

Are You a Good Guesser?

Simulation Plan:

<i>TRIAL #</i>	<i>RESULTS</i>	<i># CORRECT</i>
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

Group Data

<i># Correct</i>	<i>Frequency</i>	<i>Fraction</i>	<i>Relative Frequency</i>	<i>Cumulative Frequency</i>
0				
1				
2				
3				

Whole Class Data

<i># Correct</i>	<i>Frequency</i>	<i>Fraction</i>	<i>Relative Frequency</i>	<i>Cumulative Frequency</i>
0				
1				
2				
3				

Name: _____ Date: _____

Quiz

1. Dlas kjkjie lkajsolk ajdaewouen kdjflsiurd kjdlfuielj a d kjdier?
 - a. 1
 - b. 2
2. jUthlekj aks eris jeioa kerul)kjlj* kjsdlf23jkdla llkq?
 - a. 1
 - b. 3
3. Pklje;la ksduewqo sier si fioejs fioei lkjie fkug slit slietkslj?
 - a. 0
 - b. 5

Name: _____ Date: _____

Quiz

1. Dlas kjkjie lkajsolk ajdaewouen kdjflsiurd kjdlfuielj a d kjdier?
 - a. 1
 - b. 2
2. jUthlekj aks eris jeioa kerul)kjlj* kjsdlf23jkdla llkq?
 - a. 1
 - b. 3
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 - a. 0
 - b. 5

INVESTIGATION 4.1: ARE YOU LUCKY?

***Did you know** that when it comes to winning a lottery, though you may like the glimmer of hope, the odds are stacked against you? For example, take the California state lottery; the probability of a winning ticket is approximately 1 in 15 million. If you were to buy 50 Lotto tickets each week, you might win about once every 5,000 years. So you might feel lucky, but don't bank on it!*

Reference: www.pbs.org/wgbh/pages/frontline/shows/gamble/odds/california.html

In your social studies class, you are facing a major test. Several of the questions are True/False, but, unfortunately, you must guess on four of the questions. In this investigation, you will design and carry out a simulation to determine how many of the four you might expect to get correct.

- A. What's your best guess as to how many questions you would get correct? Explain.
- B. Suppose you need to answer at least three of the four questions correctly. Estimate your chances for success. Write a sentence supporting your estimate.
- C. Estimate your chances for guessing correctly on all four questions. Estimate your chances for guessing incorrectly on all four questions. Again, write a sentence supporting your estimate.
- D. Design a simulation using coins, spinners, and/or number cubes. Conduct 5 trials for each of the three. Record the number correct for each of these 15 trials.
- E. Design a simulation using the random integer function on your calculator. Carry out the simulation for 35 additional trials and record the number correct for each of these trials.
- F. Summarize the results from the fifty trials in a frequency table. Include a relative frequency and cumulative frequency column.
- G. Calculate the summary statistics from the simulation. Discuss what you find.

- H. Create a histogram that displays your results. Comment on the histogram.
- I. Based on your results, what is your probability of guessing 0 correct? 1 correct? 2? 3? All 4? At least 2? At least 3? How do these compare with your estimates in parts A and B?
- J. Compare your results with other classmates. Then combine the class results.
- K. Suggest at least three other real world examples that are mathematically equivalent to this problem.

SAMPLE SOLUTION: ARE YOU LUCKY?

A. What's your best guess as to how many questions you would get correct? Explain.

This question should help teachers determine whether students understand the scenario presented. Because for any one question we have a 50% probability of getting it correct, in the long run we should also expect to get 50% of the questions correct. Consequently, the best guess as to how many questions we will get correct is two. However, students should be aware that there is variability here. We will not always get 50% right when we guess on True/False questions. Sometimes we'll do better, and sometimes we'll do worse. In the long run, it should average out at 50% of the questions, which, in this scenario, would mean getting 2 out of the 4 correct.

B. Suppose you need to answer at least three of the four questions correctly. Estimate your chances for success. Write a sentence supporting your estimate.

