Young Children’s Learning Performance and Efficiency when Using Virtual Manipulative
Mathematics iPad Apps

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Part of a larger initiation mixed methods study (Greene, Caracelli, & Graham, 1989), this paper discusses the changes in young children’s learning performance and efficiency (one element of the quantitative portion of the larger study) during clinical interviews in which each child interacted with a variety of virtual manipulative mathematics iPad apps. Researchers interviewed 100 children ages 3 to 8 using a protocol format with two pre-assessment apps, four learning apps, and two post-assessment apps. Following 30-40 minute interviews where children interacted individually with the mathematics apps, results showed that children in the Preschool group increased efficiency while maintaining performance, children in the Kindergarten group increased performance while maintaining efficiency, and children in the Grade 2 group increased their performance and efficiency in skip counting, but not in place value. Overall, children in different age groups responded in different ways to the apps and some apps had a greater influence on children’s learning performance and efficiency than others. To conduct this study, researchers created a variety of research tools that were not readily available because research on children’s interactions with iPad-based virtual manipulative mathematics apps is still in its infancy. The paper describes the processes our research team used to develop new tools for novel and unique research situations.
Young Children’s Learning Performance and Efficiency when Using Virtual Manipulative Mathematics iPad Apps

Until recently, virtual manipulatives have only been available through mouse-driven apps for the computer. In the past few years, touch-screen devices have emerged as a platform for thousands of virtual manipulative mathematics apps. Researchers use a variety of terms to identify these mathematical objects that children interact with on touch-screen devices. Some researchers call them computer-based mathematical cognitive tools (MCTs) that contain interactive visual mathematical representations (VMRs) (Sedig & Liang, 2006). This paper uses the term virtual manipulatives (VMs), defined by Moyer, Bolyard, and Spikell (2002) as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373), to refer to the interactive dynamic mathematics objects that we selected for research in this project.

An important goal for mathematics education is the design and selection of mathematics “apps” (i.e., applications for mobile devices with a touch screen; Gröger, Silcher, Westkämper, & Mitschang, 2013) for use during instruction that result in positive learning benefits for students. An important goal for parents is to engage children in the use of learning apps that have positive learning benefits for their children. A recent analysis of the Education Category of Apple’s App Store commissioned by the Joan Ganz Cooney Center at Sesame Workshop (Shuler, 2012) revealed that, of the over 500,000 apps available on iTunes and over 300,000 on Android, 72% of the top selling apps target preschool or elementary aged children, and the second most popular subject for apps is mathematics (13%). Yet research on how these apps benefit children’s learning has not kept pace with the explosion of apps being created. This study sought
to contribute to the research on how mathematics apps for young children influence learning. The purpose of this study was to examine young children’s learning performance and efficiency during clinical interviews in which each child interacted with a variety of virtual manipulative mathematics iPad apps (which we will call “VM iPad apps” throughout this paper). The use of touch-screen devices in preK-12 mathematics teaching and learning will have broad applications and implications for mathematics instruction in the next decade.

**Theoretical Perspective**

Support for the use of virtual manipulatives on touch-screen devices is drawn from theories about the use of representations (e.g., internal and external representations) and how physical interactions with representations leads to learning (Manches & O’Malley, 2012). External representations in mathematics are “physically embodied, observable configurations such as words, graphs, pictures, equations, or computer microworlds” (Goldin & Kaput, 1996). Internal representations “do not only encode or represent what is external…” (p.277), they may also include “sensations and perceptions visualized or otherwise imagined objects or symbols, or even emotional feelings” (Goldin, 2003, p. 277). In theory, external representations are thought to bridge virtual or physical objects with abstract symbols. Bruner (1964) proposed that during cognitive development children make sense of their world through enactive means (i.e., the manipulation of physical objects) that then connect with iconic (visual images, pictures) and symbolic (words, numbers, symbols) representations. Providing young children with opportunities to interact with external representations is important to children’s cognitive development (Martin & Schwartz, 2005).

External representations have advantages for learning based on how they support computational offloading, re-representation, and graphical constraining (Belenky & Schalk,
Combining external representations through technology can benefit learners who understand the individual representations and relationships among the representations (Ainsworth, 2006). Ainsworth (1999, 2006) suggests the three functions of multiple external representations (MERs) are to complement (i.e., combine different processes or information), constrain (i.e., one representation can clarify ambiguous information provided by another), and construct (i.e., combine to help foster insights difficult to achieve with a single representation). A virtual manipulative, or dynamic interactive representation of an object, is an external representation. In a VM iPad app, the external representation is housed in a technological platform with a variety of features that provide affordances (Gibson, 1986) for the user that allow the user to complement, constrain, and construct with the virtual manipulative.

However, proponents of embodied cognition argue that, if we describe children as drawing upon their physical manipulations of externalized representations to make connections to abstractions, then we are creating an artificial separation between representation and abstraction (Lakoff & Nunez, 2000), and we agree with this concern. Our position on the use of virtual manipulatives for touch-screen devices is that tool fluency and mathematical understanding are not simply a process of internalizing concrete artifacts into mental representations (Nemirovsky, Kelton, & Rhodehamel, 2013). We concur with Nemirovsky et al. that when children are interacting with representations on touch screen devices, there can be no separation between mind and body – that action and thought are intertwined. Interacting with externalized representations is thought and abstraction, and children’s interactions with representations simultaneously generate thought, just as children’s physical gestures with the
objects on a touch-screen device activate embodied processes. Manipulating objects on a touch-screen device is not a precursor to mathematical thought – it is, itself, mathematical thought.

