MEANINGFUL & MOTIVATING ALTERNATIVE ALGORITHMS

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Compensatory Addition

Adding two digit numbers is a skill taught in the early primary years. The **Compensatory Addition** technique can help students add two 2 digit numbers when regrouping is required. Applying the zero pairs concept discussed in the signed numbers section of this book will help to eliminate regrouping as an issue. In fact, regrouping will never occur.

Compensatory Addition is based upon a very simple principle. When a number is added to one addend of an addition problem and the same number is subtracted from the second addend, the sum is unchanged, for example, $8 + 3 = (8 + n) + (3 - n)$. If $n = 2$, then $8 + 3 = (8 + 2) + (3 - 2)$ or $8 + 3 = 10 + 1$.

This strategy can be very helpful in finding sums from 11 to 18, particularly the plus nine facts. You can use the **Compensatory Method** to help find the answer to a variety of addition facts. But it is important to know your ten facts first, 5 + 5, 6 + 4, 7 + 3, 8 + 2 and 9 + 1. Let's explore the strategy with the example below.

$$
\begin{array}{r}
9 + 1 = 10 \\
+\ 8 - 1 = +7 \\
\hline
17
\end{array}
$$

So, if we don't know the sum of 9 + 8, we could find it by adding 1 to 9 and subtracting 1 from 8. To repeat, the sum of two addends remains unchanged if we add and subtract the same number from the addends. The number when added must form a sum of 10 (9 + 1 or 8 + 2 and so on).

The **Compensatory Method** works with two digit numbers as well and eliminates regrouping, but a little reasoning is needed. Suppose we want to find the sum of 26 + 31. First, look to see which addend is closest to a multiple of 10. In the example 26 + 31, 31 is closer to 30 than 26 is to 30. Next, we ask ourselves, "How much do I need to add to or subtract from 31 to get 30?" The answer, of course, is 1. So, I will subtract 1 from 31 to achieve 30 and add 1 to 26 for 27. Now add 27 + 30 = 57, therefore 26 + 31 = 57. Let's try a more difficult two digit problem to demonstrate how regrouping is eliminated by using the **Compensatory Method.** Follow the steps below:

$$\begin{array}{rcr} 25 - 3 & = & 22 \\ \underline{+\,57} + 3 & = & \underline{+\,60} \\ & & 82 \end{array}$$

OTHER WAYS TO ADD AROUND THE WORLD

$$
\begin{array}{r}
346 \\
238 \\
+\,165 \\
\hline
\end{array}
$$

0/6	1/3	1/9

$$749$$

Try this one.

$$
\begin{array}{r}
619 \\
+\,586 \\
\hline
\end{array}
$$

/	/	/

The **No Exchange Subtraction Method** eliminates several steps and is based upon an easy to understand principle discussed in our previous discussion on **European Subtraction**. We have learned that we can add the same term to both the minuend and subtrahend without changing the difference. To help students understand this concept, use a number line as a visual prop. To find 7 – 2, place a marker on seven and another marker on two. Now move both markers up one. This will show that the difference between 8 and 3 is the same as the difference between 7 and 2. We can now apply our knowledge of this important principle to a two digit subtraction problem which requires exchanging.

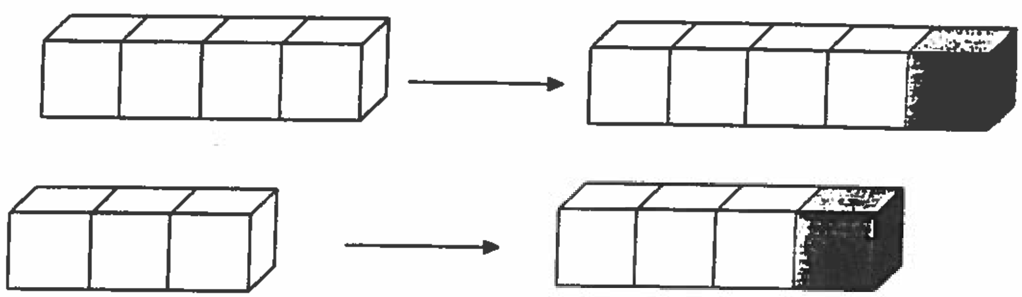
$$\begin{array}{r} 42 \\ 29 \\ \hline \\ \end{array} \longrightarrow \begin{array}{r} 43 \\ -30 \\ \hline 13 \end{array}$$

How much do I need to add to 29 to get to 30? Answer 1. Add 1 to 29 and then add 1 to 42, producing a new problem (43 – 30) with the same result as 42– 29.

$$\begin{array}{r} 84 \\ 17 \\ \hline \\ \end{array} \longrightarrow \begin{array}{r} 87 \\ -20 \\ \hline 67 \end{array}$$

How much do I need to add to 17 to get to 20? Answer 3. Add 3 to 17 and then add 3 to 84 producing a new problem (87 – 20) which yields the same result as 84 – 17.

