Turning College and Career-Ready Standards into Student Learning: What It Takes

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Teaching and Learning Beliefs Survey

SD = Strongly Disagree **D** = Disagree **A** = Agree **SA** = Strongly Agree

Belief	SD	D	A	SA
1. Mathematics learning should focus on practicing procedures and memorizing basic number combinations.				
2. The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.				
3. All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.				
4. The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.				
5. Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.				
6. An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.				
7. Students can learn to apply mathematics only after they have mastered the basic skills.				
8. Students can learn mathematics through exploring and solving contextual and mathematical problems.				
9. An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.				
10. The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.				
11. The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.				
12. Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.				

Adapted from *Principles to Actions: Ensuring Mathematical Success for All*, NCTM, 2014, p. 11.

Exploring Proportional Relationships:
The Case of Mr. Donnelly[1]

Mr. Donnelly wanted his students to understand that quantities that are in a proportional (multiplicative) relationship grow at a constant rate and that there were three key strategies that could be used to solve problems of this type – scaling up, scale factor, and unit rate. He selected the Candy Jar task for the lesson since it was aligned with his goals, was cognitively challenging, and had multiple entry points.

> A candy jar contains 5 Jolly Ranchers (JRs) and 13 Jawbreakers (JBs). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know.

As students began working with their partners on the task, Mr. Donnelly walked around the room stopping at different groups to listen in on their conversations and to ask questions as needed (e.g., How did you get that? How do you know that the new ratio is equivalent to the initial ratio?). When students struggled to figure out what to do he encouraged them to look at the work they had done the previous day that included producing a table of ratios equivalent to 5 JRs: 13 JBs and a unit rate of 1 JR to 2.6 JBs. He also encouraged students to consider how much bigger the new candy jar must be when compared to the original jar.

As he made his way around the room Mr. Donnelly also made note of the strategies students were using (see reverse side) so he could decide which groups he wanted to have present their work. After visiting each group, he decided that he would ask Groups 4, 5, and 2 to share their approaches (in this order) since each of these groups used one of the strategies he was targeting and the sequencing reflected the sophistication and frequency of strategies.

During the discussion he asked the presenters (one student from each of the targeted groups) to explain what their group did and why and he invited other students to consider whether the approach made sense and to ask questions. He made a point of labeling each of the three strategies, asking students which strategy was most efficient in solving this particular task, and asking students questions that helped them make connections between the different strategies and to the key ideas he was targeting. Specifically he wanted students to see that that the scale factor identified by Group 5 was the same as the number of entries in the table created by Group 4 (or the number of small candy jars that it would take to make the new candy jar) and that the unit rate identified by Group 2 was the factor that connected the JRs and JBs in each row of the table.

Below is an excerpt from the discussion that took place around the unit rate solution that was presented by Jerry from Group 2.

Jerry: We figured that there was 1 JR for 2.6 JBs so that a jar with 100 JRs would have 260 JBs. So we got the same thing as the other groups.
Mr. D.: Can you tell us how you figured out that there was 1 JR for 2.6 JBs?
Jerry: We divided 13 by 5.
Mr. D.: Does anyone have any questions for Jerry? Danielle?
Danielle: How did you know to do 13 ÷ 5?
Jerry: See we wanted to find out the number of JBs for 1 JR. So if we wanted JRs to be 1, we needed to divide it by 5. So now we needed to do the same thing to the JBs.
Danielle: So how did you then get 260 JBs?
Jerry: Well once we had 1 JR to 2.6 JBs it was easy to see that we needed to multiply each type of candy by 100 so we could get 100 JRs.

[1] This case is adapted from two sources: Smith, M. S., Silver, E. A., Stein, M. K., Boston, M., & Henningsen, M. A. (2005). *Improving instruction in rational numbers and proportionality: Using cases to transform mathematics teaching and learning, Volume 1*. New York: Teachers College Press; and Leinwand, S., Brahier, D., Huinker, D., Berry, R., Dillon, F., Larson, M., Leiva, M., Martin, G., & Smith, M.S. (2014). *Principles to Actions: Making Mathematics Work for All.* Reston, VA: National Council of Teachers of Mathematics.

