Relating Hypothesis Tests, Confidence Intervals, Means, and Standard Deviations

By Lance Revennaugh

Frostburg State University

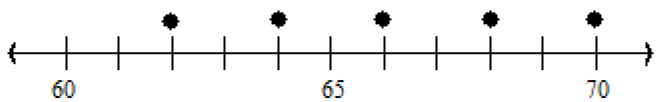
lrevennaugh@frostburg.edu

One Sample t-tests

The mean for each sample is the same but notice how the standard deviation of each affects the outcome of the one-sample t-test.

$$H_0: \mu = 60$$
$$H_A: \mu \neq 60$$
$$\alpha = .05$$

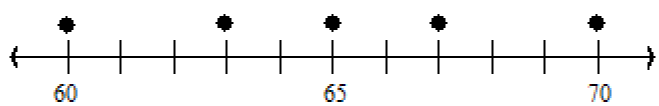
a) 62, 64, 65, 66, 68



$\bar{x} = 65$
$s = 2.24$
$t = 5.00$
$p = .0075$

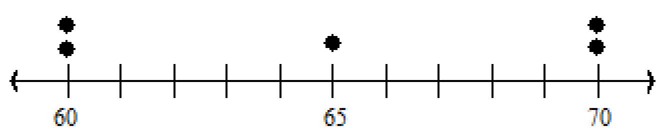
95% confidence interval (62.22, 67.78)
Reject H_0

b) 60, 63, 65, 67, 70



$\bar{x} = 65$
$s = 3.81$
$t = 2.94$
$p = .0426$
95% confidence interval (60.27, 69.73)
Reject H_0

c) 60, 60, 65, 70, 70



$\bar{x} = 65$
$s = 5.00$
$t = 2.24$
$p = .0890$
95% confidence interval (58.79, 71.21)
Cannot Reject H_0

What do you notice about the confidence intervals and the hypothesis tests?

Two Sample t-tests

Notice how the standard deviations of each sample affect the outcome of the two-sample tests.

$$H_O : \mu_1 = \mu_2$$
$$H_A : \mu_1 \neq \mu_2$$
$$\alpha = .05$$

a) Sample one: 73, 74, 75, 76, 77 | sample two: 63, 64, 65, 66, 67

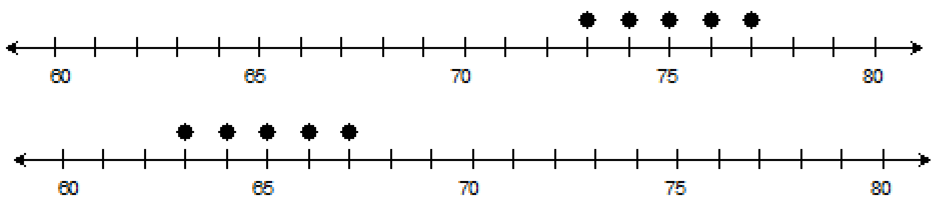
$\bar{x}_1 = 75$ | $\bar{x}_2 = 65$

$s_1 = 1.58$ | $s_2 = 1.58$

95% CI (73.04, 76.96) | 95% CI (63.04, 66.96)

one:

two



$t = 10$

$p = .0000$

Reject H_O

95% CI for $\mu_1 - \mu_2$ (7.69, 12.31)

b) Sample one: 65, 70, 75, 80, 85 | sample two: 55, 60, 65, 70, 75

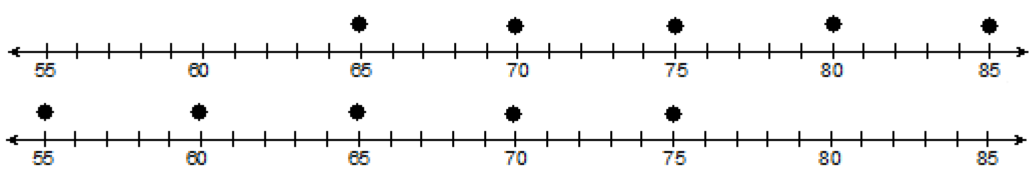
$\bar{x}_1 = 75$ | $\bar{x}_2 = 65$

$s_1 = 7.91$ | $s_2 = 7.91$

95% CI (65.18, 84.82) | 95% CI (55.18, 74.82)

one:

two:



$t = 2$

$p = .0805$

Cannot Reject H_O

95% CI for $\mu_1 - \mu_2$ (-1.53, 21.53)

What do you notice about the confidence intervals and the decisions?