After practicing with small numbers, review the principle involved to build understanding of how to work with larger numbers. Show this principle visually using blocks. Compare four blocks and three blocks to show the difference of one. See the diagram below.



Add on one gray block to each group to show that the difference did not change. The difference is still one block. Now try **471 – 289**. What must we add to both terms to eliminate exchanging?

Two Digit Addition with Regrouping

Try this method of addition with regrouping.

$$\begin{array}{r} 54 \\ \underline{+\,28} \end{array}$$

- Think 4 + 8 =12
- Record the 12 under the sum bar.
- Now add 5 + 2 + 1 in the tens column for 8
- Record the 8 and scratch out the 1.

So 54 + 28 = 82

See below:

$$\begin{array}{r} \mathbf{54} \\ \underline{\mathbf{+\,28}} \\ \cancel{1}\mathbf{2} \\ \mathbf{8} \end{array}$$

Two digit subtraction with exchange:

Try this method of subtraction with exchange.

$$\begin{array}{r} 57 \\ \underline{-\ 29} \end{array}$$

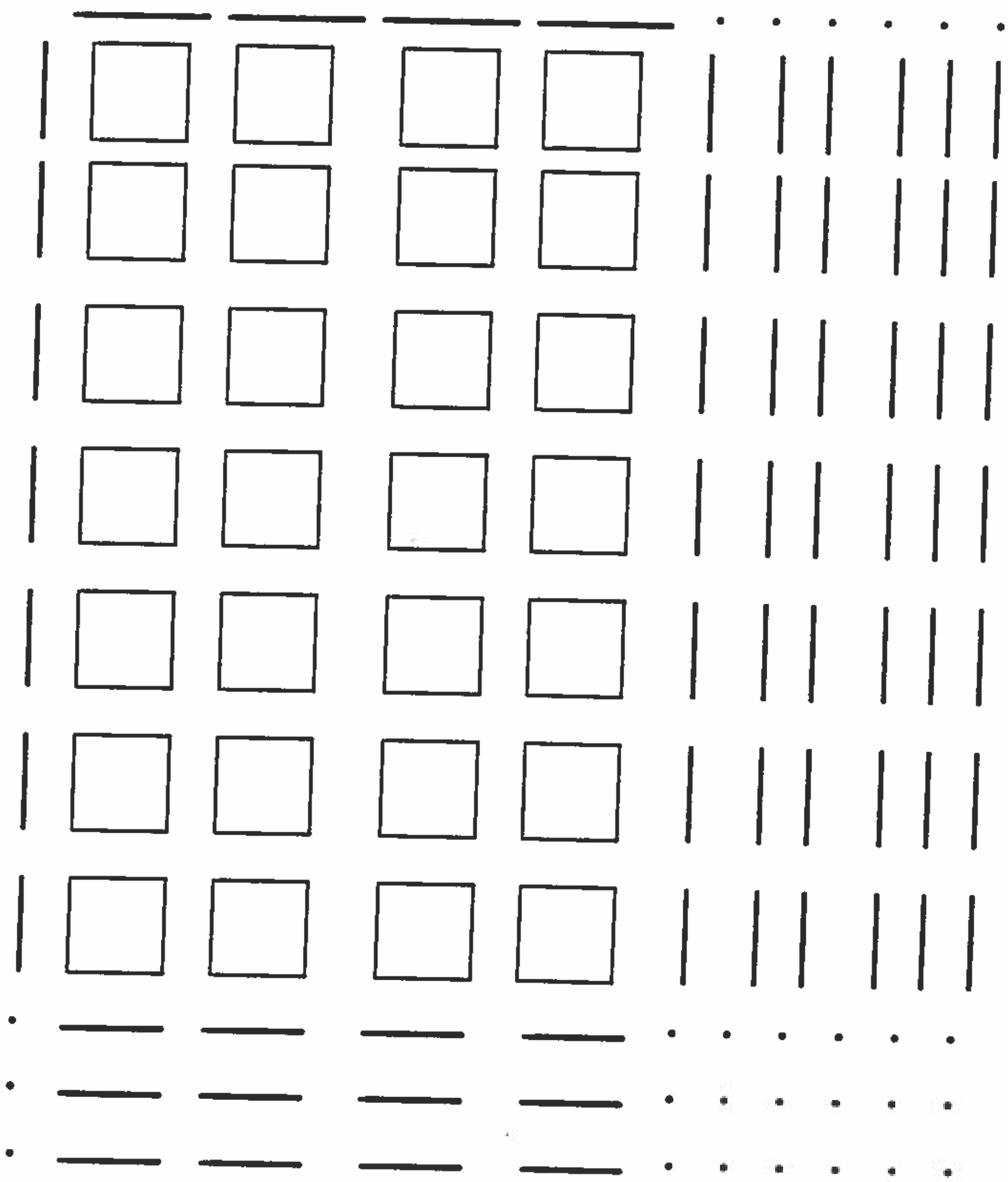
- Think $9 + x = 10$. $X = 1$.
- Add 1 to 7. $7 + 1 = 8$.
- Record 8 in the ones column under the 9.
- Add 1 to the 2 tens in 29, making it 30

