FROGS, FAUCETS, AND FRENCH FRIES

EXAMINING PROPORTIONS THROUGH MULTIPLE LENSES

NCTM Regional Conference
November 20, 2014
Houston, TX
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WHY ARE WE INTERESTED?

Increased emphasis on ratio and proportional relationships for students

CCSS for Grade 6 [6.RP],

Grade 7 [7.RP], and

Grade 8 [8.EE, 5 and 6]

Texas Expectations - TEKS

Grade 6 6b.4 A – H

Grade 7 7b.4 A – E, 7b.5 A – C, 7b.6 A – I

Grade 8 8b.3 A – C, 8b.4 A – C, 8b.5 A - I

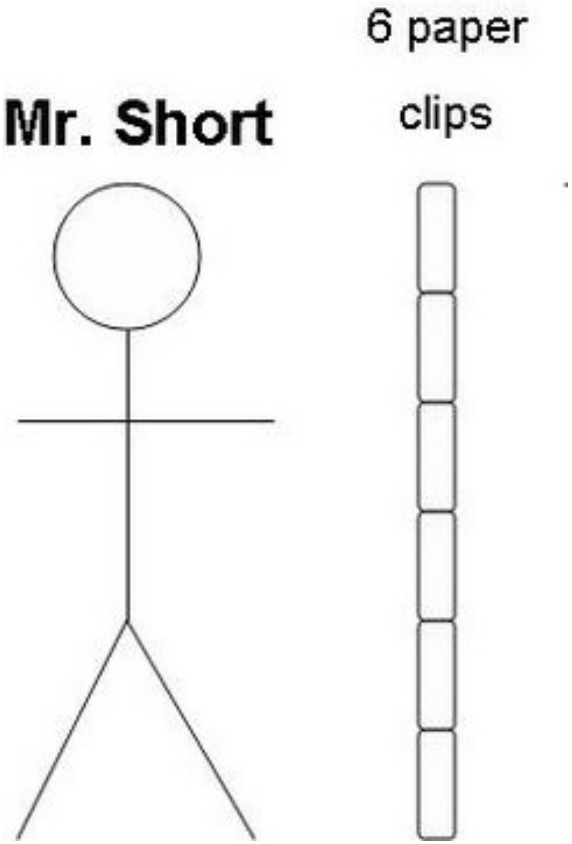
CCSS – Mathematical Practices

TEKS – Mathematical Process Standards

Important role of problem solving, representations, reasoning, tools, modeling, communication, relationships

Mr. Tall and Mr. Short – Assessing Basic Understanding

In the diagram, you can see the height of Mr. Short measured with paperclips. Mr. Short has a friend Mr. Tall. When we measured their heights with buttons, Mr. Short's height is 4 buttons and Mr. Tall's height is 6 buttons.