**Literature Review**

An extensive search of the literature reveals that most studies that examine mathematics learning on touch-screen devices, like Apple’s iPad, are fairly recent. Paek and colleagues (Paek, 2012; Paek, Hoffman, & Black, 2013; Paek, Hoffman, Saravanos, Black, & Kinzer, 2011) conducted the most detailed series of quantitative studies of a virtual manipulative mathematics app. Paek reported results from three iterations of studies connecting feedback (i.e., audio and visual) and interaction types (i.e., touch versus mouse) in first- and second-grade students’ learning with a virtual manipulative app. The virtual manipulative, Puzzle Blocks, focused on introducing multiplication and was designed to have similar interactions for both the touch- and mouse-based environments. Far-transfer comparisons revealed that the presence of audio feedback was a statistically significant factor on the mid-test, while the use by students of the touch-screen interaction was a statistically significant factor for the post-test. Students reported high motivation regarding learning mathematics, multiplication, and playing Puzzle Blocks throughout the study. Generally, the results of this series of studies suggest that audio feedback might be important at first, but that the interaction type (i.e., using the touch-screen over the mouse) may have more influence long-term.

Two studies, both using a repeated measures cross-over design, measured performance and motivation related to specific app use. In a study involving 160 Pre-K students in eight classes, Spencer (2013) found that using the Know Number Free app led to statistically significant growth in performance relative to traditional instruction, while children’s motivation levels remained high throughout the study. Riconscente (2012) measured the performance and
attitudes of 122 fifth-grade students playing the fraction app MotionMath. Riconscente reported statistically significant growth in performance scores on fraction tasks after students played MotionMath. There were also improvements in attitude measures for self-reported fraction knowledge and attitude toward fractions. In each study, VM iPad apps led to statistically significant growth in students’ performance on mathematics assessments and children’s attitude scores remained high.

Barendregt, Lindström, Rietz-Leppänen, Holgersson, and Ottosson (2012) designed and piloted an iPad app intended to help develop conceptual subitizing skills, and concluded that the app helped different students develop different skills in subitizing. Kiger, Hierro, and Prunty (2012) examined the use of iPod Touch devices as supplemental practice tools for children to use at home. They found that the mobile learning interventions led to a statistically significant difference in performance over students who used the standard curriculum materials.

Other studies are much more general in nature and focus simply on the use of the iPad for mathematics instruction. For example, Liu (2013) found that students using iPads in mathematics had higher achievement and motivational scores than students that did not use iPads. However, other studies (e.g., Carr, 2012; McKenna, 2012; Wilson, Nash, Wissinger, & Leidman, 2013; Zimmerman & Howard, 2013) contradict Liu’s findings showing no significant differences when students use iPads for mathematics learning.

Another body of research on iPad apps includes reviews and evaluations of the apps. For example, Highfield and Goodwin (2013) reviewed 53 mathematics apps, concluding that the vast majority of apps focused on drill and practice, whereas none of the apps allowed for creation of content. Larkin (2013) classified 142 apps by content and knowledge focus, and evaluated their technological and pedagogical characteristics, classifying seven apps in the top ten on both
measures. Other reviewers describe characteristics of fewer apps, focusing on technological aspects such as multi-touch interactivity (Rick, 2012), evaluating feedback (Bartoschek, Schwering, Li, & Münzer, 2013; Blair, 2013), or classifying them by structure (Zanchi, Presser, & Vahey, 2013). Although these studies investigated general use, they did not connect specific apps with student outcomes. These studies show that the use of touch-screen devices for preK-12 mathematics learning is in its infancy.

**Methods**

This paper asserts that children’s interactions with externalized representations (i.e., VM iPad apps) is an embodied process, and as such, is not a precursor to mathematical thought – it is mathematical thought. To investigate this assertion, our larger study, from which the results presented here were drawn, used an initiation mixed methods research design as analysis of quantitative data and qualitative data differed in researchers’ perspectives (Greene, Caracelli, & Graham, 1989). Researchers’ perspectives of the analysis of children’s performance and efficiency (quantitative) when engaging with VM iPad apps were meant to broadly explain the embodied relationships between children’s internal and external representations, which is the subject of this paper. Analysis of children’s words, actions, and gestures (qualitative) provided researchers with a more in-depth perspective of the nuances present in the embodied aspects of the children’s internal and external representations, and are presented in a separate paper (Moyer-Packenham, Anderson, Shumway, Tucker, Westenskow, Boyer-Thurgood, Bullock, Mahamane, Baker, Gulkilik, Maahs-Fladung, Symanzik, & Jordan, forthcoming).

The research question reported in this paper was: How do interactions with VM iPad apps immediately impact young children’s learning performance and efficiency? Learning performance was defined as accuracy on pre- and post-assessment tasks using VM iPad apps.
Learning efficiency was defined as changes in the speed with which children completed a variety of learning tasks using the VM iPad apps.

Participants

One hundred children (Preschool, ages 3-4, N = 35; Kindergarten, ages 5-6, N = 33; Grade 2, ages 7-8, N = 32) were recruited from local public and charter elementary schools, the university campus lab school, and the university campus preschools using informational brochures and letters. Children’s ethnicities included Asian (1%), Caucasian (89%), Hispanic (2%), and Mixed Race (8%). Thirty-four percent of children’s parents reported them receiving free- or reduced-lunch at school. Children’s parents reported on the use of personal touchscreen devices (PTDs) in the home with 11% having more than five PTDs, 78% with between one and four, and 8% with none. Thirteen percent of the children had access to their own PTD at home. Parents reported that the children used the PTDs every day (45%), 4–5 days per week (2%), 1–3 days per week (40%), and never (10%).

