THE MATHEMATICS MENTORSHIP PROJECT

Developing Mathematical Writing

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Topics for today

About the project

- Background
- Details
- Problem and Sample Solutions

Why writing?

Development of a framework for math writing

What students said about the project

Summer 2012…

“Mathematics of Geodesic Domes” Course at the UVA Summer Enrichment Program (SEP)

- A program of the University of Virginia Curry School of Education
- Started in 1977
- Serves “gifted/high ability students” entering grades 5 – 11
- 12-day residential program
  - Academic and social components

“I like math…

I just don’t like

school math.”

“I have math questions my teachers don’t have time to answer.”

"If school offers a math enrichment program, the person running it needs to know about math."

My question...

How can we offer ongoing math enrichment throughout the year?

Mathematics Mentoring Project

Students would receive a monthly open-ended math problem and…

- have 2 to 3 weeks to work on it;
- solve it however they felt was best for that problem; and
- submit and receive written feedback.

Could be done by email or regular mail

Mathematics Mentoring Project

A pilot project for the 2012-2013 school year

- Students who had expressed an interest in math enrichment
- Invited 15 students, most from my summer class
- Students 5th and 6th grade age (two were moved into 7th grade)
  - 2 girls, 13 boys

Continued in the 2013-2014 school year

- 80 students invited (40 girls, 40 boys)
- 27 students accepted (12 girls, 15 boys)

To sum things up so far…

Let's give kids interesting math problems to solve, and give them feedback and encouragement.

Mathematics Mentoring Project

Problem Set 1

The problem

Eric the Sheep is in line to be shorn. He is last in line. Each time the sheep shearer takes one sheep from the front of the line, Eric sneaks past two sheep to get closer to the front. If there are 50 sheep in front of Eric to start with, how many sheep will be shorn before Eric gets to the front?

What happens if there are a different number of sheep in front of Eric?

Questions to think about

- How do you know?
- Can you prove it?
- Does it always work this way?

Extending the problem - Some other things you might investigate

Patterns and Relationships

Mathematicians are interested in patterns and relationships. Do you notice any relationship between the number of sheep in front of Eric and the number of sheep shorn before Eric gets to the front? What are some other patterns or relationships you notice?

What happens if...?

Mathematicians are also interested in the question, "What happens if...?" Now, try to take to problem further. You might want to investigate what happens if Eric sneaks past 3 sheep, or 4 sheep, or more. You might want to investigate what happens if there are 2 shearers, or 3 shearers, or more. You might have a different idea about something to change in the problem and see what happens!

(Note: Problem taken from http://www.blackdouglas.com.au/taskcentre/045eric.htm)

rmation for you

ing on the problem

u work on this problem, you may want to read it once or twice before you start working.
t immediately have an idea how to start or you may need to think about it a while first.
e things you try might not work. That's okay! You might have to start over. That's okay!
lf, "What if...?" and investigate what happens when you change the problem (such as
e sheep or more shearers). Remember, you aren't getting a grade for this – the purpose
an interesting problem and to see what happens when you try different strategies.
e to limit yourself to the ideas I have included – feel free to come up with your own
o extend the problem!

e problem and asking questions

his problem with other people. Mathematicians often discuss things with each
when they aren't sure about something or if they want feedback on their work.
contact me if you have any questions or if you want me to look at something
You don't have to wait until you submit your solution to contact me!

problem

n the problem, you will need to write up your solution. This can be in any
the problem and your solution (text, graphs, charts, tables, diagrams, etc.).
t you have good, clear explanations, not just an answer. Here are a few
t what you have written:

e question?
ution? Have I shown that my solution really is correct?
nized so that people understand it?
? Could someone else understand what I did and how I did it?
re than just the basic problem? Have I looked for patterns? Have I

Send

lution, send it to me! You can send it by email or regular mail. I
s. These extra things don't have to be part of your solution –
end me your solution by **Friday, September 21, 2012.**

call you – I have the list from this summer but I want to make
ne you prefer!)
(not the day you were born!)

u get stuck? (Was it understanding the problem? How to
hat to do next? Or was it something else?)
Too boring? What kind of problem might you prefer?
ally interested in or would like to learn more about?

carefully and send you feedback. The feedback is not
w the parts that you did really well on plus ways to
Soon you will receive Problem Set 2!

Problem 1…

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Questions to think about

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- Can you prove it?
- Does it always work this way?