Subtract 5 tens – 3 tens equals 2 tens.

$57 - 29 = 28$

PARTIAL PRODUCT MULTIPLICATION
USING BASE 10 PIECES

$$\begin{array}{r} 46 \\ \underline{\times 73} \end{array}$$



4 x 7 = 28 hundreds

7 x 6 = 42 tens

4 x 3 = 12 tens

6 x 3 = 18 ones

$$\begin{array}{r} 2{,}800 \\ 420 \\ 120 \\ \underline{+\ 18} \\ 3{,}358 \end{array}$$

PARTIAL PRODUCT MULTIPLICATION

```
   46
X  73
```

	40	6
70	2800	420
3	120	18

Step 1: Fill in zeros.
Step 2: Multiply. (Just Facts!)
Step 3: Add all the parts.

```
 2,800
   420
   120
+   18
 3,358
```

Try this one: 68 x 57

PEASANT MULTIPLICATION

This is a great mental math method!
1. Halve and double.
2. Add the factors with odd "leaders".

*37 x 46	46
18 x 92	184
*9 x 184	+ 1472
4 x 368	1702
2 x 736	
*1 x 1472	

Try this one:

20 x 33 (Easy example to start with!)

EGYPTIAN MULTIPLICATION

1) List easy multiples of a factor that can be done mentally by doubling each time (x1, x2, x4, x8, x16).
2) Add enough multiples to add up to the first factor.

<u>30 x 53</u>

1 x 53 = 53		
2 x 53 = 106	16	848
4 x 53 = 212	8	424
8 x 53 = 424	4	212
16 x 53= 848	+ 2	+ 106
	30	1,590

<u>46 x 67</u>

1 x 67 = 67	16	1,072
2 x 67 = 134	16	1,072
4 x 67 = 268	8	536
8 x 67 = 536	4	268
16 x 67 = 1,072	+2	+134
	46	3,082

Try this one:

57 x 74

JELLY BEAN DIVISION

6 (bowls)) 347 (number of jelly beans)

- 60 (put 10 in each)

287

- 60 (put 10 in each)

227

- 120 (put 20 in each)

107

- 60 (put 10 in each)