Pre-Study Piloting Activities

Selecting the VM iPad apps. Prior to initiating the study, researchers carried out a variety of piloting activities that included selecting the VM iPad apps, designing instruments, and developing data collection protocols for the study. First researchers reviewed hundreds of mathematics apps to select for inclusion. To be included, apps needed to contain a virtual manipulative (i.e., interactive visual representation of a dynamic object) with some of the affordances linked to positive learning effects identified by Moyer-Packenham and Westenskow (2013):

*focused constraint* (i.e., VMs focus and constrain student attention on mathematical objects and processes), *creative variation* (i.e., VMs encourage creativity and increase the
variety of students’ solutions), simultaneous linking (i.e., VMs simultaneously link representations with each other and with students’ actions), efficient precision (i.e., VMs contain precise representations allowing accurate and efficient use), and motivation (i.e., VMs motivate students to persist at mathematical tasks) (p. 35).

Following app selection, members of the research team tested the apps with children in local schools in informal one-to-one interactions. During the interactions, researchers observed how children interacted with the apps and took notes on app features, difficulties children had, ease of use, and learning opportunities presented by the apps.

Six apps were selected for each age group (i.e., Preschool, Kindergarten, Grade 2), a total of 18 apps. Apps were selected so that each group used three apps for the base-10 quantities tasks and three apps for additional mathematical concept tasks (e.g., seriation for Preschool; subitizing for Kindergarten; skip counting for Grade 2). Seriation, the ability to sort and order objects according to a defined attribute (often by length or size), is a logical reasoning task that is commonly used in early numeracy assessments. Conceptual subitizing is the ability to perceive amounts without counting, and combine amounts to name a total amount. These skills are interdependent with later mathematics skills, such as skip counting (e.g., 2, 4, 6, 8, 10, …) and understanding the base-10 place value system (Sarama & Clements, 2009). Appendix A shows the VM iPad apps selected for the study.

Creating the interview and observation protocols. Next researchers created and refined three different interview protocols for each of the three age groups (Preschool, Kindergarten, and Grade 2). Schubert (2009) suggests that the development of these protocols be based on current theories related to the phenomenon of interest and the researcher's own experience with observing the phenomenon. Members of the research team piloted the interview
protocols with children in local schools and revised the protocols based on tasks that were too easy or too difficult. Appendix B shows a portion of the Kindergarten protocol for quantity.

The research team also developed an observation protocol for the purpose of capturing information in real time during the interview and supplementing the video data. The observation protocol asked observers to make note of verbal utterances (including terminology), counting and number strategies used, purposeful or unhelpful movements (hand gestures or virtual manipulatives), errors and correction of errors, affective responses, and interviewer actions and responses, and anything unique or noteworthy that happened during the interview. This protocol was piloted during 10 clinical interviews with children.

**Practicing data collection procedures.** Next, members of the research team were trained as interviewers and observers using a final draft of each interview and observation protocol. Training sessions were conducted in clinical rooms at the university where the interviews in the study would take place. Some members of the research team took the interviewer’s role in the training sessions; other members of the team took the child’s role. This allowed team members to practice interviewing skills and become familiar with the protocols.

An interaction protocol was developed to guide members of the team on interactions with parents and children during data collection. The interaction protocol guided researchers on what to do and say while interfacing with parents and children, ensuring that each interview was conducted in a similar manner. The protocol required the team to explain interview procedures to the parent and child, show the parent and child the interview room and observation booth, and ask the parent to read and sign consent forms. Parents had the option of staying with their child in the interview room or observing their child from the observation booth. At the end of the interview, researchers gave parents a one-page summary of the research project and gave each
child a small toy for participating in the project. The final step in preparing for data collection was to conduct pilot interviews with 10 children in the clinical interview rooms. These interviews allowed everyone to practice and finalize all procedures and protocols. It also allowed the technical team opportunities to test the camera recording equipment.

**Study Procedures**

Children participated in 20-35 minute one-to-one clinical interviews in a room equipped with two-way mirrors, audio observer booths, and built-in wall-mounted video cameras. During the interview, one member of the research team conducted the interview with the child while another member of the research team recorded observations from the observation booth. The members of the research team that conducted the interviews with children all had teaching experience and expertise conducting mathematics clinical interviews. It took four months to conduct the 100 interviews.

During interviews each child used six different VM iPad apps. Interviews included two learning sequences on two different mathematics concepts. Table 1 shows the progression of each interview with each age group and includes the mathematics app names that were used during the interviews. The entire interview sequence took place during one interview session with each child. For example, during the Preschool interview, the interviewer used the *Pink tower: free moving* app as a pre-assessment of seriation, children completed several learning tasks on seriation using the *Pink tower: tapping* and the *Red rods* apps, and the interviewer used the *Pink tower: free moving* app as a post-assessment of seriation – this covered the first half of the interview for Preschool. Next, the interviewer used the *Base-10 blocks* app as a pre-assessment of quantity, children completed several learning tasks on quantity using the *Base-10
blocks: 1-5 and Base-10 blocks: numerals apps, and the interviewer used the Base-10 blocks app as a post-assessment of quantity – this covered the second half of the interview.