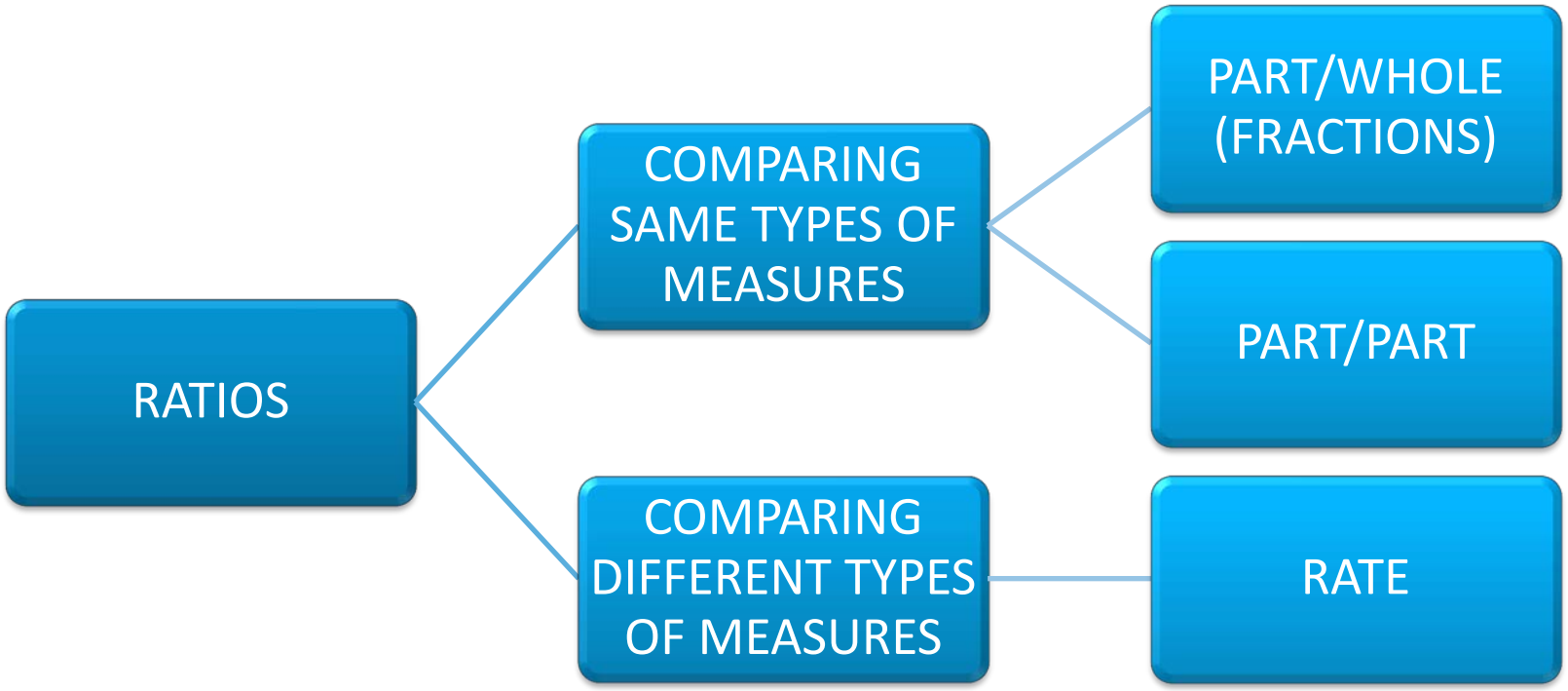


How many paperclips in height is Mr. Tall?

What do you think students gave as their response?

8 paperclips

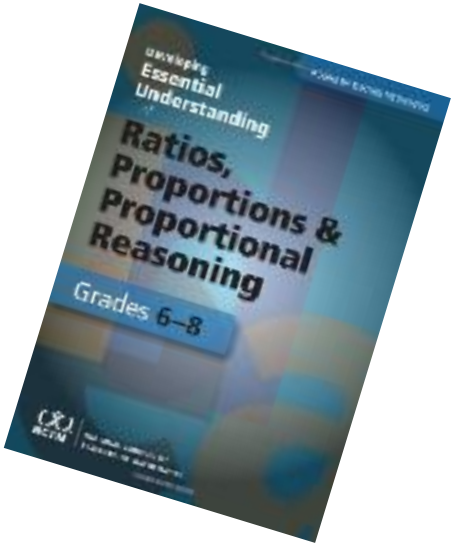
Why are students giving this answer?
What is the misunderstanding?



Van de Walle & Lovin, 2006, *Teaching Student-Centered Mathematics Grades 5-8,* page 155

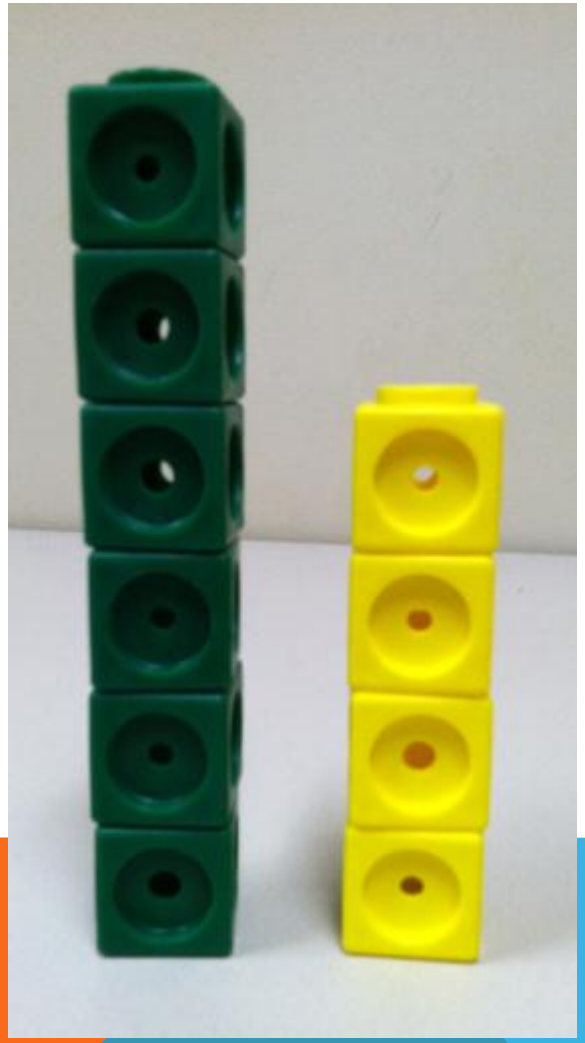
Essential Understanding #2

What is a ratio?