ANOVA'S

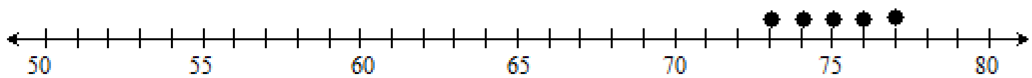
For these one-way ANOVA's, notice how the standard deviation of each sample changes the outcome of the F-test

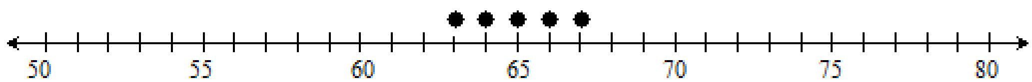
$$H_0 : \mu_1 = \mu_2 = \mu_3$$
$$H_A : \text{at least one} \neq$$
$$\alpha = .05$$

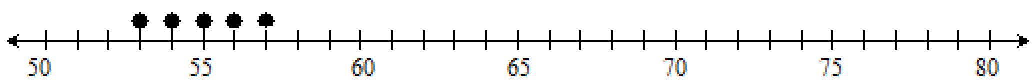
a) Sample one:
73, 74, 75, 76, 77
$\bar{x}_1 = 75$
$s_1 = 1.58$
95% CI (73.04, 76.96)

Sample two:
63, 64, 65, 66, 67
$\bar{x}_2 = 65$
$s_2 = 1.58$
95% CI (63.04, 66.96)

Sample three:
53, 54, 55, 56, 57
$\bar{x}_3 = 55$
$s_3 = 1.58$
95% CI (53.04, 56.96)

one:


two:


three:


Source	df	SS	MS	F
Between	2	1000	500	200 (.0000)
Within	12	30	2.5	
Total	14	1030		

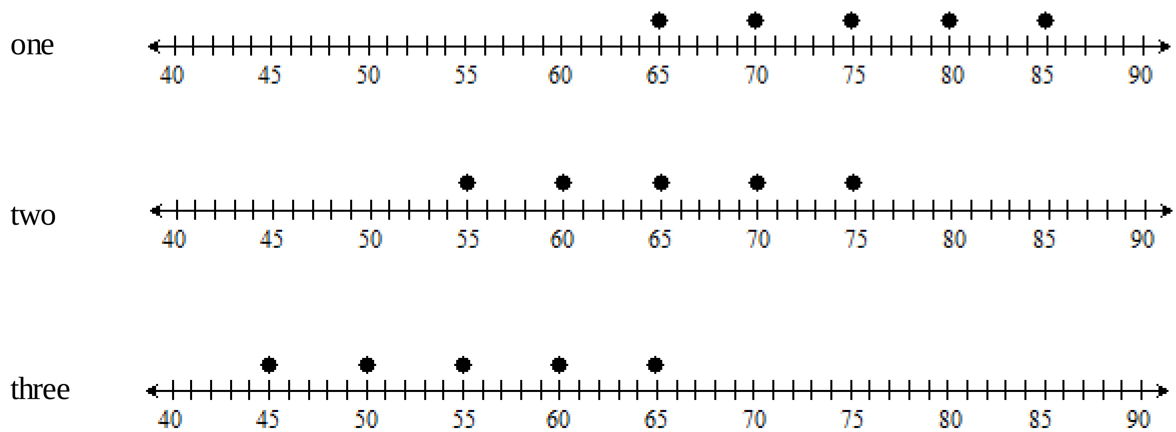
Reject H_O

$$H_0: \mu_1 = \mu_2 = \mu_3$$
$$H_A: \text{at least one} \neq$$
$$\alpha = .05$$

b) Sample one:
65, 70, 75, 80, 85
$\bar{x}_1 = 75$
$s_1 = 7.91$
95% CI (65.18, 84.82)

Sample two:
55, 60, 65, 70, 75
$\bar{x}_2 = 65$
$s_2 = 7.91$
95% CI (55.18, 74.82)

Sample three:
45, 50, 55, 60, 65
$\bar{x}_3 = 55$
$s_3 = 7.91$
95% CI (45.18, 64.82)