Table 1

Sequence of the Interviews

<table>
<thead>
<tr>
<th>Interview</th>
<th>Preschool</th>
<th>Kindergarten</th>
<th>Grade 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>App #1 (pre)</td>
<td>Pink tower: free moving</td>
<td>10 Frame</td>
<td>100s chart</td>
</tr>
<tr>
<td>App #2 (learning)</td>
<td>Pink tower: tapping</td>
<td>Hungry guppy</td>
<td>Frog number line</td>
</tr>
<tr>
<td>App #3 (learning)</td>
<td>Red rods</td>
<td>Fingu</td>
<td>Counting beads</td>
</tr>
<tr>
<td>App #1 (post)</td>
<td>Pink tower: free moving</td>
<td>10 Frame</td>
<td>100s chart</td>
</tr>
<tr>
<td>App #4 (pre)</td>
<td>Base-10 blocks</td>
<td>Base-10 blocks</td>
<td>Base-10 blocks</td>
</tr>
<tr>
<td>App #5 (learning)</td>
<td>Base-10 blocks: 1-5</td>
<td>Base-10 blocks: 11-20</td>
<td>Zoom number line</td>
</tr>
<tr>
<td>App #6 (learning)</td>
<td>Base-10 blocks: numerals</td>
<td>Base-10 blocks: numerals</td>
<td>Place value cards</td>
</tr>
<tr>
<td>App #4 (post)</td>
<td>Base-10 blocks</td>
<td>Base-10 blocks</td>
<td>Base-10 blocks</td>
</tr>
</tbody>
</table>

Instruments/Data Sources

The primary instruments used to collect data during the study were pre- and post-task-based assessments, and two methods of capturing video data: (a) one wall-mounted camera in the interview room, and (b) one wearable GoPro camera attached to the child that captured an up-close perspective of the child’s interactions with the VM iPad apps. The iPad-based assessments were created for each grade-level group and for each mathematics app (i.e., 18 iPad-based assessments). Children completed one to five iPad-based tasks with each app. Each child was equipped with a wearable GoPro camera that was positioned to be pointed at their interactions on
the iPad touch screen. This screen-capture recording process was positioned approximately six inches away from the iPad and captured all of the on-screen motions of the VM iPad apps initiated by the children. (See Figure 1.)

![Figure 1. Child with wearable GoPro camera.](image)

Wall-mounted video recordings of children and the interviewer were positioned approximately six feet away from participants and captured all actions and interactions during the interviews, providing a second perspective (Roschelle, 2000).

**Analysis**

All session videos were digitized and accessed through a password-protected site. Children’s pre and post videos were coded for various elements of learning performance (i.e., changes in accuracy during the tasks) and efficiency (i.e., changes in speed during the tasks). Coding protocols were developed by groups of researchers who met to identify actions in the video data indicating measures of learning performance and efficiency that could be identified in both the pre- and post-assessment portions of the interviews. Six unique pre-post coding
protocols were developed for each concept (2) and each age group (3) to accommodate the different apps and tasks that were used. Researchers tested the analysis tools by examining three interviews within each age group. After testing each analysis tool and making adjustments to improve the precision of the tools, the protocols were used to code the entire set of 100 interviews. To ensure reliable and accurate coding, 20% of the pre-post assessment videos were double-coded by two researchers.

After coding the pre and post assessment data from the videos, researchers explored the data using SPSS. Because data were not normally distributed, pre- and post-assessments were analyzed using the Wilcoxon Signed Ranked Test. This non-parametric statistical test uses the median of related samples (e.g., pre- and post-assessment scores) to compare data sets. Our results in this paper focus specifically on children’s learning performance (i.e., accuracy) and efficiency (i.e., speed) during the pre- and post-assessment portions of the interviews. Other papers from the larger research project detail children’s learning progressions, explore app affordances and their influence on learning, and describe specific strategies children used during their interactions with the VM iPad apps (Boyer-Thurgood, Moyer-Packenham, Tucker, Anderson, Shumway, Westenskow, & Bullock; 2014; Moyer-Packenham et al., forthcoming; Tucker, Moyer-Packenham, Boyer-Thurgood, Anderson, Shumway, Westenskow, & Bullock, 2014; Tucker, Moyer-Packenham & Jordan, 2014).

**Results**

Our overarching research question in this study focused on how interactions with VM iPad apps immediately impact young children’s learning performance and efficiency. The results presented below are reported by grade level (Preschool, Kindergarten, and Grade 2) and mathematical content tasks (e.g., seriation, subitizing, skip counting, quantity). Each section
presents the means and standard deviations of children’s learning performance and efficiency results, as well as a calculation of the change in children’s scores on the pre- and post-assessment tasks. We hypothesized that learning performance scores would increase, meaning children would improve in terms of their accuracy and performance on the tasks. We also hypothesized that efficiency scores would decrease, meaning children would become faster in their performance during the tasks.

**Preschool Seriation Pre- and Post-Assessment Tasks**

Table 2 displays the results for Preschool during their completion of the Seriation task.

Table 2

*Preschool Means, Standard Deviations, and Change in Learning Performance and Efficiency on the Seriation Task*

<table>
<thead>
<tr>
<th>Learning Measure</th>
<th>Pre Mean (SD)</th>
<th>Post Mean (SD)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (number of correct blocks)</td>
<td>.56 (.38)</td>
<td>.59 (.36)</td>
<td>+ .03</td>
</tr>
<tr>
<td>B (number of correct moves)</td>
<td>.56 (.38)</td>
<td>.56 (.37)</td>
<td>--</td>
</tr>
<tr>
<td>C (how far off)</td>
<td>.67 (.31)</td>
<td>.73 (.28)</td>
<td>+ .06</td>
</tr>
<tr>
<td>D (seconds per block)</td>
<td>5.86 (3.28)</td>
<td>5.23 (4.16)</td>
<td>- .63*</td>
</tr>
</tbody>
</table>

** < .01; * < .05

Measure A recorded the number of blocks correct until a permanent error occurred. Measure B recorded the number of correct moves out of total moves. Measure C recorded how far away from the correct sized block each block was placed. Measure D, the efficiency measure, recorded the average seconds per block it took the child to complete the task. As Table 2 shows,
Preschool children’s learning performance did not significantly increase on any of the performance measures. However, on the efficiency measure, children showed a significant decrease in speed between the pre and post assessments, indicating that children became significantly faster on the task while maintaining the same levels of performance accuracy.