A ratio is a multiplicative comparison of two quantities, or it is a joining of two quantities in a composed unit.

Ratio as a multiplicative comparison of TWO quantities



The GREEN tower is 1 ½ times the height of the YELLOW tower.

Compare the height of the YELLOW tower to that of the GREEN tower.

Ratio as a ***Composed Unit***

➔ *pre-ratio* reasoning

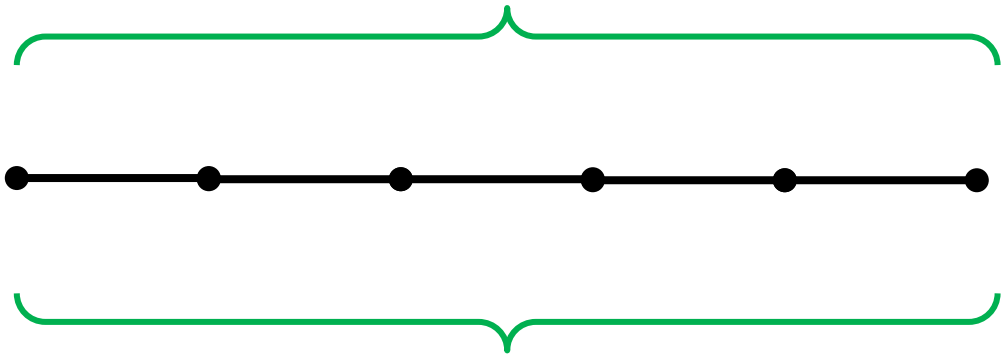
(Lesh, Post and Behr, 1988)

➔ not sophisticated

Form a ratio by joining two quantities to create a new unit

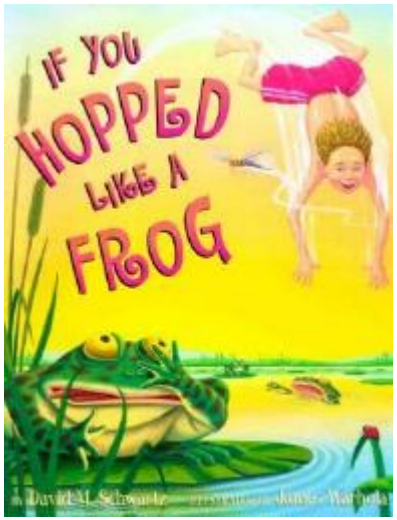
EXAMPLE:

During the growing season, the diameter of a lily pad increases 2 cm each week.



