A Sixth Grader Reinvents the Pythagorean Theorem

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CAMMP-MATH Method

At Socrates Academy a K – 8 charter school in Charlotte, NC

CAMMP-MATH Method

Comprehensively

Applied

Manipulative

Mathematics

Program

Dr. Michael G. Green & Dr. Jack A. Piel from UNC-Charlotte developed this method

Four Levels of Instruction

- **Concrete**
- **Representational**
- **Transitional**
- **Symbolic**

CAMMP-MATH Method

At Socrates Academy
a 6th grader reinvents
the Pythagorean Theorem

Pythagorean Theorem (1)

Students

- *cut off squares of 5x5, 4x4, and 3x3 from grid paper*
- *also cut off squares of 10x10, 8x8, and 6x6* (Concrete-Representational Levels)
- *glue the squares onto a piece of paper forming triangles with the three different sides of each set of squares; identify the type of these triangles* (Representational – Transitional Levels)

Pythagorean Theorem (2)

Students

- *write a mathematical expression to describe the relationship of the areas of squares in both cases* (Symbolic Level)
- *write a general expression of this relationship (ideally, the Pythagorean Theorem)*

Pythagorean Theorem (3)

Πυθαγόρειο Θεώρημα

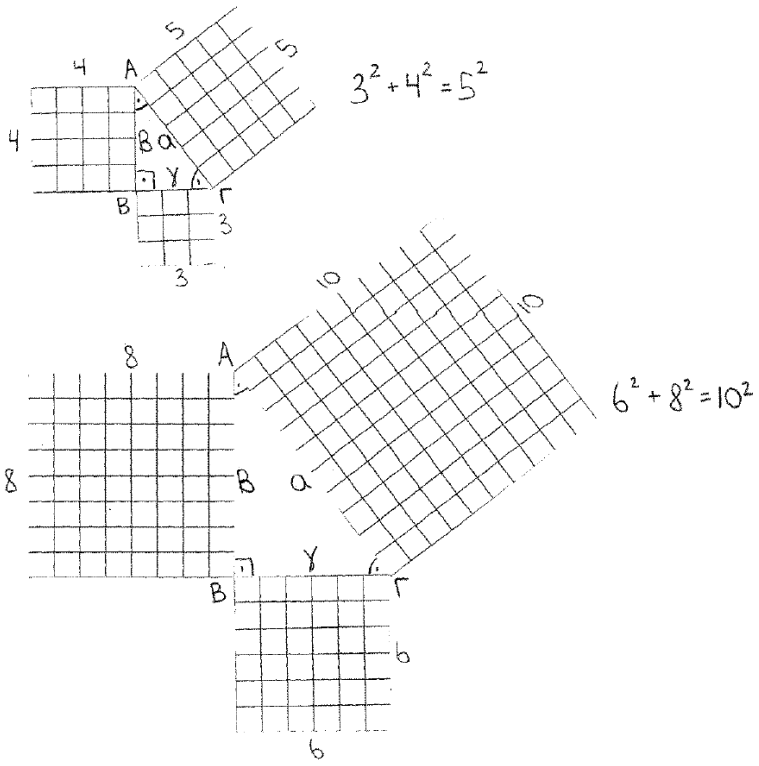
Το τετράγωνο της υποτείνουσας είναι ίσο με το άθροισμα των τετραγώνων των δύο καθέτων πλευρών.

Pythagorean Theorem

The square of hypotenuse is equal to the sum of squares of the other two perpendicular sides

$\alpha^2 = \beta^2 + \gamma^2$ - το τρίγωνο είναι ορθογώνιο
- the triangle is right

$\alpha^2 \neq \beta^2 + \gamma^2$ - το τρίγωνο δεν είναι ορθογώνιο
- the triangle is not right