Preschool Quantity Pre- and Post-Assessment Tasks

Table 3 displays the results for Preschool during their completion of the Quantity task.

Table 3

*Preschool Means, Standard Deviations, and Change in Learning Performance and Efficiency on the Quantity Task*

<table>
<thead>
<tr>
<th>Learning Measure</th>
<th>Performance</th>
<th>Pre Mean (SD)</th>
<th>Post Mean (SD)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E (verbal counting)</td>
<td>.49 (.34)</td>
<td>.52 (.33)</td>
<td>+ .03</td>
<td></td>
</tr>
<tr>
<td>F (cardinality 2-5)</td>
<td>.97 (.06)</td>
<td>.93 (.19)</td>
<td>- .04</td>
<td></td>
</tr>
<tr>
<td>G (cardinality 6-9)</td>
<td>.67 (.31)</td>
<td>.73 (.28)</td>
<td>+ .06</td>
<td></td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H (seconds for 2-5)</td>
<td>3.35 (1.59)</td>
<td>2.61 (2.63)</td>
<td>- .74**</td>
<td></td>
</tr>
<tr>
<td>I (seconds for 6-9)</td>
<td>3.34 (1.45)</td>
<td>2.69 (1.02)</td>
<td>- .65**</td>
<td></td>
</tr>
</tbody>
</table>

** < .01; * < .05

Measure E recorded the highest number the child could count in a sequence before they made a permanent error up to 25 (i.e. the interviewer stopped the child at 25). Measure F recorded how close the child was to counting the target number of blocks for quantities two, three, four, or five. Measure G recorded how close the child was to counting the target number of blocks for quantities six, seven, eight, or nine. Measure H, the first efficiency measure, recorded
the average number of seconds it took the child to count the target quantity for quantities two, three, four, or five. Measure I, the second efficiency measure, recorded the average number of seconds it took the child to count the target quantity for quantities six, seven, eight, or nine. As Table 3 shows, there were no significant changes in Preschool children’s learning performance for quantity, but their efficiency score decreased significantly, meaning that they were able to complete the task faster on the post-assessment than on the pre-assessment.

**Kindergarten Subitizing Pre- and Post-Assessment Tasks**

Table 4 displays the results for Kindergarten during their Subitizing task.

Table 4

*Kindergarten Means, Standard Deviations, and Change in Learning Performance and Efficiency on the Subitizing Task*

<table>
<thead>
<tr>
<th>Learning Measure</th>
<th>Pre Mean (SD)</th>
<th>Post Mean (SD)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A (subitize)</td>
<td>.99 (.05)</td>
<td>.96 (.16)</td>
<td>- .03</td>
</tr>
<tr>
<td>B (counting on)</td>
<td>.90 (.20)</td>
<td>.96 (.11)</td>
<td>+ .06</td>
</tr>
<tr>
<td>C (the difference)</td>
<td>.82 (.30)</td>
<td>.94 (.14)</td>
<td>+ .12*</td>
</tr>
<tr>
<td>Efficiency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D (seconds per disk)</td>
<td>2.22 (.86)</td>
<td>2.24 (1.91)</td>
<td>+ .02</td>
</tr>
</tbody>
</table>

* < .01; ** < .05

Measure A recorded how far away the child was from identifying the correct number of color disks. Measure B recorded how far away the child was from the target number after counting on (or adding) disks. Measure C recorded the difference between the subitized number and the target number, which was how many disks they moved to make the target number.
Measure D, the efficiency measure, recorded how many seconds per color disk was moved during the task. As Table 4 shows, Kindergarten children’s learning performance significantly increased on Measure C. It is also important to note that Measures A and B had relatively high pre-assessment scores. There were no significant changes in efficiency for Kindergarten children on the Subitizing task. This indicates that, with the same amount of efficiency, Kindergarten children became more accurate on Measure C for the Subitizing task.

**Kindergarten Quantity Pre- and Post-Assessment Tasks**

Table 5 displays results for Kindergarten during their completion of the Quantity task.

Table 5

*Kindergarten Means, Standard Deviations, and Change in Learning Performance and Efficiency on the Quantity Task*

<table>
<thead>
<tr>
<th>Learning Measure</th>
<th>Performance</th>
<th>Pre Mean (SD)</th>
<th>Post Mean (SD)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E (count ones/tens)</td>
<td>.77 (.35)</td>
<td>.80 (.36)</td>
<td>+ .03*</td>
</tr>
<tr>
<td></td>
<td>F (use tens rod for tens)</td>
<td>.76 (.33)</td>
<td>.74 (.35)</td>
<td>-.02</td>
</tr>
<tr>
<td></td>
<td>G (identify ones/tens)</td>
<td>.76 (.28)</td>
<td>.70 (.33)</td>
<td>-.06</td>
</tr>
</tbody>
</table>

**Efficiency**

|                  | H (seconds per block) | 2.92 (1.94)     | 2.09 (.99)      | - .83* |
|                  | I (seconds)           | 30 (32.93)      | 24 (32.07)      | - 6    |

** < .01; * < .05

Measure E recorded how close the number of blocks the child counted was to the target number of blocks. Measure F recorded how far away the child was from the most efficient number of moves. Measure G was a two part task that recorded whether or not the child
identified the correct amount of tens in a given number and whether or not the child used a tens rod. Efficiency measure H recorded how many seconds per block it took the child to build the target number. Efficiency measure I recorded, in seconds, how long it took the child to build the target number. As Table 5 shows, Kindergarteners’ learning performance significantly increased on measure E. In addition, measure H shows a significant change in efficiency from pre- to post-assessment, indicating that children became faster on the task during the post-assessment.

