

The Impact of a Dynamic Geometry-Centered Teacher Professional Development Program

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This study investigated the impact of a *dynamic geometry-centered teacher* professional development program on high school geometry teachers' change and their students' geometry learning. 64 geometry teachers were randomly assigned to two groups. The teachers in the experimental group participated in a one-week summer institute followed by six half-day workshop sessions during the school year, in which they studied the critical features of the dynamic geometry (DG) approach and the DG-oriented teaching strategies they were expected to use in their classrooms. *The classroom observation data and the teachers' responses to the implementation questionnaires revealed that most teachers in the DG group were faithful to the instructional approach assigned to them. Teachers in the experimental group scored higher in a conjecturing-proving test than did teachers in the control group, but the difference was not significant. The students of teachers in the experimental group significantly outperformed the students of teachers in the control group in a geometry achievement test.*

Dynamic geometry (DG) is an active, exploratory study of geometry carried out with the aid of interactive computer software, which has been available since the early 1990's. The major DG software packages include the Geometers' Sketchpad (Jackiw, 2001), Cabri Geometry (Laborde & Bellmain, 2005) and Geogebra (Hohenwarter, 2001). DG environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena (CCSSI, 2010). With its main features such as dragging and measuring, DG software can be used to help students to engage in both constructive and deductive geometry (Schoenfeld, 1983) as they build, test and verify conjectures using easily constructible models.

A research project funded by the National Science Foundation (NSF) conducted repeated randomized control trials to investigate the efficacy of an approach to high school geometry that utilizes DG software to supplement ordinary instructional practices. This approach was referred to as the DG approach in the project, and the DG software used by the project was the Geometers' Sketchpad (GSP). To guide the creation, selection, and implementation of DG activities the researchers developed the following operational definition of the DG approach:

As learning activities emphasized by the DG approach, students are expected to:

- *Construct dynamic geometric objects with DG tools*
- *Construct dynamic representations of problem situations with DG tools*
- *Perform actions (drag, measure, transform, and/or animate) on the constructed objects/situations*
- *Observe variants and invariants related to the characteristics of objects under different actions*
- *Investigate mathematical relationships and/or solutions in multiple ways*
- *Formulate conjectures*
- *Test conjectures*
- *Receive immediate feedback from the software*
- *Think mathematically and prove (or disprove) their conjectures*

As teaching activities recommended by the DG approach, teachers are expected to:

- *Facilitate the student use of DG software*
- *Help students construct mathematical ideas through active explorations and investigations*
- *Present prepared DG environments for students to explore mathematical relationships when the constructions of the DG environments are too complicated for students or when the constructions themselves are not the focus of the related explorations*
- *Facilitate students' argumentations by asking "why" questions – prompting students to furnish justifications for their statements and checking the validity of their justifications (Vincent, 2005)*
- *Extend students' explorations by asking "what if" questions*

Based on the operational definition of the DG approach, our basic hypothesis is that use of DG software to engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proof results in better geometry learning experiences for most students.

This paper is to report a study conducted during the second year of the project, whose purpose was to examine the impact of the professional development designed for and offered to the teachers in the experimental group. The study built upon related research studies on mathematics teachers' professional development (e.g., Carpenter et al., 1989), including those concentrating on technology-centered (and especially DG-centered) professional development (Stols et al., 2008; Meng & Sam, 2011).

THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

An integrative framework (Olive & Makar, 2009) drawing from Constructivism, Instrumentation Theory and Semiotic Mediation was used to guide the study. Within this framework, as teachers and students interact with, and in, DG environments these interactions with the technology tool influence the next acts by each person, and continue in an interplay between the tool and user. For example, as students "drag" an object and observe outcomes from that act, the user (teacher or student) adjusts her or his thinking, which in turn influences the next interaction with the tool. Because DG tools allows users to adjust their sketches and the relationships within them, users are transforming the tool, their use of the tool, and their thinking.

This study addresses the following research questions:

- 1) Did teachers in the experimental group develop stronger conjecturing and proving abilities than did teachers in the control group?

- 2) Did the students of teachers in the experimental group perform significantly better in geometry learning over a full school year than did the students of teachers in the control group?
- 3) How did the instructors implement the DG approach in their classrooms with fidelity?