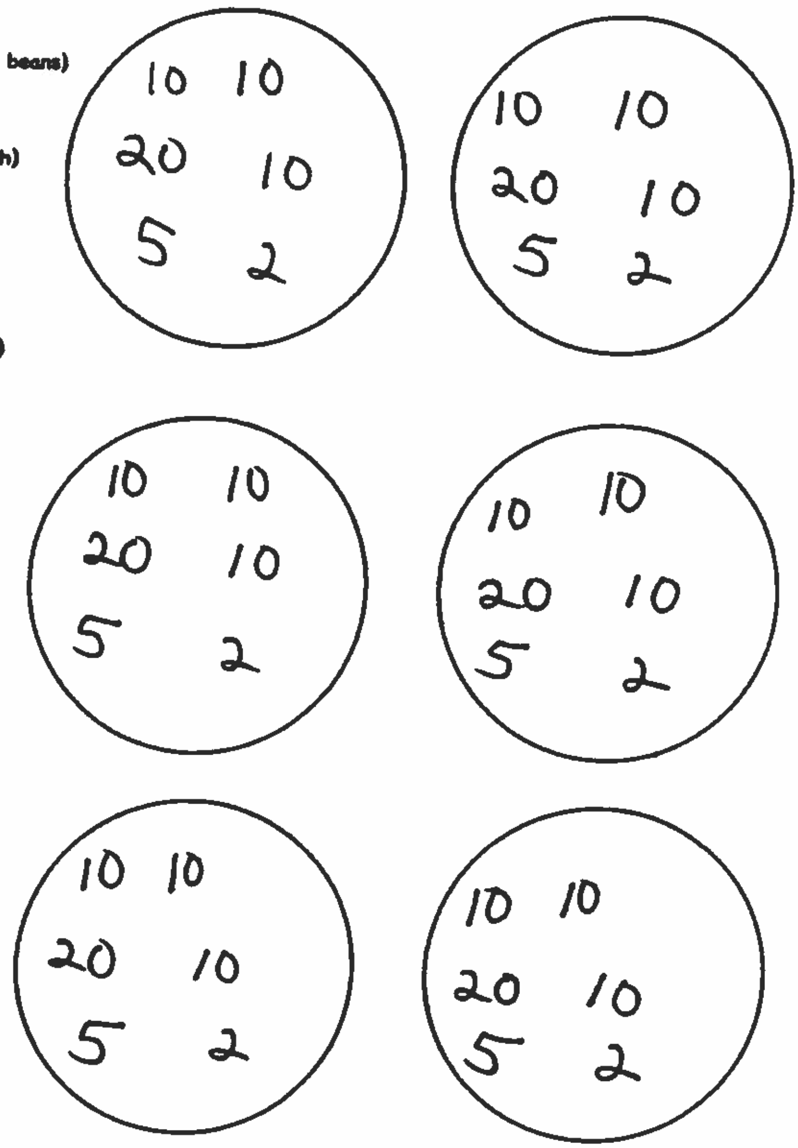
47

- 30 (put 5 in each)

17

- 12 (put 2 in each)

5 (leftover jelly beans)



Answer: 57 jelly beans in each bowl,
with 5 left over

JELLY BEAN DIVISION

LADDER DIVISION

Think Box

1 x 32 = 32
2 x 32 = 64
4 x 32 =128
10 x 32 =320
20 x 32 =640
40 x 32=1280
100x32=3200

32	4,356	
	-3,200	100
	1,156	
	- 640	20
	516	
	- 320	10
	196	
	- 128	4
	68	
	64	2
	R. 4	136 (add this column)

LADDER DIVISION

Think Box

Division of Fractions

As a student, I can recall vividly my first introduction to dividing fractions. My fifth grade teacher, Mrs. Peterson entered the classroom and announced, "Today we are going to learn how to divide fractions." There were a few groans, including my own. Mrs. Peterson was swift and to the point. Without teaching any conceptual understanding at all and in less than five minutes, she showed us **how** to divide. "Flip it upside and multiply," she chirped as she pointed to one of the terms. Remarkably, she concluded by giving us a set of chalkboard problems to practice. As hard as I tried, I could never remember which term to invert. Unfortunately, I have heard several of my college students describe similar episodes. I certainly hope the students are exaggerating their stories. Needless to say, the inversion method remains alive in our schools today as the main (if not the only) method of dividing fractions. Several other division techniques **could** be used.

Common Division Method

This is an easy technique to use and relates directly to the multiplication algorithm. For multiplication most students learn the standard algorithm, multiply the numerators and multiply the denominators. If we cement this technique for multiplication, why not teach division similarly? In other words, divide the numerators and divide the denominators. Sadly, we do not use this approach. Instead, we teach division using the "inversion" method, a separate and unrelated concept. In the **Common Division Method** we divide the numerators and denominators just as we multiply them during multiplication. Study the example and you will clearly understand the method.

Example: $$\frac{16}{24} \div \frac{4}{8} = \frac{4}{3}$$

Divide the numerator $16 \div 4 = 4$ and divide the denominator $24 \div 8 = 3$.That's how easy it is! No inversion is necessary. In this example, the numerators and denominators can be divided exactly without any remainders. When division is inexact, we need to find an equivalent fraction for the first term (the dividend). See the example below.