**Grade 2 Skip Counting Pre- and Post-Assessment Tasks**

Table 6 displays the results for Grade 2 during their completion of the Skip Counting task.

**Table 6**

*Grade 2 Means, Standard Deviations, and Change in Learning Performance and Efficiency on the Skip Counting Task*

<table>
<thead>
<tr>
<th>Learning Measure</th>
<th>Performance</th>
<th>Pre Mean (SD)</th>
<th>Post Mean (SD)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A (skip counting by 4s)</td>
<td>.75 (.36)</td>
<td>.86 (.31)</td>
<td>+ .11*</td>
<td></td>
</tr>
<tr>
<td>B (skip counting by 6s)</td>
<td>.95 (.17)</td>
<td>.98 (.12)</td>
<td>+ .03</td>
<td></td>
</tr>
<tr>
<td>C (skip counting by 9s)</td>
<td>.77 (.33)</td>
<td>.85 (.28)</td>
<td>+ .08*</td>
<td></td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D (seconds per touch; 4s)</td>
<td>2.21 (.74)</td>
<td>1.80 (.94)</td>
<td>- .41**</td>
<td></td>
</tr>
<tr>
<td>E (seconds per touch; 6s)</td>
<td>2.37 (1.44)</td>
<td>2.16 (1.26)</td>
<td>- .21</td>
<td></td>
</tr>
<tr>
<td>F (seconds per touch; 9s)</td>
<td>4.24 (3.83)</td>
<td>3.56 (4.61)</td>
<td>- .68*</td>
<td></td>
</tr>
</tbody>
</table>

** < .01; * < .05

Measure A recorded how many numbers were correct when skip counting by 4s to 28 until a permanent error occurred. Measure B recorded how many numbers were correct when
skip counting by 6s to 18 until a permanent error occurred. Measure C recorded how many numbers were correct when skip counting by 9s to 36 until a permanent error occurred. Measure D recorded how many seconds per touch it took the child to touch each number on the hundreds board app when skip counting by 4s. Measure E recorded how many seconds per touch it took the child to touch each number skip counting by 6s. Measure F recorded how many seconds per touch it took the child to touch each number skip counting by 9s. As Table 6 shows, Grade 2 children’s learning performance increased significantly when skip counting by 4s and 9s. There was a significant decrease in speed from pre- to post-assessment when skip counting by 4s and 9s, meaning that children became more efficient at skip counting during these tasks. It is also important to note that Measure B (skip counting by 6) had relatively high pre-assessment scores.

**Grade 2 Quantity Pre- and Post-Assessment Tasks**

Table 7 displays the results for Grade 2 during their completion of the Quantity task.

Table 7

<table>
<thead>
<tr>
<th>Learning Measure</th>
<th>Pre Mean (SD)</th>
<th>Post Mean (SD)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G (model numbers)</td>
<td>.80 (.30)</td>
<td>.84 (.31)</td>
<td>+ .04</td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H (seconds per block)</td>
<td>1.70 (.95)</td>
<td>1.38 (.55)</td>
<td>- .32</td>
</tr>
<tr>
<td>I (seconds)</td>
<td>18 (10.57)</td>
<td>15 (7.22)</td>
<td>- 3</td>
</tr>
</tbody>
</table>

** < .01; * < .05
Measure G was a seven-item composite measure where children modeled numbers with the blocks and measured whether a response was correct or incorrect (e.g., identify the number of tens in a given three-digit numeral, model given numbers using the base-10 blocks, and model numbers greater than and less than given numbers). Measure H, an efficiency measure, recorded how many seconds per block it took to build the number 181. Measure I recorded how many seconds it took to build 181. As Table 7 shows, Grade 2 children’s learning performance did not significantly increase on measure G. Although there was a numerical decrease in the time it took children to build 181, the results were not statistically significant.

Discussion

The purpose of this study was to examine young children’s learning performance and efficiency during clinical interviews where each child interacted with a variety of VM iPad apps. Examining mathematics learning on personal touch-screen devices is an emerging field, with some studies showing that PTD use leads to growth in performance (e.g., Liu, 2013; Risconscente, 2012; Spencer, 2013), while other research shows no significant differences when students use PTDs for mathematics learning (e.g., Carr, 2012; McKenna, 2012; Wilson et al., 2013; Zimmerman & Howard, 2013). Our results reflected some of the trends in this emerging research, showing that children in different age groups responded in different ways to the apps and different apps had a greater influence on children’s learning performance and efficiency.

Preschoolers’ Embodied Efficiency and Facility

Preschool children’s learning performance scores on the Seriation and Quantity tasks remained relatively constant, while they become more efficient with both tasks. Their improved efficiency on both tasks could be the result of improved understanding of the tasks or it could be a function of learning the technology and more comfortably working with the apps on the post-
assessments. While their learning performance remained constant, Preschoolers seemed to learn the physical mechanics needed to complete the tasks in a more efficient manner resulting in improved overall efficiency for both mathematical concepts. This explanation seems especially plausible when examining the similarity of the VM iPad apps that were used for each part of the interview. For example, on the Preschool Quantity task, the pre- and post-assessment apps and the two learning apps were all variations of the base-10 block virtual manipulative. Similarly, on the Preschool Seriation task, the pre- and post-assessment apps and one of the learning apps were all variations of the pink tower blocks virtual manipulative. Working with VM iPad apps with similar representations of objects and similar app features may have allowed the Preschoolers to develop greater efficiency with the objects as their interactions became a more embodied process resulting in their improved efficiency on the tasks. Another possible explanation is that preschoolers small motor skills are less developed, and while participating in the tasks, they were able to refine their fine motor skills because of features in the apps.