Problem 1…

Extending the problem - Some other things you might investigate

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Example 1

How many sheep? 17 Sheep

What happens with more? You divide by 3

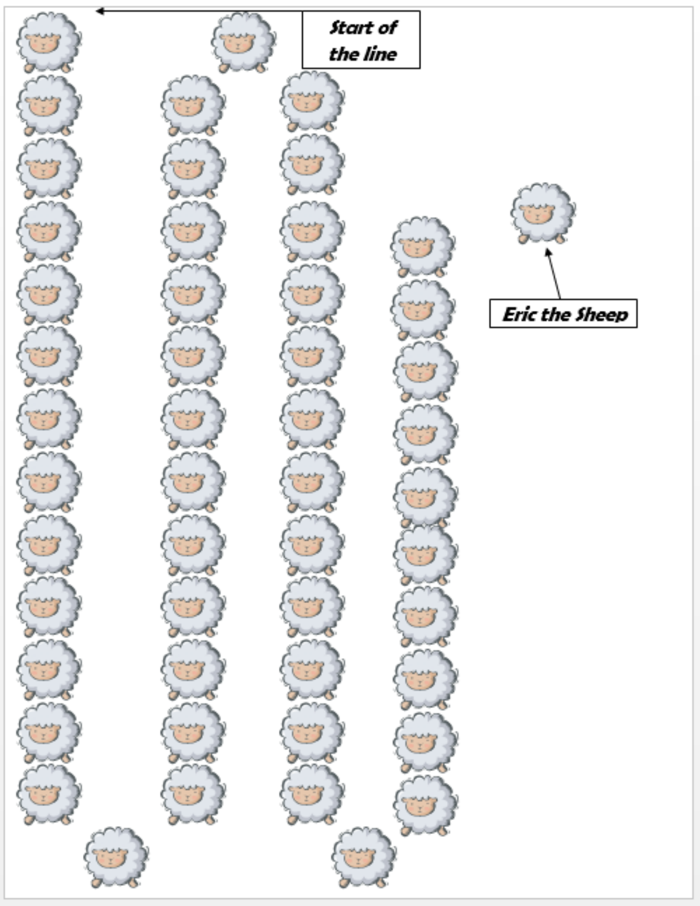
How do you know? I tried it by drawing it out

Can you prove it? Tried it with multiple numbers

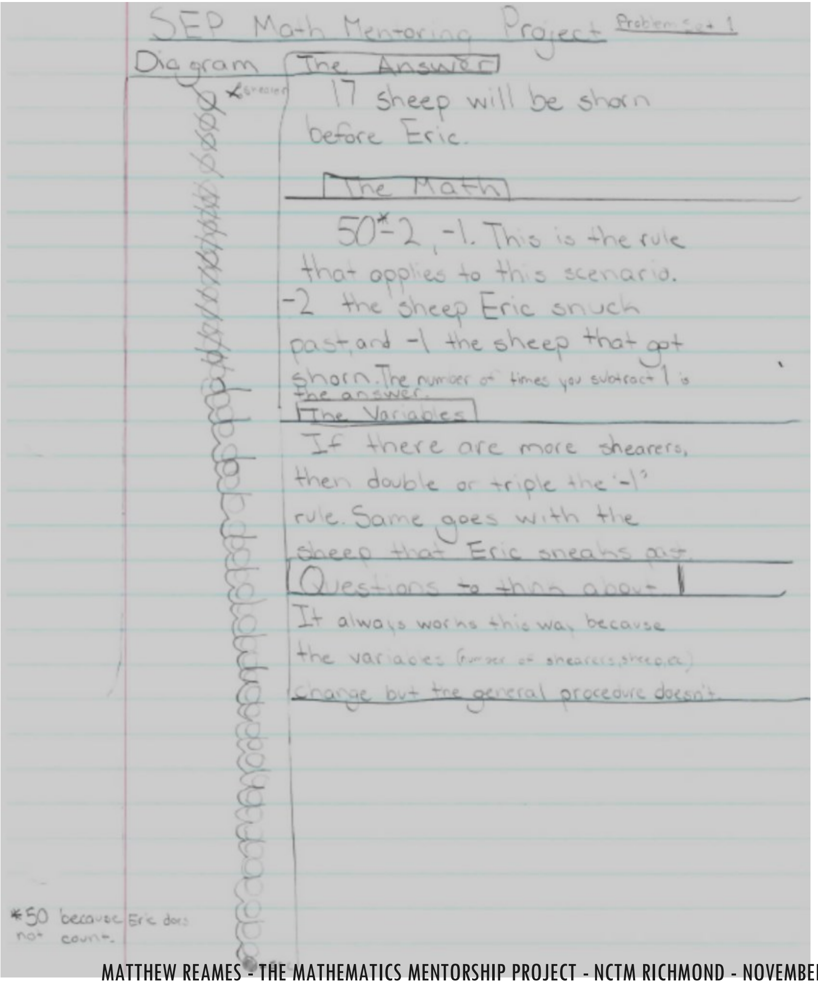
Does it always work like this? Unless it's not divided by 3

A pretty average problem but I had a hard time explaining my work.

