 73.48 \cdot 0.51^x$.

E Compare your experimental equation with your theoretical equation.

In theory, our value for a should have been 78; we were not that far off the mark. Our value for b, in theory, should have been 0.50, and here we were even closer. However, students should not believe that they were wrong if their regression model does not come as close to their theoretical model as ours. Perhaps we were just having a lucky day. In any case, we believe it is worth class time exploring how much variability there is in the models from each group. Developing a sense of variability is essential for statistical literacy, something we believe is critical for all students.

F If the M&M's represent carbon-14 atoms, what does one trial represent? Explain.

One trial would represent 5,700 years, since we expect approximately half of graphically and with a table.

Sherlock Holmes: Does Mary Survive?

Sketch Mary's motion:

What are the variables?

What assumptions should we make?

What information do you need to know & how can we find it?

Possible solution:

You've been entered into The Game. Instead of using your own cunning and skill, however, **random chance** controls your **fate**. Survive all three rounds and live. There is one twist that might offer you an advantage. You get to pick which game you want to enter.
Do you want to enter DEADLY DIE or FATAL CARDS?

DEADLY DIE

Round 1: Roll a die. ONE side is fatal: "1"
Roll anything but the number 1 and you survive to round 2.

Round 2: Roll a die. TWO sides are fatal: "1" and "2"
Roll anything but the number 1 or 2 and you survive to Round 3.

Round 3: Roll a die. THREE of the numbers are fatal: "1" , "2" , "3"
Roll any of the three and you're done.
Roll anything but those numbers and you survive the game.

Theoretical probability of **surviving** *Deadly Die*:

You've been entered into The Game. Instead of using your own cunning and skill, however, **random chance** controls your **fate**. Survive all three rounds and live. There is one twist that might offer you an advantage. You get to pick which game you want to enter.
Do you want to enter DEADLY DIE or FATAL CARDS?

FATAL CARDS

You're given a suit of cards. Pair up with a neighbor who can administer the rounds. Then switch.

Round 1: You pick one card. Two of the 13 cards are fatal: 2 and 3
Pick either of the fatal cards and you're done.
Pick any other card and you survive to round 2.

Round 2: The cards from Round 1 are replaced.
Two more cards are now fatal.
There are now four fatal cards: 2, 3, 9, and 10.
You pick one card.
Pick any of the four fatal cards and you're done.
Pick any other card besides those four and you survive to round 3.

Round 3: The cards from Round 2 are replaced.
Two more of the cards are now fatal.
There are now six fatal cards: 2, 3, 9, 10, J, A
You pick one card.
Pick any of those six cards and you're done.
Pick any other card besides those six and you SURVIVE!

Probability of **surviving** *Fatal Cards*:

One Versus Many

Devise and carry out an experiment to test the theory

- What are the variables?
- What information do you need to know & how can we find it?
-

One Versus Many

Option 1: By Myself!! One person builds an object with *n* pieces and runs the distance formed by *n* people.

# people/ # of blocks	time

Option 2: Team Work: *n* people build an object with *n* pieces while passing the object down a line formed by *n* people.

# people/ # of blocks	time

Pit and the Pendulum

What variables affect the period of a pendulum?

Constants:

Data Collection:

Length	Period (sec)			

Analysis

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