METHOD

The population from which the participants of the study were sampled was the geometry teachers at high schools and some middle schools in Central Texas school districts. The study followed a randomized cluster design. 64 teachers who were randomly assigned to two groups (an experimental group and a control group) received relevant professional development, implemented the instructional approaches respectively assigned to them, helped the project staff in administering the pre- and post-tests of the participating students, and participated in other data collection activities of the project.

Professional Development and the DG Treatment

In order to effectively implement the DG approach in their classrooms, teachers need to learn the approach first. Without professional development training, “teachers often fail to implement new approaches faithfully” (Clements et al., 2011, p. 133). So teachers’ professional development (PD) was an important component of the project. For our PD to be effective, it had to be long enough, intensive enough, and relevant enough, with substantial support from the school districts. Based on these guiding ideas, a weeklong summer institute was offered to the participating teachers in the DG group, followed by 6 half-day Saturday PD sessions during the school year.

The nature of each PD session for the experimental group was interactive and emphasizing participating teachers' active involvement and conceptual understanding of mathematics. Important geometric concepts, processes, and relationships were presented or revisited through challenging problem situations, which were explored with the DG software as a tool. Teachers learned DG skills in the process of using them to tackle the problems. They came to learn, first hand, as learners of mathematics, how DG environment encourage mathematical investigations by allowing users to manipulate their geometric constructions to answer "why" and "what if" questions, by allowing them to backtrack easily to try different approaches, and by giving them visual feedback that encourages self-assessment.

Typically, each activity in a PD session consisted of the following instructional events:

- 1) Presenting a task (exploring concepts/relationships or solving a problem) to the teachers;
- 2) Requesting teachers to use DG tools to construct the related geometric object or problem situation (with help if necessary) or provide them with a prepared DG environment;
- 3) Asking teachers what conjecture(s) they can make based on their initial observation;
- 4) Requesting teachers to use dragging, measuring, and multiple, linked representations to experiment with the constructed or provided DG environment, and observe what characteristics change and what remain the same;
- 5) Asking teachers to further make and test conjecture(s);
- 6) Reminding teachers to redo 4) and 5) in a new aspect or at a higher level, as appropriate;
- 7) Asking teachers to summarize and reflect on what they have conjectured;
- 8) Helping teachers develop explanations to prove or disprove their conjecture(s).

For event #8 above, the PD facilitators allowed a significant amount of time for teachers to think on their own and share their ideas with other teachers. After that, if teachers were able to

construct their own proofs, they were encouraged to present the proofs to the large group for a discussion. For teachers who experienced difficulties, a problem solving strategy – Using Subproblems – was applied to help them. The task of proving a conjecture was scaffolded as a series of subproblems (presented as questions) organized in a logical order. As the teachers finished solving all subproblems, the goal of developing a proof was achieved.

As an example, let's consider proving the conjecture that the three perpendicular bisectors of any triangle intersect at a single point. It might be better to consider one perpendicular bisector first. Therefore, teachers were first asked to solve the following subproblem: *Make a conjecture about the property of the perpendicular bisector of a segment, and prove your conjecture.* With the dynamic moving and measurement features of the software and their knowledge of triangle congruence, it was relatively easy for teachers to solve this subproblem. Similarly, they were able to relatively easily solve the second subproblem: *What is the converse statement of the property of the perpendicular bisector of a segment, and prove it.* With these two subproblems solved, it became more straightforward for the teachers to solve the third subproblem: *Construct any two perpendicular bisectors of a triangle, and prove their intersection point being on the third perpendicular bisector of the triangle.* After combining the solutions of all three subproblems, the original conjecture was successfully proved.

In each PD lesson, teachers either worked individually in front of a computer or worked in pairs (or small groups). In either case, the facilitators encouraged teachers to share ideas and help each other. The facilitators circulated, observed (to monitor the progress), asked questions, and provided necessary help as teachers were working. At the time when a large group discussion was needed, the facilitators initiated it in a timely manner. During the whole session, assessment was an integral part of instruction that informed and guided the facilitators as they made training

decisions.

In terms of content, the summer part of the PD sessions concentrated on the most important topics of high school geometry such as triangle congruence and similarity, properties of special quadrilaterals, properties of circles, and geometric transformations, while the school year follow-up sessions aligned with the scope and sequence determined by the participating school districts.