CAMMP-MATH Method

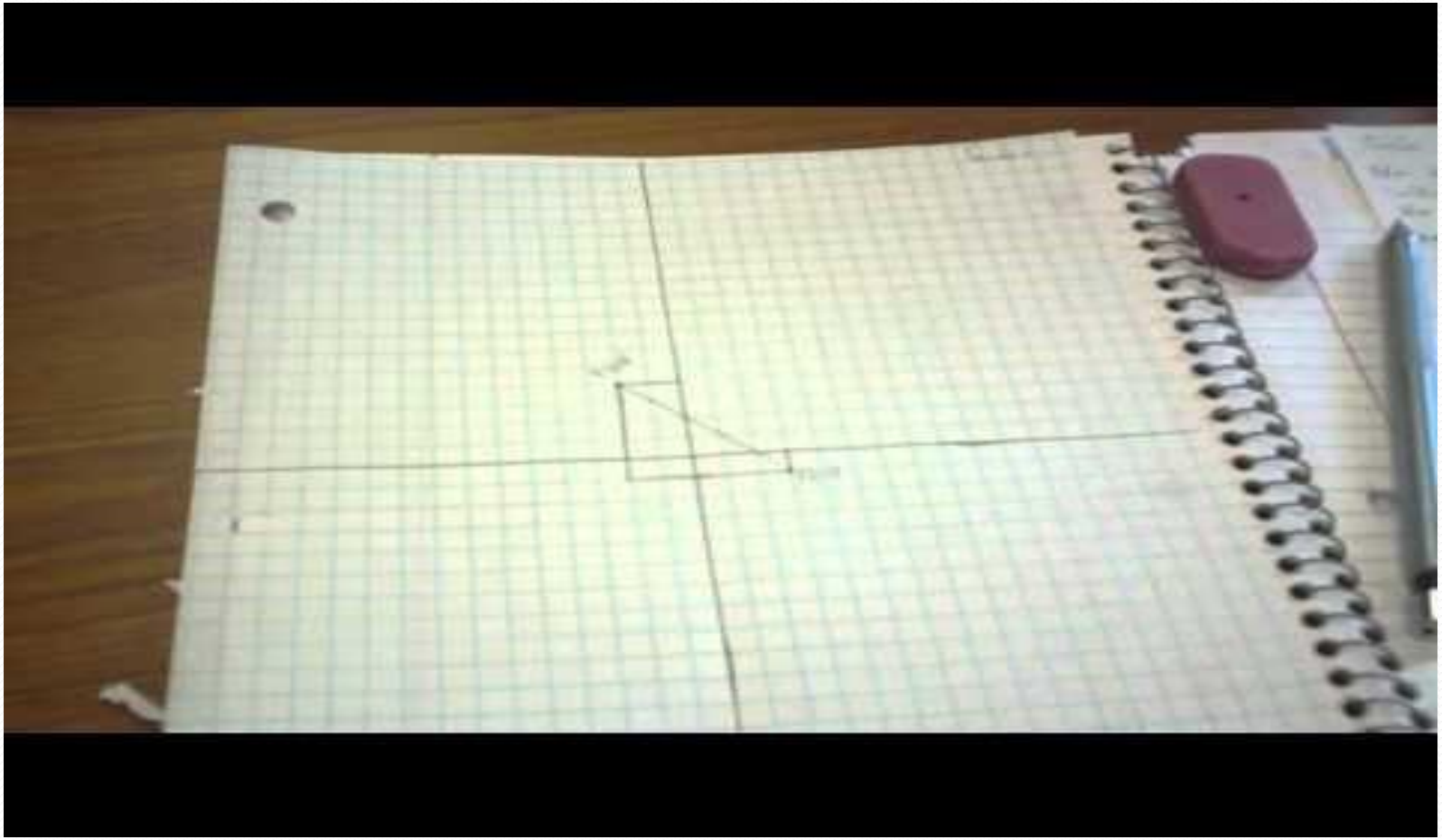
At Socrates Academy
a 6^{th} grader applies
the Pythagorean Theorem
to solve a problem about
the formula of the distance
of two points
in Cartesian coordinate system

Distance between Two Points

Students

- *put points A(-2, 4) and B(3, -1) on the Cartesian coordinate system* (Concrete – Representational Levels)
- *figure out the distance between these two points by forming a right triangle* (Transitional – Symbolic Levels)

Distance between two Points



All three sides of a triangle

Students

- *need to calculate the length of each one of the three sides of triangle ABC with vertices at A(-2, 5), B(3, -1), and C(-3, 3) on the x-y coordinate system* (Representational Level)
- *form right triangles around the three sides* (Transitional Level)
- *use the Pythagorean Theorem for each right triangle* (Symbolic Level)