Example: $\frac{6}{7} \div \frac{3}{4} = n$

$$\frac{24}{28} \div \frac{3}{4} = \frac{8}{7}$$

In the example above, 6 can be divided exactly by 3, but 7 cannot be divided exactly by 4. As a result, we must find an equivalent fraction for 6/7 so that both the numerator and the denominator can be divided exactly. For this reason, we converted 6/7 into an equivalent fraction 24/28.

When **both** the numerator and the denominator cannot be divided exactly, an equivalent fraction must be found so that **both** numerators and denominators can be divided exactly. See below.

Example: $\frac{3}{5} \div \frac{2}{3} = n$ Change 3/5 to 18/30 an equivalent fraction, so that all numbers can be divided.

$$\frac{18}{30} \div \frac{2}{3} = \frac{9}{10}$$

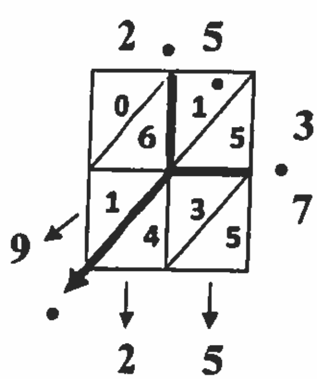
Decimals

Lattice Decimal Multiplication Algorithm

Lattice Decimal Multiplication is similar to the **Lattice Multiplication Method** for whole numbers described on page 16. First, construct the lattice boxes for the partial products and place the two factors of the problem around the outside of the boxes. See below. Enter the partial products inside the cells with the first digit of the products in the top part of the box and the second digit in the lower section.

Add along the diagonals. If there is any regrouping ("carrying'), place a black dot in the next diagonal to indicate that there has been regrouping.

Locate the decimal point by drawing two bold lines to form an intersection. Draw a bold line along the diagonal to the end. This is where the decimal point is placed in the product. Study the example below and practice some problems on your own. It will be quite easy to follow.



Solve 2.5 x 3.7 = *n*. Fill in the boxes by completing four separate multiplications: 3 x 5 = 15, 3 x 2 = 6, 7 x 5 = 35 and 7 x 2 = 14. Add along the diagonals. (The black dot inside the 3 x 5 cell indicates that "1" is carried from the second diagonal.) Draw bold lines from the decimal points in the two factors until the lines intersect and then draw a third line from the intersection to the end of the lattice boxes. This is where you place the decimal point in the answer.

Dividing Decimals

In division, we **subtract** the digits to the right of the decimal point. In the example below, we subtract 3 – 1 = 2, because there are 3 digits to the right of the decimal point in the dividend 1.244 and 1 digit to the right of the decimal point in the divisor .4.

$$\begin{array}{r} \textbf{3.11} \\ \textbf{.4)}\overline{\textbf{1.244}} \end{array}$$

To solve 1.244 ÷ .4, ignore the decimal points and complete the division as if you were dividing whole numbers. After you have divided, subtract 3 – 1 = 2 and insert the decimal point between the 3 and the 1 in the quotient.

Dividend Places – Divisor Places = Quotient Places

Try this second example to give yourself a little practice.

$$\begin{array}{r} \textbf{2.5} \\ \textbf{.25)}\overline{\textbf{.625}} \end{array}$$

There are 3 digits to the right of the decimal point in the dividend and 2 digits to the right of the decimal point in the divisor. Subtract 3 - 2 = 1. There should be one digit to the right of the decimal point in the quotient. Place the decimal point between the 2 and the 5 in the quotient.

On some occasions, there are not enough decimal point places to subtract. If this occurs, annex zeroes to the right of the digits in the dividend. Annexing zeroes to the right of a decimal does not change the value of the number.

$$\textbf{.11)}\overline{\textbf{6.6}} \longrightarrow \begin{array}{r} \textbf{60.} \\ \textbf{.11)}\overline{\textbf{6.60}} \end{array}$$

In the example above, we could not subtract 1 place – 2 places, so we annexed a zero to 6.6 to expedite subtraction. Now there are 2 digits to the right of the decimal point in the dividend and 2 places in the divisor. 2 – 2 = 0. The decimal point should be placed after the 0 in the quotient because there will be no numbers to the right of the decimal point in the quotient.