The PD facilitators' action of leading the teachers to conduct investigations modeled what teachers were expected to do with students in their classrooms. To further help teachers to consider changes in their instructional strategies, in the relation between them and their students, and in how they facilitate student learning, mathematical explorations were always followed by discussions on questions such as "How will you teach this content in your classrooms using DG software?" and "How will you lead your students in conjecturing and proving using DG software?" The PD facilitators realized the importance of teachers learning from each other and sharing ideas. Therefore, teachers were encouraged to give presentations on their important insights of DG implementation and successful stories, or to describe problems they anticipated with other teachers offering suggestions to address the concern. Teachers also worked in groups of 3 or 4 to prepare lesson plans to share with the entire group.

The control group

The teachers in the control group taught geometry as they had done before. They also participated in a PD workshop, in which the same mathematical content covered in the DG group PD sessions was introduced to them in a non-DG environment. The PD facilitators used an approach that most teachers were currently using – a lecture based, though activity based instructional methodology – to conduct their training, mainly lecturing, heavily relying on the textbook, and participants' problem solving exercises without using technology tools. The

amount of instructional time spent on this regular workshop was the same as that for the DG group PD training. The purpose of holding this non-DG workshop was to address a confounding variable. With this comparable amount of professional development, if differences appeared on the project's measures between the experimental and control groups, we would be able to rule out the possibility that the mathematics content presented in the DG group PD sessions could account for them rather than the interactive DG learning environment.

MEASURES AND DATA ANALYSIS

A measure for teachers' conjecturing-proving knowledge

A conjecturing-proving test was developed by the project team to measure teacher knowledge. After a series of intensive work (a thorough literature review, construct development, item development, Advisory Board members' review, pilot-testing, several rounds of revision, etc.), a version of the test consisting of 26 multiple-choice items and two free-response proofs was produced. The instrument was administered to the teacher participants as both a pre-test and a post-test at the PD Summer Institute.

Teachers' implementation fidelity and classroom observations

The DG approach involves using dynamic software intensively in classroom teaching to facilitate students' geometric learning. The critical features of the DG approach include using the dynamic visualization to foster students' conjecturing spirit, their habit of testing conjectures, focusing on relationships, and explaining what is observed, their logical reasoning desire and abilities, as well as their conjecturing-investigating-proving oriented learning style in exploring problem situations. To determine how to capture these critical features of the DG approach, two measures of implementation fidelity (the DG Implementation Questionnaire and the Geometry

Teaching Observation Protocol [GTOP]) were developed. The DG Implementation Questionnaire was adapted from a teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and her colleagues) in an NSF funded project, based on the critical features of the DG approach. The final version of the DG Implementation Questionnaire contains six multiple-choice items and ten open-response questions, which was administered to the teachers in the experimental group six times across the school year. A different version of the questionnaire was administered to the control group teachers (also six times) to examine how they teach geometry without using dynamic technology. The GTOP was adapted from the *Reformed Teaching Observation Protocol* (Sawada, et al., 2002), also built on the critical features of the DG approach. Consisting of 25 items in four sections (*Description of intended dynamic geometry lesson, Description of implemented dynamic geometry lesson, Assessment of quality of teaching, Assessment of engagement and discourse*), the GTOP was administered to 16 participating teachers in both of the two groups (eight selected from each group). Each teacher was observed four or five times across the school year. For the control group teachers, the GTOP was a sub-scale of the GTOP for the experimental group teachers. The items removed for the control group were items in the implementation aspect that were related to the use of software functions (dragging, dynamic measuring, etc.).

Student level measures

The instruments used for measuring students' geometry knowledge and skills were: (1) (for the pre-test) Entering Geometry Test (ENT) used by Usiskin (1981) and his colleagues at University of Chicago; and (2) (for the post-test) Exiting Geometry Test (XGT). XGT was developed by selecting items from California Standards Tests – Geometry (CSTG). The final version for XGT has 25 multiple-choice items. Figure 1 shows three examples of the items.

Item 1 is a straightforward application of the standard side-side-side triangle congruence theorem. Item 8 involves the definition of a circle. In item 11, students need to visualize the box formed by the net before calculating its volume.