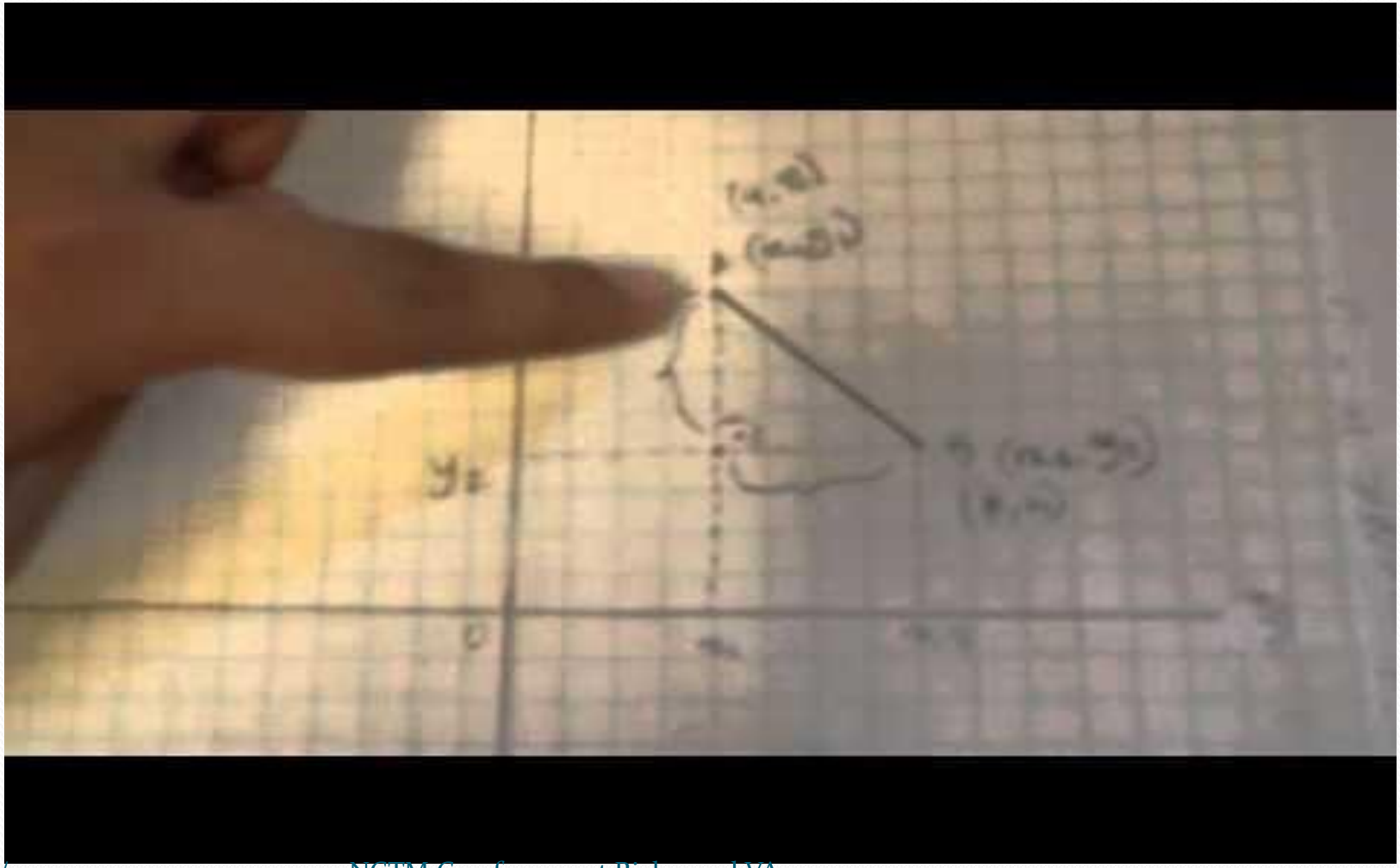
All the three Sides of a Triangle

Formula for the Distance between Two Points

Students

- *find the general formula for the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ forming a right triangle and using the Pythagorean Theorem*

Formula for the Distance between Two Points



The Student

- Had 6 years experience with CAMMP-based math instruction
- Reflected a deep understanding of Cartesian coordinates
- Spontaneously constructed a specific solution to the problem AND a general scheme for any problem
- The general scheme WAS the Pythagorean Theorem

Thank you for your attention!

… please ask your questions