I like problems with variables in them. I also like order of operations.



Example 2



SEP Math Mentoring Project — Problem Set 1

Diagram

shearer

The Answer

17 sheep will be shorn before Eric.

The Math

50*-2, -1. This is the rule that applies to this scenario. -2 the sheep Eric snuck past, and -1 the sheep that got shorn. The number of times you subtract 1 is the answer.

The Variables

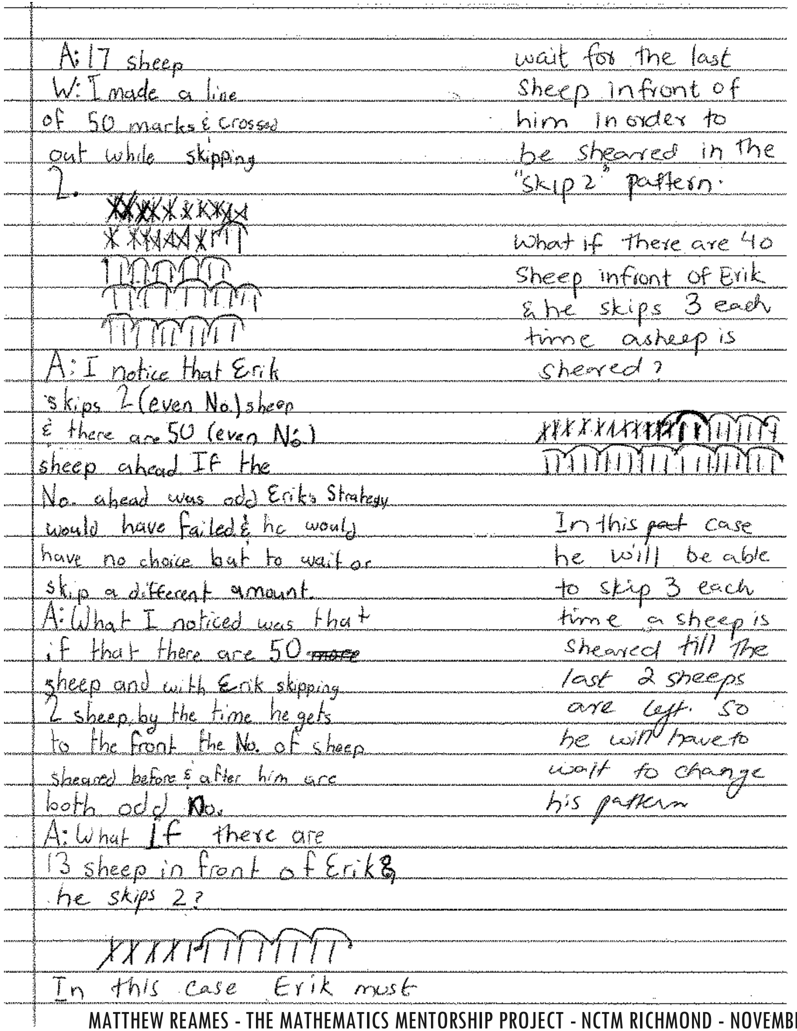
If there are more shearers, then double or triple the '-1' rule. Same goes with the sheep that Eric sneaks past.

Questions to think about

It always works this way because the variables (number of shearers, sheep, etc.) change but the general procedure doesn't.

*50 because Eric does not count.

Example 3



A: 17 sheep
W: I made a line of 50 marks & crossed out while skipping 2.

A: I notice that Erik skips 2 (even No.) sheep & there are 50 (even No.) sheep ahead. If the No. ahead was odd Erik's Strategy would have failed & he would have no choice but to wait or skip a different amount.

A: What I noticed was that if that there are 50 ~~more~~ sheep and with Erik skipping 2 sheep by the time he gets to the front the No. of sheep sheared before & after him are both odd No.

A: What If there are 13 sheep in front of Erik & he skips 2?