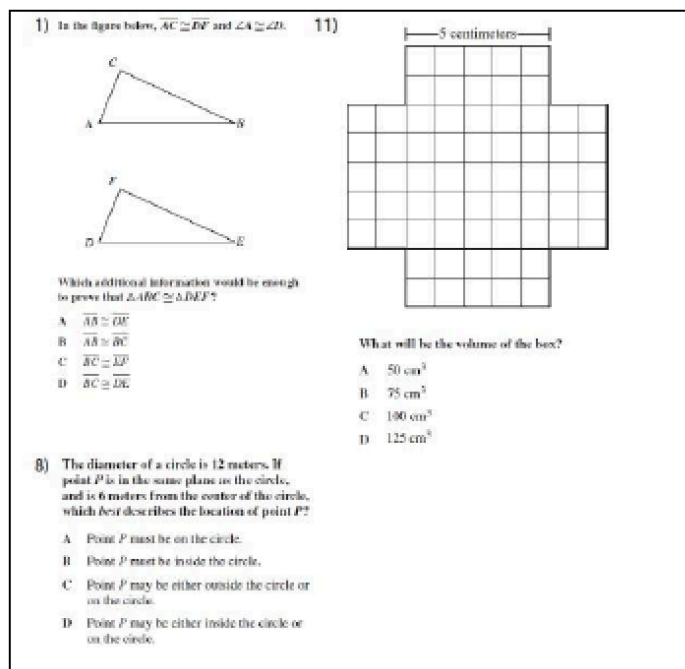


Figure 1. Three items of the geometry posttest

All research instruments mentioned above, except the student geometry pre-test, were developed by the project staff. For all project-developed measures, the Cronbach's Alpha statistical values were within the acceptable ranges for reliability. Item Response Theory (IRT) scoring routines were applied to the DG Implementation Questionnaire and Students' post-test data and provided evidence for the instruments' construct validity.

Two-level hierarchical linear modelling (HLM), other statistics methods, and the constant comparison method (Glaser & Strauss, 1967) were employed to analyse the quantitative and qualitative data respectively. Below is a summary of the main findings from the data analysis.

RESULTS

Findings from the Conjecturing and Proving Test

This instrument was administered to project participating teachers at the beginning and end of the weeklong Summer PD Institute. A statistic for overall competence on the instrument was calculated by adding the number of correct multiple-choice responses with points from free-response items. Teachers averaged 20.49 on the pre-test and 21.86 on the post-test with an average gain of 1.37. A paired-sample t-test showed that this gain was statistically significant ($p = .003$). This suggests that the professional development had an effect on teachers' capabilities. The average gain was greater for the experimental group (1.56) than the control group (1.18). This difference was not statistically significant ($p = .670$).

Findings from the Classroom Observations

The results for the observations in both the DG group and the control group are summarized in Table 1. The main result that stands out is that the DG teachers demonstrated an intention to teach a lesson with the DG approach and they also demonstrated, to some extent, the knowledge required to do so. These were the aspects with the highest scores (Good Lesson Design, Use of DG Features, and Teachers' Knowledge). Overall, teachers were implementing the DG approach at a medium level [2.28] (the maximum score was 4). Considering the challenges (the inaccessibility of a computer lab in the first several weeks, the pressure of the intensive state testing, etc.) that the teachers experienced during the school year, most of them should be regarded as being faithful to the DG approach.

Table 1
Comparison between DG and control groups

Aspect	Sub-aspect	Mean DG	Mean Control	p-value
Intended Dynamic Lesson	Good Lesson Design	2.81	1.85	.032*
	Use of Dynamic Features	2.75	0.70	.000*
Implementation	Actions beyond Use of Software	2.06	1.33	.095
Quality of Teaching	Cognitive Demand	2.30	1.78	.113
	Teachers' Knowledge	2.89	2.84	.924
	Conjecture/Proof	1.93	1.40	.206
Engagement and Discourse		2.37	2.29	.735
Overall GTOP Score		2.28	1.68	.088

Table 1 also shows the comparison between the results for the two groups. The p-values tested the significance of the treatment effect and were obtained using a mixed effect ANOVA. As expected, the results showed that the control group did not use dynamic features of the approach to teaching geometry. Further, the intended lessons for the DG group were significantly better designed than the control group. In particular, lesson plans for the DG group had in general appropriate objectives and they were designed to move students from initial conjecture, to investigation, to more thoughtful conjecture, to verification and proof. Although teachers in the DG group and teachers in the control group did not differ significantly in other aspects, the GTOP scores of the DG teachers in these aspects are all higher than those of the control teachers. (More thorough analysis of the classroom observation data is ongoing.)

