

# Apportionment Seven Roads to Fairness

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## 2010 Apportionment of U.S. House of Representatives

State	Population	Apportionment	Change from 2000 Census	State	Population	Apportionment	Change from 2000 Census
Alabama	4,802,982	7	0	Montana	994,416	1	0
Alaska	721,523	1	0	Nebraska	1,831,825	3	0
Arizona	6,412,700	9	+1	Nevada	2,709,432	4	+1
Arkansas	2,926,229	4	0	New Hampshire	1,321,445	2	0
California	37,341,989	53	0	New Jersey	8,807,501	12	-1
Colorado	5,044,930	7	0	New Mexico	2,067,273	3	0
Connecticut	3,581,628	5	0	New York	19,421,055	27	-2
Delaware	900,877	1	0	North Carolina	9,565,781	13	0
Florida	18,900,773	27	+2	North Dakota	675,905	1	0
Georgia	9,727,566	14	+1	Ohio	11,568,495	16	-2
Hawaii	1,366,862	2	0	Oklahoma	3,764,882	5	0
Idaho	1,573,499	2	0	Oregon	3,848,606	5	0
Illinois	12,864,380	18	-1	Pennsylvania	12,734,905	18	-1
Indiana	6,501,582	9	0	Rhode Island	1,055,247	2	0
Iowa	3,053,787	4	-1	South Carolina	4,645,975	7	+1
Kansas	2,863,813	4	0	South Dakota	819,761	1	0
Kentucky	4,350,606	6	0	Tennessee	6,375,431	9	0
Louisiana	4,553,962	6	-1	Texas	25,268,418	36	+4
Maine	1,333,074	2	0	Utah	2,770,765	4	+1
Maryland	5,789,929	8	0	Vermont	630,337	1	0
Massachusetts	6,559,644	9	-1	Virginia	8,037,736	11	0
Michigan	9,911,626	14	-1	Washington	6,753,369	10	+1
Minnesota	5,314,879	8	0	West Virginia	1,859,815	3	0
Mississippi	2,978,240	4	0	Wisconsin	5,698,230	8	0
Missouri	6,011,478	8	-1	Wyoming	568,300	1	0
				<b>Total</b>	<b>309,183,463</b>		

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## Constitution of the United States

### Article 1. Section 3. Apportionment of Representatives and Direct Taxes

Representatives [and direct taxes] shall be apportioned among the several states which may be included within this Union, according to their respective numbers... The actual enumeration shall be made within three years after the first meeting of the Congress of the United States, and within every subsequent term of ten years, **in such manner as they shall by law direct**. The number of Representatives shall not exceed 1 for every 30,000, but each state shall have at least 1 Representative;...

### This Means:

- ✓ The seats in the House of Representatives shall be apportioned to the states on the basis of their respective populations.
- ✓ The populations of the states, apportionment method, and total number of seats that will be apportioned shall be determined every 10 years.
- ✓  $\frac{\# \text{ people}}{\text{seat}} \geq 30,000$
- ✓ Each state should get at least one seat.

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## Definitions

$$\textit{standard divisor} = \frac{\textit{total population}}{\textit{total seats}}$$

$$\textit{standard quotient} = \frac{\textit{state population}}{\textit{total population}} \times \textit{total seats} = \frac{\textit{state population}}{\textit{standard divisor}}$$

## Apportionment Algorithms

### Quota Methods

Hamilton, Alexander (used from 1852 to 1900)

1. Calculate the standard quota for each state.
2. Round down, or truncate.
3. Temporarily assign each state that number. Add.
4. Assign surplus seats on a basis of descending fractional parts until the number of surplus seats is exhausted.

Lowndes, William (proposed in 1822, never adopted)

1. Calculate the standard quota for each state.
2. Round down, or truncate.
3. Temporarily assign each state that number. Add.
4. Divide the fractional part of the standard quota by the integer part of the standard quota to get the **relative** fractional part.
5. Assign surplus seats on a basis of descending **relative** fractional parts until the number of surplus seats is exhausted.

**Apportionment Algorithms continued on next page**

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## Apportionment Algorithms (continued)

### Divisor Methods

**Jefferson, Thomas** (used from 1792 to 1840)

1. Calculate a standard divisor, or ideal district size.
2. Divide each state's population by this divisor to get a modified quota.
3. Round **down**, or truncate any fractional part. Add.
4. If the sum of the seats does not equal the number of seats sought, adjust the divisor up or down and go to step 2.

**Adams, John Quincy** (proposed in 1832, never adopted)

1. Calculate a standard divisor, or ideal district size.
2. Divide each state's population by this divisor to get a modified quota.
3. Round **up** any fractional part. Add.
4. If the sum of the seats does not equal the number of seats sought, adjust the divisor up or down and go to step 2.

**Webster, Daniel** (used from 1842 to 1850 and again from 1911 to 1940)

1. Calculate a standard divisor, or ideal district size.
2. Divide each state's population by this divisor to get a modified quota.
3. Round **off** any fractional part. Add.
4. If the sum of the seats does not equal the number of seats sought, adjust the divisor up or down and go to step 2.

**Huntington-Hill** (used from 1941 to present)

1. Calculate a standard divisor, or ideal district size.
2. Divide each state's population by this divisor to get a modified quota.
3. Calculate the geometric mean of the two whole numbers that are closest to the modified quota.
4. If the modified quota is greater than or equal to the geometric mean, round up.
5. If the modified quota is less than the geometric mean, round down.
6. If the sum of the seats does not equal the number of seats sought, adjust the divisor up or down and go to step 2.

**Dean**

1. Calculate a standard divisor, or ideal district size.
2. Divide each state's population by this divisor to get a modified quota.
3. Calculate the harmonic mean of the two whole numbers that are closest to the modified quota.
4. If the modified quota is greater than or equal to the harmonic mean, round up.
5. If the modified quota is less than the harmonic mean, round down.
6. If the sum of the seats does not equal the number of seats sought, adjust the divisor up or down and go to step 2.

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## References

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