The aims of this study were to investigate how students compare the volume of rectangular prisms of equal volume and how their comparisons relate to students’ levels of sophistication in volume measurement. The study was designed in an exploratory nature and cross-sectional data were collected from participants at two different grades: Grades 4 and 6. Structured task-based clinical interviews (Goldin, 2000) were conducted with 14 upper-elementary-grade students. The interview tasks required participants to compare 3-dimensional objects (i.e., rectangular prisms or containers) by various aspects of volume. The tasks, which involved the pairs of objects with equal or nearly equal volume, were prepared to explore students’ reasoning about volume invariance. A constant comparative method (Merriam, 1998) was used to develop codes regarding students’ strategies and correctness for exploring volume invariance tasks. The frequency of each strategy and correctness for each task were determined per task, per aspect of volume, per grade level. Students’ responses to learning trajectory placement tasks were analyzed according to the levels of sophistication stated in the learning trajectory for volume measurement. According to the results, while reasoning in volume invariance tasks, three main categories, highly effective, effective, and ineffective, were observed in students’ strategies. Students exhibiting specific levels of the learning trajectory also applied similar strategies for the volume invariance tasks. Implications for instruction and research are discussed.
Overview

The focus of this report is to present the results of a qualitative research study that explored students’ reasoning about volume invariance and how their reasoning related to levels of sophistication in volume measurement. A body of research has shown that children, and even adults, often have difficulty in fully grasping the concept of volume measurement. In all aspects of measurement, like volume measurement, many children apply formulas to solve problems without understanding their meaning (Clements & Battista 1992). Findings in previous studies within and outside of mathematics education have helped explain why students have these difficulties. Notable studies outside of mathematics education research (e.g., Piaget, Inhelder, & Szeminska, 1960) focused on the conservation of volume as a physical property of an object. The term “conservation” was defined as the ability of an individual to be aware of the invariance of the quantitative value or properties of objects while the object is transformed into a qualitatively different one (Piaget et al., 1960). Piaget and his colleagues (1960) indicated that children are unable to conserve volume before the age of 11. Around the age of 11 or 12, children begin to understand volume in relation to spatial mediums and to associate volume with three dimensions. Some later studies, however, do not support Piaget’s claim about students associating volume with three dimensions of objects (e.g., Ebersbach, 2009).

Research Questions

1. How do students compare the volume of rectangular prisms in situations in which the prisms have equal volumes when students do not know they are equal?

2. How do students’ levels of sophistication in volume measurement relate to their reasoning in the comparison of the volumes of rectangular prisms of equal volumes?
Theoretical Framework

The theoretical perspective guiding this study is described by the framework of *hierarchic interactionalism* (Clements & Sarama, 2007). Of its 12 tenets, the hypothetical learning trajectory is the most relevant one of this study. Hypothetical learning trajectory for volume measurement was utilized in order to develop volume measurement tasks and to analyze students’ reasoning through their responses.

Methods and Data Sources

The study was planned according to a qualitative research design (Merriam, 1998). Seven grade four and seven grade six students from an elementary and middle school in the Midwest region of the United States were chosen through convenient and purposive sampling (Merriam, 1998). Each student was interviewed twice with two sets of interview protocols. Students’ reasoning and thinking was probed through structured, task-based clinical interviews with each individual student (Goldin, 2000). The tasks, which involved the pairs of objects with equal or nearly equal volume, were prepared to explore students’ reasoning about volume invariance. The tasks, which involved the pairs of objects, which had overtly different volume, were prepared to explore students’ reasoning in volume measurement as well as to place their reasoning into the levels of learning trajectory for volume measurement (Sarama & Clements, 2009).