Answers will vary, but students' responses should give teachers insight into their sense of probability. Most should at least recognize that the probability of getting any particular question correct is 50%, but they will likely have difficulty extending their thinking into multiple questions. Answers, of course, should be between 0 and 1, perhaps expressed as percents. If not, then they are likely not quite ready to tackle this simulation, so some basics about probability may be in order. If students understand these basics, then it may be worthwhile to have discussions about whether they think their chances are pretty good of getting at least three of the four answers right. Try to build their intuition so that they understand that their chances are less than 50%. Though we do not encourage discussion of this at this point in the investigation, some teachers may want to know that the theoretical probability is 31.25% of getting at least three correct.

- C. Estimate your chances for guessing correctly on all four questions. Estimate your chances for guessing incorrectly on all four questions. Again, write a sentence supporting your estimate.

Again answers will vary, but students should recognize that their chances of getting all four correct are less than their chances for getting at least three correct. You may wish to ask them explicitly whether their chances of getting all four correct are greater than, less than, or equal to their chances of getting three of the four correct. If they are not alert to some reasonable expectations, then they will likely not recognize whether the simulation they are about to design and conduct is sound.

Because the probability on any one question is 50%, students should also recognize that the probability of getting all four wrong is the same as getting all four right. In either case, all four responses must be right or all four responses must be wrong. Though we do not advocating discussion of this at this point, teachers may wish to know that the probability of getting all four answers correct (or of getting all four wrong) with random guessing is 6.25%.

It may, however, be worth some discussion to help students understand that the probability of getting all answers right is not the same as the probability of getting them all wrong when the probability is not 50% for any particular question. For example, the probability of getting all questions right on multiple choice questions is less than the probability of getting all questions wrong (assuming there are more than two choices).

Similarly, depending on the goals of the teacher and the background of the students, you as the teacher might want students to understand that the probability of getting exactly three right is equal to the probability of getting exactly three wrong. Again, this is not the case if the probability of getting any one question right (or wrong) is not 50%.

D. Design a simulation using coins, spinners, and/or number cubes. Conduct 5 trials for each of the three. Record the number correct for each of these 15 trials.

Before we discuss our strategies, an important point must be clear. Students are not simulating whether an answer was True or False, but whether they guessed correctly or incorrectly. We don't care whether the answer was True or False, just whether we were right or wrong.

Many tools can be used for a simulation as long as they can generate a 50% probability of success for each question. With coins, students might decide that Heads represents a correct guess and Tails represents a wrong guess (not whether Heads represents True and Tails represents False). Flipping a coin four times would model one trial. Students would count the number of heads to indicate how many they got right in that particular trial.

More efficient students may use four coins and flip all four at one time for each trial, again counting the number of heads to indicate how many correct answers they had on that trial. Recall that they should conduct five trials using coins. A result we obtained from this strategy is shown in the table below. Make sure that students record their data and their results, although you may wish to give them some flexibility as to how to do this.

TRIAL #	RESULTS	# CORRECT
1	T, T, T, T	0
2	T, T, H, T	1
3	H, H, H, H	4
4	T, H, T, T	1
5	H, H, T, T	2

With spinners, the areas should be divided up. For example, if a spinner has eight sectors of equal area, four sectors can be used to represent a correct guess and the other four sectors, a wrong guess. If the spinner has an odd number of sectors, then one of the sectors can be considered a "non-guess," requiring a re-spin. The key is that two equal areas are used for correct and wrong guesses. Our spinner had eight sectors

of equal areas and we determined ahead of time that odds would represent correct guesses and evens would represent incorrect guesses. The data and results from five of our trials are shown below.

<i>TRIAL #</i>	<i>RESULTS</i>	<i># CORRECT</i>
6	8, 4, 6, 8	0
7	6, 2, 7, 3	2
8	4, 8, 3, 7	2
9	5, 7, 6, 5	3
10	3, 2, 3, 4	2

A strategy with number cubes would be similar to that used with spinners. Select three of the six faces to represent correct guesses and the other three to represent wrong guesses. We determined ahead of time that 2, 3, and 5 (the prime numbers) would represent correct guesses and 1, 4, and 6 would represent incorrect guesses. Data and results from five of our trials are again shown below.