In this case Erik must wait for the last sheep infront of him in order to be sheared in the "skip 2" pattern.

What if there are 40 sheep infront of Erik & he skips 3 each time a sheep is sheared?

In this ~~part~~ case he will be able to skip 3 each time a sheep is sheared till the last 2 sheeps are left. So he will have to wait to change his pattern

Example 4

I started of with the question by finding a way to answer it. I tried to use the fact the 50 sheep are in front of Eric. I did the number of sheep in front of him (50) than subtract the one sheep that had gotten shorn. Then I subtracted the two sheep that he skipped. I labeled that as turn 1. I repeated this as turn two. I continued doing this until I got to 0, turn 17. Next, I tried to find a pattern. I used 17 as my answer. I made myself a chart of questions and answered them.

# of sheep in front	Total sheep	# of sheep shorn		# of moves up		total	answer	
50	51	1	+	2	=	3	17	51 divided by 3 = 17
99	100	1	+	3	=	4	25	100 divided by 4 = 25
125	126	2	+	4	=	6	21	126 divided by 6 = 21
95	96	1	+	3	=	4	24	96 divided by 4 = 24
1,232	1,233	3	+	6	=	9	137	1,233 divided by 9 = 137

I realized a new pattern. Sheep sheared (1) + # of moves up (2) = total (3). Then you divide total sheep (51) by total (3) to get your answer (17).

Example 5

The answer that I got for the base problem was 17. I got this answer by making a visual model with Eric, a shearer and 50 dots representing sheep between Eric and the shearer. I crossed out three dots each time meaning either that Eric had passed them or that they had been sheared, and then made a tally mark. I did this repeatedly until all the sheep and Eric had been crossed out, and then added up my tallies. To check myself, I then did 51/3, which was my answer. I also figured out that each time the number of sheep in front of Eric increases by 1 the number of sheep that have to be shorn increases by 1/3, meaning that if the number of sheep in front of Eric increases by 3, the number of sheep that have to be shorn before he is shorn increases by 1.

Example 6 – part 1

ANSWER: **17**

I got this answer with a very simple table. All I did was put the number of sheep shorn in the x column, and then the number of sheep left in front of Eric (after he sneaks in front of two.) So in the X column, I put numbers starting with 1, and counting up. In the Y column, I put numbers starting with 50 (across from the zero) and counted down by threes. That was the easy part.

X	Y
0	50
1	47
2	44
3	41
4	38
5	35
6	32
7	29
8	26
9	23
10	20
11	17
12	14
13	11
14	8
15	5
16	2
17	-1

Example 6 – part 2

Now, I had to try to come up with an equation that connected the numbers across. I thought about it for a while and I eventually turned the Y column upside-down, and then multiplied the X column by three and subtracted one.

Y	X	X * 3 - 1
-1	0	-1
2	1	2
5	2	5
8	3	8
11	4	11
14	5	14
17	6	17
20	7	20
23	8	23
26	9	26
29	10	29
32	11	32
35	12	35
38	13	38
41	14	41
44	15	44
47	16	47
50	17	50

Example 6 – part 3

This led to another equation, The equation is: $W/(X+Y)=Z$

W = the total number of sheep in line (including Eric) (50 +1)

X = the number of sheep that are shorn before he cuts in front of Y (1)

Y = the number of sheep Eric cuts in front of when X number of sheep is shorn (2)

Z = answer

I got stuck coming up with this equation. I took a 15-minute break and then it just came to me. I can use this equation if any of the W, X, or Y variables change, and I tested it to make sure.

Feedback

Dear ____,

Thanks for sending in your solution to the first problem. I like how you described the way you solved the initial problem. I do have a question, though. You said that you would circle one sheep and then erase three – when I followed your instructions, I only got 13. Did you mean that you erased two sheep instead of three?

I like the idea of calculating how long Eric stayed in the line, plus the idea that alternate sheep took 10 minutes. I think both 1 hour 25 minutes and 2 hours 5 minutes are both a lot of waiting!

I was also pleased to see that you tried asking, "What if…?" with 3 people shearing and Eric jumping 2 sheep. Your explanation of the method you used to solve the problem was easy for me to follow.

Something to think about for future problems is how you might describe the situation using algebra. For example, could you use variables to figure out an equation for the number of sheep shorn before Eric?

I hope school is off to a great start!

Problem 2 is almost ready!

Matthew Reames

Addition of a Mega Problem

I recently read online that the Fibonacci series can be used to approximate conversions between miles and kilometers.