All interviews were videotaped and transcribed. A constant comparative method (Merriam, 1998) was used to develop codes regarding students’ strategies and correctness for exploring volume invariance tasks. The frequency of each strategy and correctness for each task were determined per task, per aspect of volume, per grade level. Students’ responses to learning
trajectory placement tasks were analyzed according to the levels of sophistication stated in the learning trajectory for volume measurement (Sarama & Clements, 2009). The dominant level for each student was determined. Students exhibiting the same dominant level were grouped together. Students’ reasoning in volume invariance was not included in the learning trajectory for volume measurement. Therefore, it was an “outside concept” for the trajectory. In order to integrate an outside concept into the existing learning trajectory, the students’ strategies for the volume invariance tasks were reexamined for each group separately. Similarities and patterns, which appeared across students in a group were determined. These similarities and patterns suggested additions to the existing learning trajectory. Figure 1 illustrates the design of data analysis.

![Diagram](image)

**Figure 1.** Design of Data Analysis

**Results**

Three main categories, highly effective, effective, and ineffective, were observed in students’ strategies in volume invariance tasks. Ineffective strategies were accompanied incorrect answers. These strategies are a) considering-one-linear-dimension, b) counting-faces, and c) adding-lengths. When students applied these strategies they obtained a correct answer just 11%
Highly effective strategies were more likely to lead a correct answer when applied. They led a correct answer 58% of the time. The strategies in this category were a) compensating and b) morphing. The third category is for the effective strategies, which lead to correct answers specific to tasks and students. These strategies are a) considering-two-linear-dimensions and b) considering-three-linear-dimensions. Neither considering-two-linear-dimensions nor considering-three-linear-dimensions necessarily leads to correct answers in volume comparisons.

Based on students’ solutions to the volume measurement tasks, students were organized into four different groups. The students in the same group showed similar levels of sophistication in volume measurement (Sarama & Clements, 2009). Figure 2 represents how each level of learning trajectory mapped to each group (see gray shaded areas). The darker shading represents the predominant level of the group. The lighter shading represents the other levels exhibited by the students in each group. These levels were observed at least for one learning trajectory placement task for any student in the group.

![Figure 2. Level mapping for groups](image-url)
Conclusions

These findings extend the findings of Ebersbach (2009), which indicated that that 42% of the kindergartners integrated at least two dimensions of the prisms and 26% integrated all three dimensions multiplicatively. Students’ levels of sophistication in volume invariance tasks develop by age; however, each individual student’s level of sophistication may vary independent from their age or grade level. The results demonstrated that students might not fully grasp the concept of volume even in the upper elementary grades. The results are consistent with the findings of previous research (Battista & Clements, 1992; Campbell, Watson, & Collis, 1992). Moreover, according to Piaget and his colleagues, a child around the age of six to eight still may not conserve volume. However, the results of this study indicated that students at the sixth grade (around age 12) were sometimes unable to conserve the number of cubes in a prism when the prism is transformed into another prism.

This research suggests that students who exhibit less-sophisticated reasoning in volume measurement struggled to address volume invariance tasks correctly. Students’ reasoning in volume invariance is interwoven with their reasoning and understanding in volume measurement. These two reasoning types cannot be prioritized or separated completely as two disjoint sets of reasoning. Students’ reasoning for one set may be a priority or result of their ability to reason in another set.

Results of the study indicated that students exhibiting specific levels of the learning trajectory also exhibited a corresponding set of strategies for the volume invariance tasks. These patterns of correspondence between the existing trajectory levels and emergent sets of strategies on volume invariance tasks suggest additions to the existing learning trajectory for volume measurement developed by Sarama and Clements (2009).
Educational Importance of the Research

This study extended the literature on student thinking of volume measurement related to invariance of volume as a quantity. The study indicated that there is a need for instructional focus on this sub-domain to improve students’ understanding of volume measurement. Observing and understanding students’ strategies when coping with the tasks involving with the rectangular prisms with equal volume provides a basis to inform and extend the instructional decisions about and teaching methods in volume measurement related domains. The results showed that students’ reasoning in volume measurement develop parallel with their reasoning in volume invariance. While designing instructional activities, this result should be considered and volume invariance tasks should be included in volume measurement instruction.

References


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