<i>TRIAL #</i>	<i>RESULTS</i>	<i># CORRECT</i>
11	6, 6, 1, 5	1
12	5, 1, 1, 4	1
13	3, 4, 5, 1	2
14	4, 4, 6, 6	0
15	5, 3, 5, 3	4

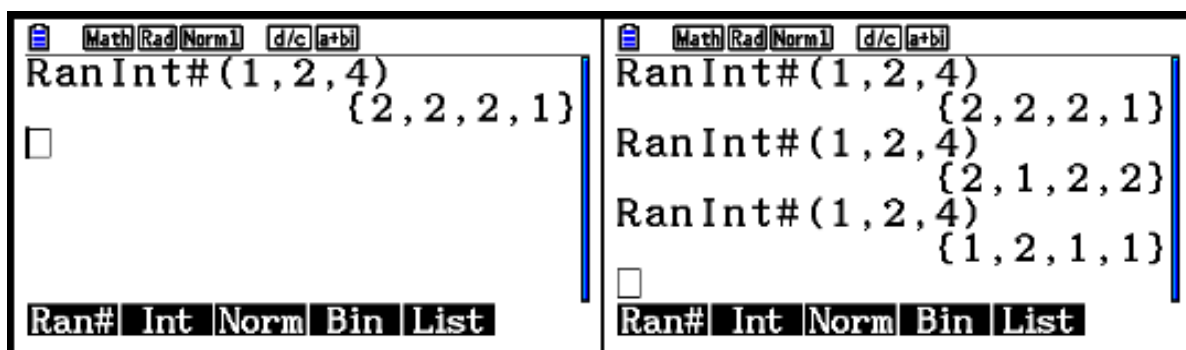
- E. Design a simulation using the random integer function on your calculator. Carry out the simulation for 35 additional trials and record the number correct for each of these trials.

The calculator can produce random integers quite quickly for us, and we have several options. For example, we could have the calculator produce four random integers between 1 and 100, and we could let 1-50 represent correct guesses and 51-100 represent incorrect guesses. We could also let odds represent correct guesses and evens represent incorrect guesses, or vice versa. The important consideration is that the probability of getting any particular question right (or wrong) is 50%.

We have chosen to let the calculator produce either a 1 or 2 and have decided ahead of time that 1 will represent a correct guess and 2 will represent an incorrect guess. From the Main Menu, choose Run-Matrix. Then,

☞ Press OPTN, F6 for more options, and F3 (PROB).

☞ Select F4 (RAND) and F2 (Int) for Random Integers. Type 1 for the smallest integer, a comma, 2 for the largest integer, another comma, 4 to indicate you want four of these integers, and close the parentheses. Then press EXE. See below left. This trial indicates we missed the first three questions but were correct on the last one. Continue to press EXE for additional trials as shown below right.



The raw data from our trials are shown below.

<i>TRIAL #</i>	<i>RESULTS</i>	<i># CORRECT</i>
16	2, 2, 2, 1	1
17	2, 1, 2, 2	1
18	1, 2, 1, 1	3
19	2, 1, 2, 2	1
20	2, 2, 2, 1	1
21	1, 1, 1, 1	4
22	1, 2, 2, 2	1
23	1, 1, 2, 1	3
24	1, 2, 1, 1	3
25	2, 1, 2, 2	1
26	1, 2, 2, 2	1
27	1, 2, 1, 1	3
28	1, 2, 2, 2	1
29	2, 1, 1, 1	3
30	2, 2, 2, 2	0
31	1, 1, 1, 1	4
32	2, 2, 1, 2	1
33	1, 2, 1, 2	2
34	2, 1, 1, 2	2
35	1, 2, 1, 1	3
36	2, 1, 1, 2	2
37	1, 2, 1, 1	3
38	1, 2, 1, 1	3
39	2, 1, 1, 1	3
40	2, 2, 1, 2	1
41	1, 1, 1, 2	3
42	1, 2, 1, 1	3
43	2, 2, 2, 1	1
44	2, 1, 1, 2	2
45	2, 1, 1, 1	3
46	1, 2, 2, 1	2
47	1, 1, 1, 2	3
48	1, 1, 2, 1	3
49	1, 1, 1, 2	3
50	1, 2, 1, 2	2