Source	df	SS	MS	F
Between	2	1000	500	8 (.0062)
Within	12	750	62.5	
Total	14	1750		

Reject H_O

$H_0 : \mu_1 = \mu_2 = \mu_3$

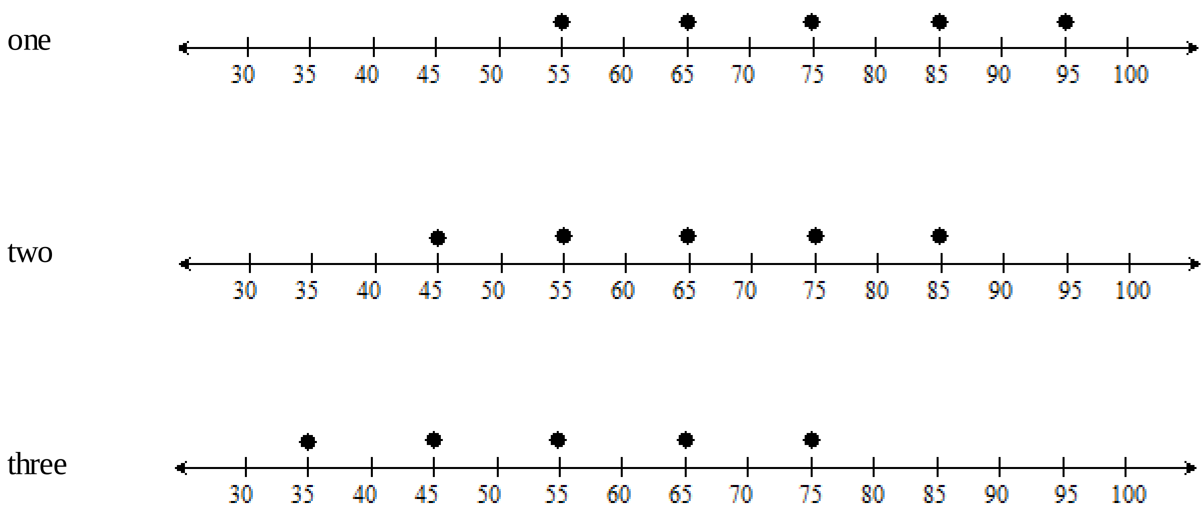
H_A : at least one $\neq$

$\alpha = .05$

c) Sample one:
55, 65, 75, 85, 95
$\bar{x}_1 = 75$
$s_1 = 15.81$
95% CI (55.37, 94.63)

Sample two:
45, 55, 65, 75, 85
$\bar{x}_2 = 65$
$s_2 = 15.81$
95% CI (45.37, 84.63)

Sample three:
35, 45, 55, 65, 75
$\bar{x}_3 = 55$
$s_3 = 15.81$
95% CI (35.37, 74.63)



Source	df	SS	MS	F
Between	2	1000	500	2 (.1780)
Within	12	3000	250	
Total	14	4000		

Cannot Reject H_O

Notice that the sums of squares (SS) is the same for the Between source. Why? This next example changes the Between SS, but the F-value is the same as example b. Why?

$H_0 : \mu_1 = \mu_2 = \mu_3$

H_A : at least one $\neq$

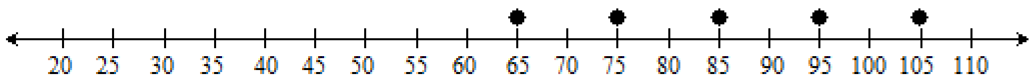
$\alpha = .05$

d) Sample one:
65, 75, 85, 95, 105
$\bar{x}_1 = 85$
$s_1 = 15.81$
95% CI (65.37, 104.63)

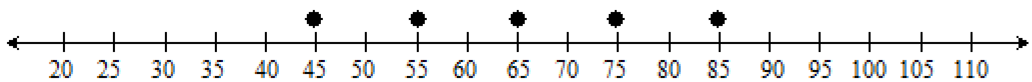
Sample two:
45, 55, 65, 75, 85
$\bar{x}_2 = 65$
$s_2 = 15.81$
95% CI (45.37, 84.63)

Sample three:
25, 35, 45, 55, 65
$\bar{x}_3 = 45$
$s_3 = 15.81$
95% CI (25.37, 64.63)

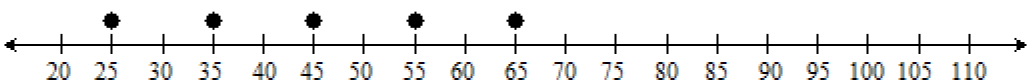
one



two



three



Source	df	SS	MS	F
Between	2	4000	2000	8 (.0062)
Within	12	3000	250	
Total	14	7000		

Reject H_O

SSB is a measure of "variance" between the means of the three samples and SSN is a measure of variance within each sample.

Source	df	SS	MS	F
Between Within	k – 1 n - k	SSB SSW	$MSB = \frac{SSB}{k-1}$ $MSW = \frac{SSW}{n-k}$	$\frac{MSB}{MSW}$
Total	n - 1	SSTo		

SS = sum of squares
MS = mean square
k = number of groups
n = total sample size

MORE

Sam Statistic is in a class that has 5 chapter exams and a final exam. The final exam counts twice as much as a chapter exam. If Sam scores 90, 84, 76, 82, and 77 on the chapter tests, what should his grade be on the final exam to get an average of 83 for the class?

Find 4 numbers whose mean is 50 and whose median is 60.

Find mean, median, and mode for

Which has a larger standard deviation (scales are the same)?

Find five numbers whose mean is 50 standard deviation is zero.

Use $s = \sqrt{\dfrac{\sum\left(x - \bar{x}\right)^2}{n-1}}$

Find five numbers whose mean is 50 and whose standard deviation is 10.