ACTIVITY – *ITERATING A COMPOSED UNIT*

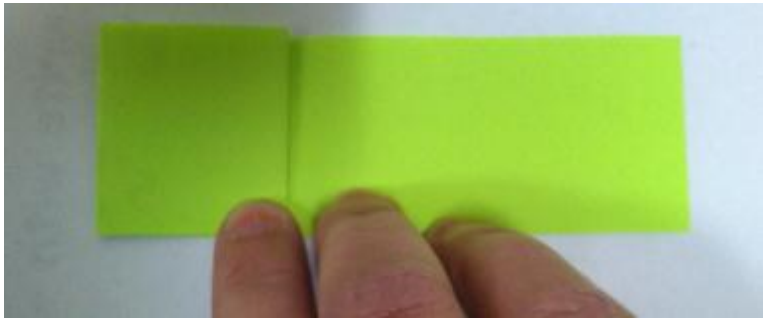
If You Hopped Like a Frog

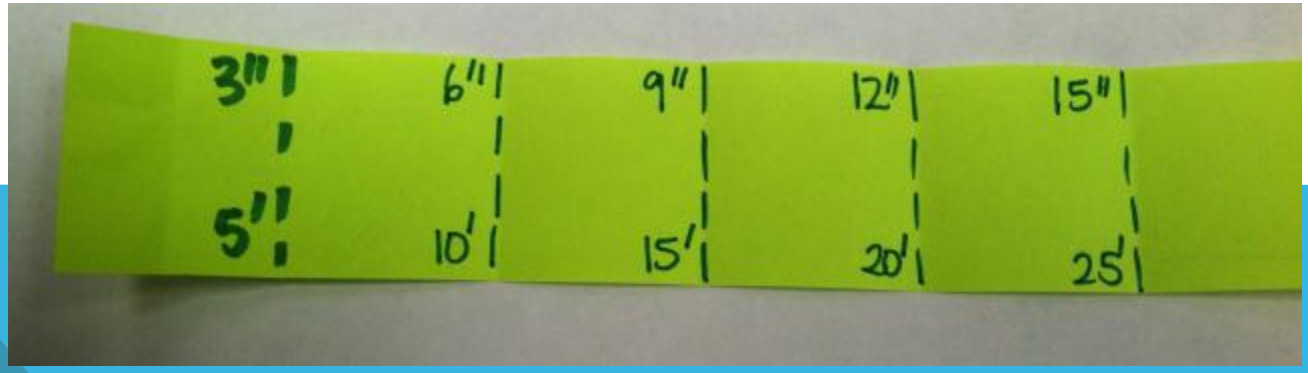


Problem

A 3” frog can jump 20 times its body length. How far could you jump if you hopped like a frog?

Using paper strips:





Use your strip to answer the following questions:

1) About how far could a ***newborn baby*** leap if she could jump like a frog? _____

2) About how far could a ***kindergartener*** leap if he could jump like a frog? (Assume the kindergartener is 42" tall).
_____

3) If you were ***15 times longer*** than a frog, then how far could you leap if you could jump like a frog? _____

4) About how far could ***you*** leap if you could jump like a frog?
_____

5) About how far could a ***7' tall basketball player*** leap if he could jump like a frog? _____

Use your strip and multiplicative reasoning to complete the table.

Body Length	1 ½ ”	3”	60”	120”	180’		300”	
Leaping Distance		5’				400’		5280’

Explain what strategies you used to fill in the table.

ACTIVITY – *REASONING UP AND DOWN*

PROBLEM

A small order of McDonald's french fries weighs about 75 grams. Complete the ratio table to determine the number of calories in a small order of fries if each fry weighs about 2 grams and contains 6.4 calories.

Serving size, g

Explain the strategies you used to complete the table.

Each value meal comes with a medium order of fries, weighing in at around 110 grams. Determine the calorie count in a medium order of fries.

Serving size, g	
s	6.4

Super-size it! Determine the number of calories in a large order of fries (178 g) using a ratio table.

Serving size, g	

gies.

Talk with a tablemate – What strategies did you use to complete each table? What additional strategies might we expect students to use?

What key ideas about ratio and proportion are promoted by these two activities?

Essential Understanding #7

What are the key aspects of proportional reasoning?

Pages 36 - 41

➔ Equivalent ratios can be created by iterating and/or partitioning a composed unit.