The findings for Preschool may provide emerging evidence for children’s beginning development of an internal representation for the block tower. It is likely that these preschoolers had little prior experience with seeing pictures of block towers. Because of this they did not yet have a clearly formed image of what they were trying to build and were initially hesitant about moving the blocks. The first pink tower learning app constrained their movement of the blocks to build the tower because it would only allow the child to move the correct block into place. This constraining feature may have helped children to form an image about how the pink towers should look upon completion. Children’s efficiency improved because children learned to use the app and because they had a clearer image of what they were building. Although not significant, children improved (measure C) at getting the blocks closer to the correct position.
On the Quantities tasks, we propose that children’s thinking was grounded in their motoric actions. Physical movement of the blocks in the VM iPad app, one at a time, by its very nature, may generate a scheme for one-to-one correspondence of the accompanying numbers that represent the block amounts (Goldin-Meadow, 2000). In fact, the movement of the blocks may have activated, in children, certain embodied processes from children’s previously enacted motor schemes, helping them to connect prior counting experiences with the motor actions of counting with the app.

Changes in Performance and Efficiency for Kindergarteners

While Preschoolers showed improvements in efficiency, Kindergarteners showed significant increases in learning performance on some assessment measures for both mathematical concepts (i.e., subitizing and quantity). Kindergarteners also increased their efficiency between the pre- to post-assessment on the Quantity task, indicating that they improved their learning performance and also became faster at this task. It seemed that for Kindergarteners, they demonstrated improvements in their learning performance while also learning to use the technology efficiently, especially for the quantity task. The Kindergarten Quantity task included pre- and post-assessment apps and two learning apps that were all variations of the base-10 block virtual manipulative, which may have allowed the children to become familiar with the app and its features. Similar to preschoolers, the kindergarteners’ small motor skills may have become more refined as they interacted with each base-10 blocks app. For the Kindergarteners, the use of the base-10 block virtual manipulative may have allowed them to develop meaning and understanding at the same time they were developing greater efficiency.

On the Subitizing task, where Kindergarten children showed significant increases in performance but not efficiency, there were three very different apps for the pre/post-assessment
and for the learning activities (e.g., Friends of Ten, Hungry Guppy, and Fingu). While this improved children’s learning performance on the Friends of Ten task during the post-assessment, it did not lead to an increase in Kindergarteners’ efficiency. This may have been due to the fact that children were not as familiar with the Friends of Ten app during the post assessment (which they saw only one time on the pre-assessment) as they were with the base-10 blocks during the post-assessment (which they saw on the pre-assessment and on the two learning activities). An additional observation to consider was that the Kindergartners who had difficulty arriving at an accurate response did not use the ten frame to visualize the two addends that resulted on the frame. Unless children were already familiar with the tens frame, they did not use two different color chips for the two different addends. This meant that their focus was on counting the sum or using their fingers to count on to obtain the sum, rather than on the two addends that were formed. The fact that children’s performance improved may show that the external representation helped children by maintaining a visual image of the number of color disks, thereby reducing the amount of cognitive effort required to identify the difference between the number of disks (measure C).

**Differences between App Sequences for Grade 2**

The Grade 2 results showed that children significantly improved their learning performance and their learning efficiency on the Skip Counting task. Once again, these results could be due to improved skill in skip counting after working through the learning apps, greater facility with the apps, or a combination of improved content understanding and efficiency with the technology. In addition, the results could have been influenced by the similarity of the Skip Counting tasks because in each task children were asked to count by 4’s, 6’s, and 9’s. In contrast, there were not significant improvements in Grade 2 children’s learning performance or learning
efficiency on the Quantity task, but these pre- and post-assessment tasks were structured differently from the learning tasks with the apps. For example, in our previous explanations for the improvement of learning efficiency for Preschool and Kindergarten, we noted that children’s familiarity with the base-10 blocks may have been a contributing factor in increasing children’s efficiency (i.e., they used a similar virtual manipulatives app repeatedly and became more efficient with its use). However, Grade 2 children used three very different virtual manipulatives (i.e., base-10 blocks, number line, and number cards) as representations of quantities, and they did not increase their learning performance or their efficiency. But conversely, for Grade 2 on the Skip Counting tasks, children used three very different virtual manipulatives (i.e., hundreds board, number line, and counting beads; See Appendix A) as representations of skip counting and they did increase their learning performance and their efficiency, despite the fact that the virtual manipulatives were very different.

One possible explanation for these seemingly contrasting results for Grade 2 was that the learning sequence with the Skip Counting apps included three variations of the process of skip counting in three different contexts (e.g., skip counting on a hundreds board, skip counting where the initial count number did not always begin at zero, and skip counting with equal groups of beads). Scaife and Rogers (1996) call this idea *re-representation*, where different representations have the same abstract structure, thereby making it easier to understand the concept across the representations. However, in contrast the Quantities apps required children to model numbers with blocks, model numbers with cards, and place the numbers on a number line. The Skip Counting learning sequence required children to skip count with the same numbers while changing the representations they used to skip count, thereby reconstructing their understanding of the meaning of skip counting. Perhaps the broader focus on place value concepts with the
Quantities apps did not allow children to connect and develop their understanding of the relationships among ones, tens, and hundreds, which was a requirement of the assessment tasks for this concept. In the case of Grade 2, the combination of the three Skip Counting apps may have been an optimal combination for learning this content, whereas the combination of the three Quantities apps was not an optimal combination for learning this content.