Is this statement correct?

Questions to think about

- Is the statement correct? Is it always correct or only sometimes?
- How do you know?
- Can you prove it?
- Can you use this for numbers that are not in the Fibonacci series?

Photo by matthew_reames on flickr.

Why Focus on Writing?

The questions to think about:

Why is mathematical writing important?

What does mathematical writing look like?

How is it different from other types of writing?

NCTM Process Standards

Communication: Instructional programs from pre-kindergarten through grade 12 should enable all students to:

Organize and consolidate their mathematical thinking through communication;

Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;

Analyze and evaluate the mathematical thinking and strategies of others; and

Use the language of mathematics to express mathematical ideas precisely.

Common Core Math Curriculum

Standard 3: Construct viable arguments and critique the reasoning of others

*…They **justify** their conclusions, **communicate** them to others, and respond to the arguments of others. They **reason** inductively about data, making plausible **arguments** that take into account the context from which the data arose.*

Why is mathematical writing important?

One goal of written mathematical communication is to help students **explain their thinking** and **consider how and why they solve problems as they do.** (Haltiwanger & Simpson, 2013)

Another goal is for students to **reflect on their own learning**, or to **explore additional mathematical ideas.** (Burns, 2004)

More crucial, however, is that the way mathematical language is used helps to determine how children see themselves mathematically:

‘The **language** used in mathematical practices, both in and out of school, **shapes the ways of being a mathematician** and the conceptions of the nature of mathematical knowledge and learning that are possible within those practices.’ (Burton & Morgan, 2000, p. 429)

What does mathematical writing look like?

Initial Framework:
Components of Mathematical Communication

Communication About Mathematics	Communication In Mathematics	Communication With Mathematics
Reflection on cognitive processes: Descriptions of procedures, reasoning. Metacognition - giving reasons for procedural decisions.	Mathematical register: Special vocabulary. Particular definitions of everyday vocabulary. Modified uses of everyday vocabulary. Syntax, phrasing. Discourse.	Problem-solving tool: Investigations. Basis for meaningful action.
Communication with others about cognition: Giving point of view. Reconciling differences.	Representations: Symbolic. Verbal. Physical manipulatives. Diagrams, graphs. Geometric.	Alternative solutions: Interpretation of arguments using mathematics. Utilisation of mathematical problem-solving in conjunction with other forms of analysis.
Communications Framework for Mathematics (Brenner, 1998, p. 155).		

What actually happened

Mismatch between the original framework and the data.

Eventually became an inductive approach (Bazeley, 2013).

Began to develop a framework grounded in the data.