F. Summarize the results from the fifty trials in a frequency table. Include a relative frequency and cumulative frequency column

We need to determine how many times we had 0 correct, how many times we had 1 correct, how many times we had 2 correct, how many times we had 3 correct, and how many times we had 4 correct. The reported results should include the number correct, the frequency with which that occurred, the relative frequency of the response, and the

cumulative frequency of that response. The results from the 50 trials reported above are summarized below. However, students should use their own data, not the data reported here; consequently, answers will vary.

# CORRECT	FREQUENCY	REL FREQ	CUM FREQ
0	4	8%	8%
1	15	30%	38%
2	11	22%	60%
3	16	32%	92%
4	4	8%	100%
TOTAL	50	100%	

These results may be somewhat surprising. We would have expected more 2's than anything else, but here we obtained more 1's and more 3's than 2's. As we hope students would expect, the number of times we missed all of the questions or got all of them right was significantly less than the number of times for the other possibilities.

For those who are interested, the theoretical probabilities, which you might compare to the relative frequencies, are as follows:

$$P(0) = 0.0625$$

$$P(1) = 0.25$$

$$P(2) = 0.375$$

$$P(3) = 0.25$$

$$P(4) = 0.0625$$

Teachers may wish to aggregate their class data to see how close they come to these theoretical probabilities. The Law of Large Numbers suggests that if the total number of trials is large, the probabilities (or relative frequencies) from the simulation (which are called the empirical results) will approximate the theoretical probabilities.

G. Calculate the summary statistics from the simulation. Discuss what you find.

Before having students complete this part, teachers might wish to lead a discussion that indicates to students that even though we will use two lists, we are working with a single variable, specifically, the number correct. Rather than beginning our list by entering 0, 0, 0, 0, we have chosen to enter 0 in List 1 and then put a 4 in List 2 to indicate that we had 0 right 4 times in our simulation.

From the Main Menu, choose 2 (Statistics). If necessary, delete the data from List 1 and List 2, switch to a different File (using the SET UP), or find two empty lists that are available. We will be using List 1 for the number correct and List 2 for the frequencies.

- ☞ *Type in the numbers 0, 1, 2, 3, 4 into List 1 and their frequencies into List 2. Students should again use their own data. Our results, along with names in the subject fields, are shown below left, though the last values (4 right answers with a frequency of 4) are not shown.*
- ☞ *We'll now generate the statistics. Select F2 (CALC), and F6 (SET). We do not care what is entered into the 2Var fields, so we can ignore them. We are only concerned about the 1Var fields. The values are in List 1, so, if necessary, on the top line (1VarXList) press F1 (LIST), type in 1, and press EXE.*
- ☞ *Highlight 1Var Freq. Press F2 (LIST), type in 2, and press EXE. See below right.*

	List 1	List 2	List 3	List 4
SUB	RIGHT	FREQ		
1	0	4		
2	1	15		
3	2	11		
4	3	16		

1Var XList	:List1
1Var Freq	:List2
2Var XList	:List1
2Var YList	:List2
2Var Freq	:1

To generate the statistics,

From the screen shown above right, press EXIT and then F1 (1-VAR). You can scroll through the statistics as desired. The statistical results from our 50 trials are shown below.