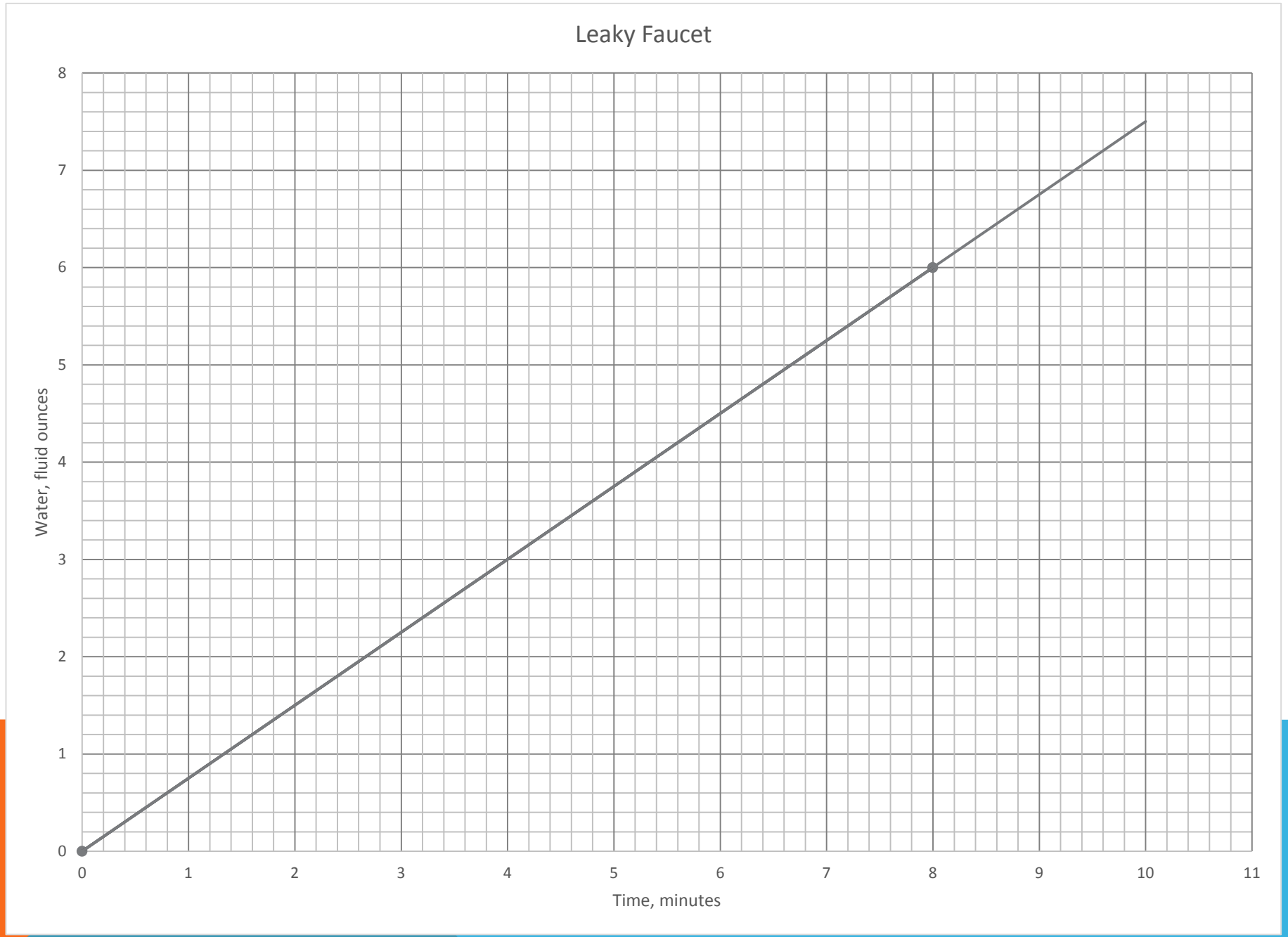
➔ If one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

➔ The two types of ratios – composed units and multiplicative comparisons – are related.

ACTIVITY – *Making Connections*

PROBLEM

A faucet is dripping in the bathroom. Jason placed a measuring cup under the faucet to capture the water as it leaked. After 8 minutes, he noticed 6 fluid ounces of water had accumulated in the cup. Knowing that the cup was empty when he began his experiment, and that the water was dripping at a constant rate, he created the following graph.


Leaky Faucet
Water, fluid ounces
0
1
2
3
4
5
6
7
8
Time, minutes
0
1
2
3
4
5
6
7
8
9
10
11

Use the graph to complete the table of values. You may have to use reasoning to fill in some of the values.

Time, minutes	

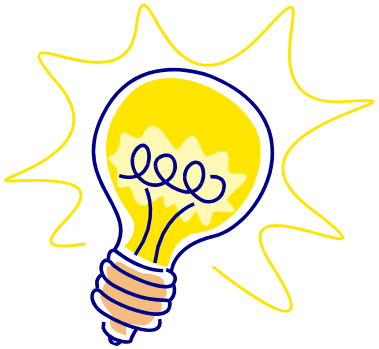
om the faucet? Explain how that rate is represented in the table.

Let's find the connections!

What rate was the water dripping from the faucet? Explain how that rate is represented in the table.

Interpret the slope in the context of the problem.