Mr. D.: So Jerry's group multiplied by 100 but Danielle and her group (Group 5) multiplied by 20. Can they both be right? Amanda?

Amanda: Yes. Jerry's group multiplied 1 and 2.6 by 100 and Danielle and her group multiplied 5 and 13 by 20. Jerry's group multiplied by a number 5 times bigger than Danielle's group because their ratio was 1/5 the size of the ratio Danielle's group used. So it is the same thing.

Mr. D.: Do others agree with what Danielle is saying? *(Students are nodding their heads and giving Danielle a thumbs up.)* So what is important here is that both groups kept the ratio constant by multiplying both the JRs and JBs by the same amount. We call what Jerry and his group found the **unit rate**. A unit rate describes how many units of one quantity (in this case JBs) correspond to one unit of another quantity (in this case JRs). *(Mr. Donnelly writes this definition on the board.)*

Mr. D.: I am wondering if we can relate the unit rate to the table that Group 4 shared. Take 2 minutes and talk to your partner about this. (*2 minutes pass*)

Mr. D.: Kamiko and Jerilyn (from Group 4), can you tell us what you were talking about?

Kamiko: We noticed that if we looked at any row in our table that the number of JBs in the row was always 2.6 times the number of JRs in the same row.

Mike: Yeah we saw that too. So it looks like any number of JRs times 2.6 will give you the number of JBs.

Mr. D.: So what if we were looking for the number of JBs in a jar that had 1000 JRs?

Mike: Well the new jar would be 1000 times bigger so you multiply by 1000.

Mr. D.: So take 2 minutes and see if you and your partner can write a rule that we could use to find the number of JBs in a candy jar no matter how many JRs are in it.

(After 2 minutes the discussion continues.)

Towards the end of the lesson Mr. Donnelly placed the solution produced by Group 1 on the document camera and asked students to decide whether or not this was a viable approach to solving the task and to justify their answer. He told them they would have five minutes to write a response that he would collect as they exited the room. He thought that this would give him some insight as to whether or not individual students were coming to understand that ratios needed to grow at constant rate that was multiplicative not additive.

Group 1 (incorrect additive)

100 JRs is 95 more than the 5 we started with. So we will need 95 more JBs than the 13 I started with.

5 JRs + 95 JRs = 100 JRs
13 JBs + 95 JBs = 108 JBs

Groups 3 and 5 (scale factor)

You had to multiply the five JRs by 20 to get 100, so you'd also have to multiply the 13 JBs by 20 to get 260.

(x20)
5 JRs → 100 JRs
13 JBs → 260 JBs
(x20)

Groups 4 and 7 (scaling up)

JR	JB	JR	JB
5	13	55	143
10	26	60	156
15	39	65	169
20	52	70	182
25	65	75	195
30	78	80	208
35	91	85	221
40	104	90	234
45	117	95	247
50	130	100	260

Group 2 (unit rate)

Since the ratio is 5 JRs for 13 JBs, we divided 13 by 5 and got 2.6. So that would mean that for every 1 JR there are 2.6 JBs. So then you just multiply 2.6 by 100.

(x100)
1 JR → 100 JRs
2.6 JBs → 260 JBs
(x100)

Group 6 (scaling up)

JRs	5	10	20	40	80	100
JBs	13	26	52	104	208	260

We started by doubling both the number of JRs and JBs. But then when we got to 80 JRs we didn't want to double it anymore because we wanted to end up at 100 JRs and doubling 80 would give me too many. So we noticed that if we added 20 JRs: 52 JBs and 80 JRs: 208 JBs we would get 100 JRs:260 JBs.

Group 8 (scaling up)

We drew 100 JRs in groups of 5. Then we put 13 JBs with each group of 5 JRs. We then counted the number of JBs and found we had used 260 of them.