1-Variable		1-Variable	
\bar{x}	=2.02	minX	=0
Σx	=101	Q1	=1
Σx^2	=267	Med	=2
σx	=1.12231902	Q3	=3
sx	=1.13371342	maxX	=4
n	=50	Mod	=3

1-Variable	
Med	=2
Q3	=3
maxX	=4
Mod	=3
Mod : n	=1
Mod : F	=16

The mean tells us that, from the 50 trials, we had a mean number of correct answers that was just above 2. The median for our simulation was also 2. This means that half the time we had two or fewer correct and half the time we had two or more correct. The mode, the most frequent number of correct answers, was 3, which occurred 16 times. This is a little unusual, but that's what our results produced.

The measures discussed so far all address measures of center, summary values that in some fashion indicate where the data are centered. Though extremely important and valuable, measures of center do not paint the entire picture. We should also look at the spread of the numbers. First we might notice that the minimum was 0 and the maximum was 4; consequently, we know that if we are faced with this situation in the future, we might not get any of the questions correct, but we also might not miss any of them.

Also of interest is the Interquartile Range (IQR). The first quartile, Q_1 , was 1, telling us that 25% of the time we had 1 or fewer correct. The third quartile, Q_3 , was 3, telling us that 75% of the time we had 3 or fewer correct. This also means that we had 3 or more correct 25% of the time. We determine the IQR by looking at Q_1 and Q_3 together. Here we find that our IQR, which represents the middle 50% of the distribution, was 2, ranging from 1 to 3.

We also might note that the standard deviation, denoted by s_x on the calculator, is a little more than 1. Although this is not a literal interpretation of the standard deviation, it might help us to think that, on average, the number we will get right differs from the mean of 2 by a little more than 1.

Combining the ideas of range, IQR, and standard deviation, we should begin to get a sense that even though on average we might expect to get 2 correct, we do not score 2 very consistently. The spread from 2 is significant; we should not have much confidence that in any one situation we will get 2 correct answers by random guessing. The only thing we should feel somewhat confident about is that if we guessed on four True/False questions in many, many situations, in the long run the average number we should expect to get correct is 2. We should also recognize that the chances of getting 0 or all 4 correct are not all that high.

H. Create a histogram that displays your results. Comment on the histogram.

If looking at the statistics as shown above,

☞ Press **SHIFT EXIT (QUIT)** to return to the Statistics home screen.

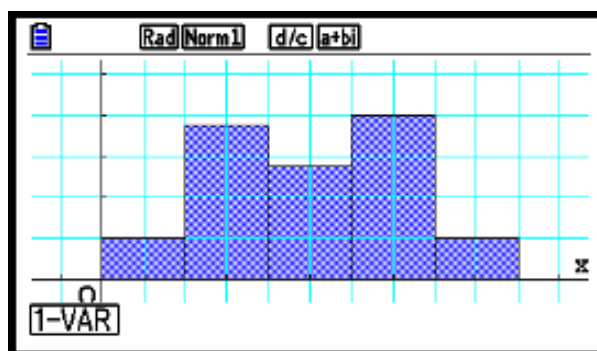
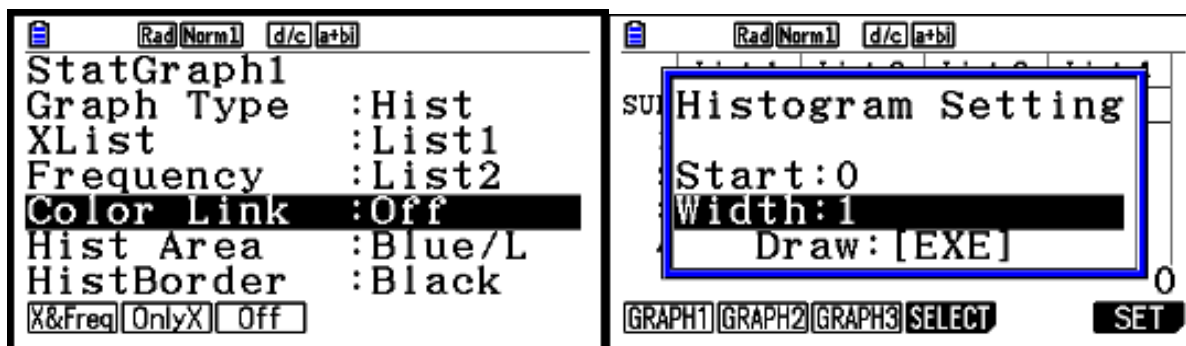
☞ Select **F1 (GRAPH)** and **F6 (SET)**.