THE BIG IDEA

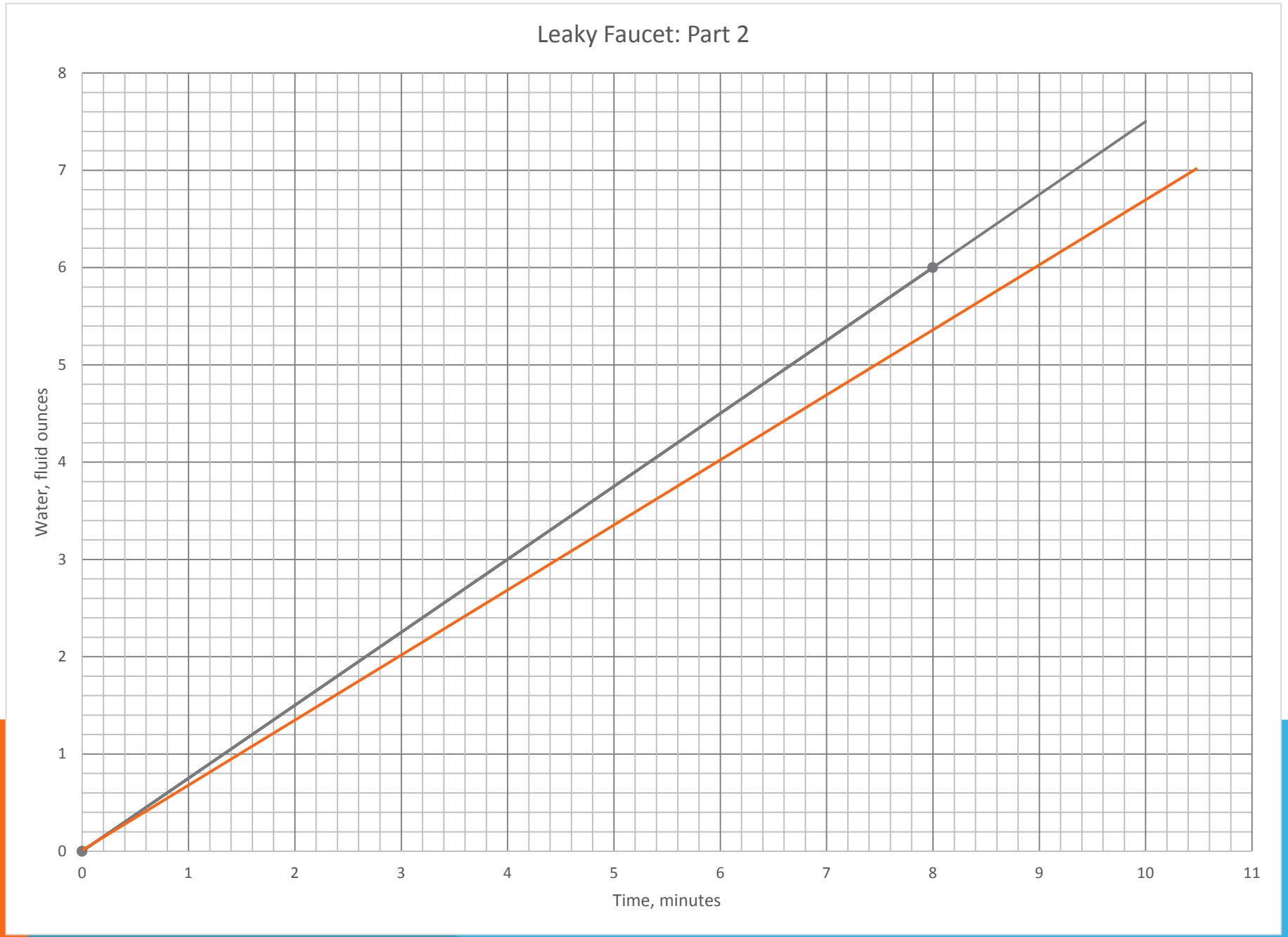


When two quantities are related proportionally, the ratio of one quantity to the other is invariant as the numerical values of both quantities change by the same factor.

A faucet in the science lab was also dripping. Jason placed a measuring cup under the faucet and recorded the total amount of water in the cup over time. Fill in any missing information in his table and then graph the data on the same axis provided above (Leaky Faucet).

Time, minutes	

dripping at a faster rate? Use both tables and graphs to justify your response.


Leaky Faucet: Part 2
Water, fluid ounces
0
1
2
3
4
5
6
7
8
Time, minutes
0
1
2
3
4
5
6
7
8
9
10
11

Looking Back, Looking Forward in Support of Student Learning

Bridge to Procedural Understanding – Cross–Multiplication Algorithm

Bridge to Algebraic Concepts

What bridges do you see?

References

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[Chapter 6 – Developing Concepts of Ratio and Proportion]

Link to McDonald's nutrition information:
http://www.mcdonalds.com/us/en/food/food_quality/nutrition_choices.html

Questions?

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