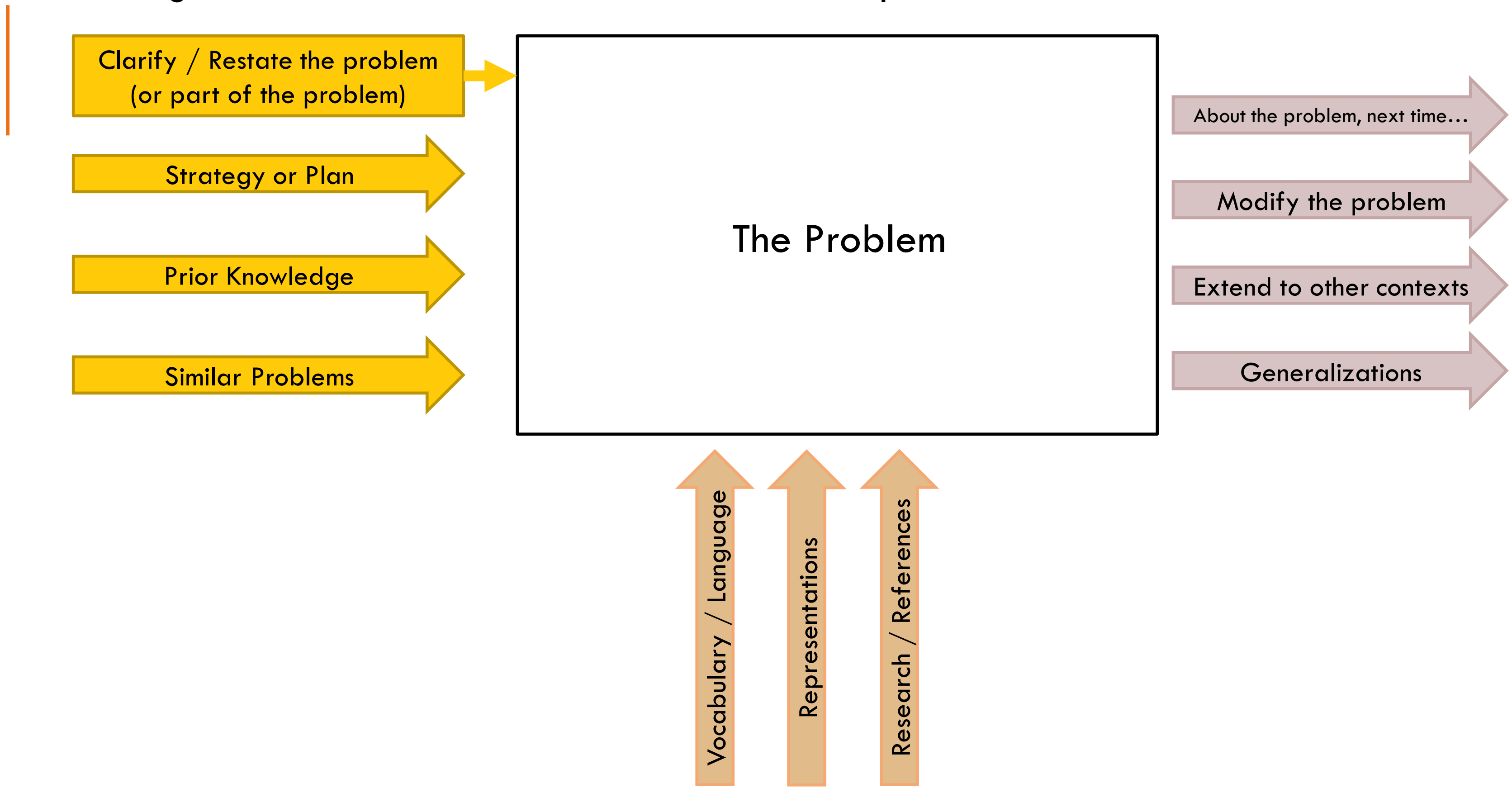
What do children do when they write about their math solutions?

A lot of things

&

A range of things

Using what I know… …to solve this problem… …now I…



…using these tools and resources…

Implications for Teaching

How teachers help children develop their math writing

How math is taught in schools

Differentiation in math

Answer

Next steps:
Quality of Mathematical Writing

Mathematical Knowledge

- Understanding of the problem's mathematical concepts and principles
- Use of appropriate mathematical terminology
- Computations are correct

Strategic Knowledge

- Use of relevant information
- Identifies the important parts of the problem and understands the relationships between them
- Use of appropriate strategy to solve the problem
- Shows clear solution process
- Solution process is complete and systematic

Communication

- Clear and effective explanation of the solution process
- May include appropriate diagram, chart, table, equation, etc.
- Strong and logical supporting arguments
- May include examples or counterexamples

(From Lane, 1993)

What students said…

- "I liked **thinking about problems diffe**rently to solve them. I also liked the different types of questions."
- "I really liked the project and I thought that the problems were challenging and **made us look at math at a new perspective**. I would like to do this again."
- "All of it (feedback) was very useful, and I liked how Mr. Reames explained it in a way I could understand. I think **it would be even better if Mr. Reames would use diagrams in his feedback**."
- "I **used the feedback on how to think of the problems differently**, using more questions and answers, and seeing whether my questions were accurate."

What students said…

- "I thought that the feedback was always helpful, but **I liked it when you told us a good thing that we did and then a bad thing that we did.**"
- "…the **only problem was that if the Internet went down** or wasn't working, then it might take a while to communicate."
- "…the problems were very different and they **needed a different perspective** to look at them."
- "I liked making up possible equations and questions. I also liked **how I couldn't really be right or wrong** and I wouldn't be getting a grade."

What students said…

- "I would because I like how **I can try anything** and I learn a lot about different ways to think."
- "It was **more difficult and interesting** (than the math I do in school)."
- "I liked that **it was like another math class, except that it was not for a grade**."
- "I **want to do it again next year**!"

Things to think about

It's not the same as being in a classroom.

Lack of immediate communication or discussion.

Tailoring problems to specific students is more difficult.

It can be difficult to sustain participation.

Email can influence the types of (or presentations of) solutions

Can rely heavily on parents.

Requires someone willing and able to provide feedback.

It's flexible – students can participate as they are able.

It is location independent.

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