We must also consider an alternative explanation for our results. It is possible that children’s performance and speed would have changed after a brief period of time between pre and post assessment, regardless of whether they used VM apps in between. In the future, it will be useful to use the same pre and post assessments but give children a different intervening task (whether a mathematics task or even just free play) to confirm that the experience with VM iPad apps specifically influenced young children’s performance and efficiency. Although we observed significant changes in children’s performance and efficiency in a relatively short period of time (i.e., one 30-minute interview), it remains unknown how long these changes would persist into the future after this brief exposure.

**Conclusion**

In this paper, we have reported on the learning performance and efficiency results of 100 children ages 3-8 who interacted with VM iPad apps during a one-to-one clinical interview. The statistical results provide information about students’ learning performance and efficiency, and changes in these constructs, from pre- to post-assessments. However, our team is now engaged in further qualitative analyses of children’s interactions with the VM iPad apps to explain and extend our understanding of children’s interactions with the apps and the influence of the external representations on children’s learning. A part of this paper also focused on the processes our research team used to develop new tools for novel and unique research situations. To conduct
this study, researchers created a variety of research tools that were not readily available because research on children’s interactions with VM iPad apps is still in its infancy. We hope that this paper provides some guidance into the processes for creating high quality tools for novel research situations.

Teachers and children in K-12 classrooms are using touch-screen devices for mathematics instruction, and yet little is known about how mathematics learning changes or is influenced when children use a touch-screen interface for mathematics learning. This research makes a unique and foundational contribution to the research base, that to date does not yet exist. Providing a clinical analysis of the nature of young children’s use of virtual manipulatives on PTDs serves as a foundation for further research on how individual differences influence how children interact with VM iPad apps, how features and affordances on the iPad impact learning, and how overall conceptions of learning may change when children interact with mathematics concepts in embodied ways.
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## APPENDIX A

### Mathematics Apps Selected for the Pre and Post Assessments and Learning Activities

<table>
<thead>
<tr>
<th>Category</th>
<th>Pre/Post</th>
<th>Activity</th>
<th>Activity</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base-10 Concepts</strong></td>
<td></td>
<td>Montessori Numbers (Quantity: 1-9)</td>
<td>Pre/Post Pink Tower (Free Moving)</td>
<td></td>
</tr>
<tr>
<td><strong>Preschool</strong></td>
<td>Activity A</td>
<td>Montessori Numbers (1 to 20: 1-5)</td>
<td>Activity A Pink Tower (Card #12)</td>
<td></td>
</tr>
<tr>
<td><strong>Activity B</strong></td>
<td>Montessori Numbers (Numerals from Quantity: 1-9)</td>
<td>Activity B Intro to Math (Red Rods)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Kindergarten</strong></td>
<td>Pre/Post</td>
<td>Montessori Numbers (Quantity Activity: 10-99)</td>
<td>Pre/Post Friends of Ten (Teaching Tool)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity A</td>
<td>Montessori Numbers (1 to 20: 11-20)</td>
<td>Activity A Hungry Guppy (Dots: four dots of the same color)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity B</td>
<td>Montessori Numbers (Numerals from Quantity: 10-99)</td>
<td>Activity B Fingu (Level 1)</td>
<td></td>
</tr>
<tr>
<td>Grade 2</td>
<td>Pre/Post Montessori Numbers (Quantity: 100-999)</td>
<td>Pre/Post 100s Board</td>
<td></td>
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<tr>
<td>---------</td>
<td>-------------------------------------------------</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity A Math Motion Zoom (Levels 2-4)</td>
<td>Activity A Number Lines (Skip Counting)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Activity B Place Value Cards (3-digit problems without zeros)</td>
<td>Activity B Skip Counting Beads</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX B

Portion of the Kindergarten Interview Protocol: Quantity Tasks

**Activity A**- Montessori Numbers-"1 to 20" Activity, Difficulty Level 11 – 20 (3 minutes)

Turn sound on. Listen to the app say each number.

**SAY:** Here we have some more ten sticks and ones. We’ll use them to build some more number models.

**SAY:** Can you build the model of the number 11 here? **SHOW**

*Let the child build with all ones if this is the method they choose.*

**SAY:** Can you build the model of 15 here? **SHOW**

*If the child begins building the 10 with ones

**SAY:** Can you think of a faster way to build 10?

**SAY:** Can you build the model of 20 here?

*If the child begins building the 10 with ones **SAY:** Can you think of a faster way to build 10?

---

**Activity B**- Montessori Numbers-“Numerals from Quantity” Activity, Difficulty Level 10 -99 (3 minutes)

**SAY:** In this activity we’ll count the base ten blocks and then move the numerals to show how many there are. I will do this first.

**SHOW:** Count the blocks aloud and touch each one.

Move the correct digit “cards” to the correct digit places.

**SAY:** Now it’s your turn. Can you count the blocks out loud? Then choose the right numerals to match the number of blocks?

*Bring up a new problem by touching “replay.” Make sure that students only work on identifying quantities equal to or less than 50. If a number higher than this is generated exit to the main menu and re-enter.*

*Student work for 4 minutes or complete 3 problems, which ever comes first, before moving to the post assessment.*

*Allow the student to work for two minutes before giving a correction. Choose from those below.*