☞ Set up **StatGraph1** as a histogram using **List 1** as the **XList** and **List 2** as the **Frequency**. The color links are not needed. See below left.

☞ Press **EXIT** to back up and **F1 (GRAPH1)** to draw the graph.

☞ Set the **Width of the bars** to 1 as shown below right.

☞ Then press EXE to draw the graph.



The scale factor for the y-axis, which represents the frequencies, is set at 4. Typically with the histogram we would find the highest bar above 2 and the heights getting lower as we move to left or right (above 1 and 3) and then even lower above 0 and 4. Here we see the lowest bars, as expected, above 0 and 4, but not the highest one at 2.

- I. Based on your results, what is your probability of guessing 0 correct? 1 correct? 2? 3? All 4? At least 2? At least 3? How do these compare with your estimates in parts A and B?

The primary purpose of this question is to make sure that students make the connection between the frequencies of what happened in the simulation and what they tell us about the probabilities of getting a certain number of correct answers. To answer the first questions here, we need look only at the relative frequencies in our table. We find that, according to our simulation, the probability of guessing 0 correct is 8%, guessing 1 correct 30%, guessing 2 correct 22%, guessing 3 correct 32%, and guessing all 4 correct 8%. Students should recognize that these percents sum to 100%.

The probability of getting at least 2 correct answers is the sum of getting 2, 3, or 4 correct answers, as “at least 2” suggests 2 or more. For our simulation, then, the probability of getting at least 2 correct, written mathematically, is $P(\text{at least 2 correct}) = 22\% + 32\% + 8\% = 62\%$. Thus we have, according to our simulation, a 62% probability of getting at least 2 right. Though we weren’t asked, we also find the probability of getting at least 2 wrong answers, written mathematically, is $P(\text{at least 2 wrong}) = 8\% + 30\% + 22\% = 60\%$.

Similarly, the probability of getting at least 3 correct answers is the sum of getting 3 or 4 correct answers. We find $P(\text{at least 3 correct}) = 32\% + 8\% = 40\%$. Again, though we were not asked, we could determine the probability of getting at least 3 wrong answers. We find $P(\text{at least 3 wrong}) = 8\% + 30\% = 38\%$. Teachers may wish to ask several questions of this nature, as we have found in our experience that students often have difficulty with phrases such as “at least” and “at most.”

As to how well the results compare to students’ estimates from the beginning of the problem, answers should vary.

- J. Compare your results with other classmates. Then combine the class results.

Again, answers will vary. However, in the long run, with the number of trials in the hundreds, the experimental probabilities should approach the theoretical probabilities, which were shown earlier and are repeated here to try to reinforce the idea of variability. They can be computed readily on the calculator using the Binomial Probability Distribution. You might wish to compare the class results with these values to see how close they come.

$$P(0) = 0.0625$$

$$P(1) = 0.25$$

$$P(2) = 0.375$$

$$P(3) = 0.25$$

$$P(4) = 0.0625$$

K. Suggest at least three other real world examples that are mathematically equivalent to this problem.

Answers will certainly vary but some possibilities follow.

- 1) *Suppose you have four children. How many of them would you expect to be girls (boys)? How likely is it that at least three are girls (boys)?*
- 2) *Suppose that the weather forecast calls for a 50% chance of rain for four different days. How many of the days would you expect it to rain? How likely is it that it rains on at most two of the days?*
- 3) *Suppose that on your trip to school you must pass through four traffic lights. Further suppose that the probability that any of the lights will be green when you reach the intersection is 50%. How many lights would you expect to be green when you reach them? What is the probability that all of the lights will be green?*