Findings from the Implementation Questionnaire

Figure 2 and Figure 3 below show how teachers rated themselves when it comes to their effectiveness and comfort in using GSP in teaching of geometry. The results reveal that in terms of the level of effectiveness in using GSP in teaching geometry, from those teachers who

completed the questionnaire (total of 31), 29% of the teachers were at the high level, 61% of the teachers were at the middle level, and 10% of the teachers were at the low level. In addition, it seems that more teachers felt more comfortable than effective in using GSP in teaching. Only one teacher did not feel comfortable using GSP in teaching of geometry. An overwhelming majority of teachers (97%) felt very comfortable or somewhat comfortable in using GSP in teaching. 22 teachers felt as effective as they felt comfortable in using GSP in teaching of geometry.

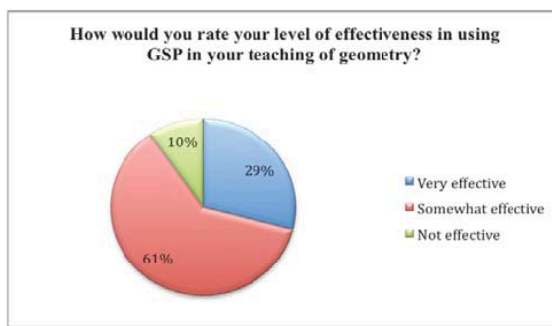


Figure 2. Effectiveness in using GSP

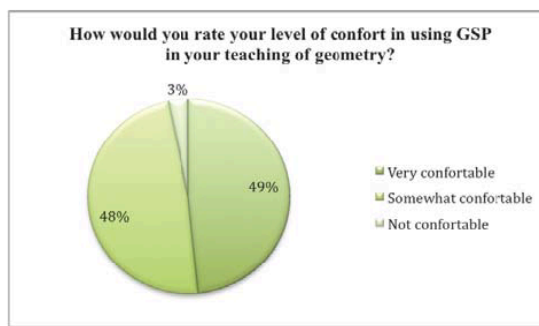


Figure 3. Level of comfort in using GSP

Figures 4 and 5 show average teacher and student use of GSP throughout the school year. Here “average teacher use of GSP” represents “average times per week the teacher uses the demonstration computer in his/her classroom to do GSP presentations and/or demonstrations to students”. The “average student use of GSP” represents “average times per week students work in a computer lab doing hands-on explorations with GSP”. Among the 31 teachers who completed the questionnaire, on average, 77% of them used GSP on the demonstration computer at least one time each week, and 38% of them at least two times. However, among the 31 teachers who completed the questionnaire, in terms of “taking students to the computer lab to do hands-on activities with GSP,” on average, only 61% of them did so at least one time each week, and only 10% of them did so at least two times each week.

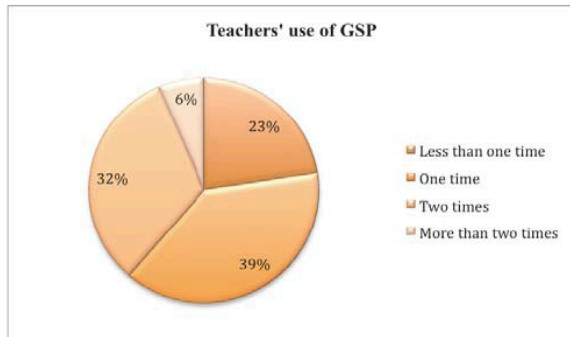


Figure 4. Average teacher use of GSP

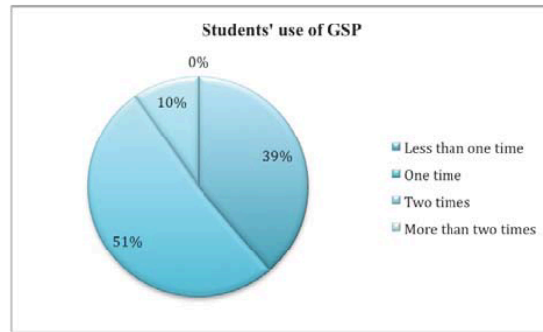


Figure 5. Average student use of GSP

These facts show that overall, and especially in terms of “taking students to the computer lab to do hands-on activities with GSP,” the teachers’ implementation of the DG approach was at the medium intensity level. This finding is consistent with that from the classroom observations. However, almost all teachers were positive or enthusiastic in using GSP in geometry teaching. Considering the challenges (the inaccessibility of a computer lab in the first several weeks, the pressure of the intensive state testing, etc.) that the teachers experienced during the school year, most of them should be regarded as being faithful to the DG approach. (More thorough analysis including qualitative analysis of the Implementation Questionnaire data is ongoing.)

Findings from the Student Geometry Achievement

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the DG approach on overall student achievement. The model was analysed using student pretest (ENT) scores as a covariate. The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectation and quality of the students in each of these levels was very different, the factor *Class Level* was included in the model. Additionally, the years of classroom experience of the teachers in the sample varied a lot, ranging from 0 years all the way

up to 35 years. It was possible that a more experienced teacher might have greater command of the classroom but be less able to implement the technology. For this reason, the covariate *Years Exp* (number of years of classroom experience) was included in the models.

The HLM model equations are:

(Level 1: Student)

$$XGT_{ij} = \beta_{0j} + \varepsilon_{ij}$$

(Level 2: Classroom)

$$\begin{aligned} \beta_{0j} = & \gamma_{00} + \gamma_{01}DG_j + \gamma_{02}Regular_j + \gamma_{03}PreAP_j + \gamma_{04}(YrsExp * Regular)_j \\ & + \gamma_{05}(YrsExp * PreAP)_j + \gamma_{06}(ENT - \overline{ENT}) + u_j \end{aligned}$$

i represents the student, *j* the classroom, ε and u are independent normally distributed error terms.

This model examines the effect of the DG intervention when taking into account Entering Geometry Test (*ENT*) as well as *Class Level* and *Years Exp*. To simplify interpretation of the other coefficients, *ENT* was centered by subtracting the overall mean. The results of the Model (shown in Table 2) indicate that the DG effect was strongly significant ($p = .002$).

As expected, *ENT* is a significant predictor of student performance on *XGT* ($p = .000$). However, even controlling for the pretest, compared with Middle School students, on average Pre-AP students scored 13.2 points lower ($p = .049$) and Regular students scored 20.1 points lower ($p = .004$). The effect of experience was significant in the Pre-AP group ($p = .012$), but not significant in the Middle School group ($p = .344$). An increase in 10 years of experience raised the scores 4.5 points for the Pre-AP group and decreased the scores by 4.1 points for the Regular group. Comparing the means, the DG group outperformed the control group in each level of Geometry and the effect size (.45) was substantially larger at the Regular Geometry level.

Table 2
HLM Results with Pretest as a Covariate HLM Results with Pretest as a Covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept	79.05	6.119	12.919	28	.000
DG Effect	5.62	1.678	3.352	43	.002
Level					
Regular	-20.10	6.119	-3.284	21	.004
Pre-AP	-13.23	6.293	-2.102	20	.049
Level*Years Exp					
Regular*Years Exp	-.4137	.1516	-2.729	53	.009
Pre AP * Years Exp	.4451	.1617	2.753	22	.012
M. School*Years Exp	1.811	1.868	0.969	20	.344
<i>ENT</i> (Mean Centered)	.4114	.0366	11.237	47	.000

Note. XGT is the response variable.

DISCUSSION

The HLM model taking pretest, class level, and teaching experience into account showed that the DG group students significantly outperformed the control group students in geometry achievement. Given that teachers were randomly assigned to the two groups and that both groups received professional development of the same duration on the same topics, the results from this study provide evidence to support the finding that the DG professional development did make a difference – it did positively impact the student geometry learning. Both DG and control group teachers have obtained significant gains on the Conjecturing-Proving Test through the one-week summer PD institute. This result suggests that both the PD sessions designed for the DG group

and those designed for the control group had an effect on teachers' conjecturing and proving. Although the DG and control teachers did not differ significantly on their mean gain scores, the DG teachers' mean gain score was still 32% higher than that of the control teachers. The classroom observation data revealed that lesson plans that the DG group teachers prepared were significantly better designed than the control group, aiming at facilitating students' conjecturing and proving abilities. The teacher GTOPI scores (overall and in each sub-scale) consistently favored the DG group although most of the differences were not statistically significant.

In summary, the results of this study suggest that the DG professional development offered to the participating teachers has had a significant positive effect on the teachers' change on both mathematics conjecturing-proving knowledge and teaching strategies. The teachers, in turn, helped their students achieve better geometry learning. This can be illustrated by the following comment made by one of the participating teachers: *"From the DG technology professional development I have gained new teaching strategies to engage my geometry students. I have found by using these learned strategies that my students are able to make their own conjectures about the figures they construct using the Geometer's Sketchpad. Many students are both kinesthetic and visual learners; I am able to reach these students with the dynamic-ness of constructing using technology."*

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