Electric Vehicle Mathematics

Philip Rash
NC School of Science & Math
rash@ncssm.edu

NCTM Regional Conference
Richmond, NC
November 13 2014

Outline

- About this talk, introduction
- Prius PSD
- Quantifying "fuel efficiency"
- Look at some real data from driving & charging an electric vehicle

Introduction - Terminology

- Hybrid – motive force may be drawn from an electric motor and/or internal combustion engine. All vehicle energy derived from combustion of gasoline, but energy may be stored & retrieved in battery pack(s).
- Plug-in hybrid – Like regular hybrid, but also allows battery to be charged from external source, thus reducing total gasoline usage.

Introduction - Terminology

- Electric Vehicle – motive force drawn exclusively from an electric motor, with energy supplied from a battery pack, charged from an external source.

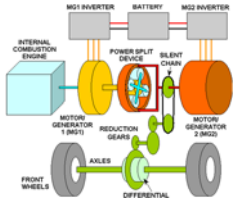
Introduction - Hybrids

- Parallel Hybrid – either internal combustion engine or electric motor may physically connect to & drive wheels
- Series Hybrid – only electric motor may physically drive the wheels, but internal combustion engine can power a generator to provide electricity to the electric motor

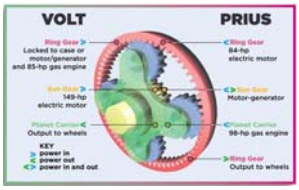
Examples

- Hybrid – Toyota Prius
- Plug-in Hybrid – Chevy Volt (and some Prius's)
- Electric Vehicle – Tesla, Nissan Leaf

Prius Drivetrain



Prius vs. Volt



Planetary Gear Math

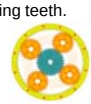
- Sun gear, Planet carrier, Ring gear
- http://leahart.com/prius/psd/
- Relationship between speed of various inputs/ outputs of PSD?
  - With ICE=0, MG1 & MG2 speeds proportional, specifically $MG1 = -2.6MG2$
  - With ICE=1011, $MG1 = -2.6MG2 + 3639.6$
  - With ICE=2022, $MG1 = -2.6MG2 + 7279.2$
  - With ICE=3033, $MG1 = -2.6MG2 + 10918.8$

Planetary Gear Math

- Relationship between speed of various inputs/ outputs of PSD?
  - So it appears that $MG1 = -2.6MG2 + 3.6\mathrm{ICE}$
- Furthermore, speed and MG2 proportional: $59.1speed(in\ mph) = MG2$
- So also, $MG1 = 3.6ICE - 153.66speed$
- Note, if $ICE = 0$, $|MG1|_{max} = 6500$ implies $speed_{max} = 42.3$

Planetary Gear Math

- One final note: the 2.6 and 3.6 coefficients can also be derived by counting teeth.
- The sun gear has 30 teeth; the ring gear has 78.



- $\frac{78}{30} = 2.6$, and $\frac{78+30}{30} = 3.6$
- It's left as an exercise for you to figure out why it's those ratios that provide the coefficients! (Hint: one source to check is the Wikipedia page for "Epicyclic gearing")

Quantifying Efficiency

- First, a thought experiment - which of the following transactions do you think would result in the greatest gasoline savings?
  - Trading in a car that gets 10 MPG for a car rated at 11 MPG
  - Trading in a car that gets 22 MPG for a car rated at 26 MPG
  - Trading in a car that gets 38 MPG for a car rated at 50 MPG

Quantifying Efficiency

- Well, let's assume we drive 12,000 miles in a year
  - 10 MPG -> 11 MPG means burning 1200 versus 1090.9 gallons – a savings of 109 gallons
  - 22 MPG -> 26 MPG means burning 545.45 versus 461.54 gallons – a savings of 83.9 gallons
  - 38 MPG -> 50 MPG means burning 315.8 versus 240 gallons – a savings of 75.8 gallons

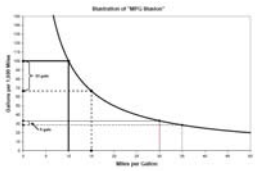
Quantifying Efficiency

- MPG vs GPHM (gallons per hundred miles)
  - If you have two cars, one which is rated 14 MPG and the other 30 MPG, what is your average MPG assuming you drive each car the same miles?
  - If you have two cars, one which is rated 5 GPHM and the other 2 GPHM, what is your average GPHM?

Quantifying Efficiency

- "MPG" illusion – from an EPA publication: "While a miles per gallon (MPG) estimate is a required feature that has appeared on the fuel economy label for several decades, this metric can be potentially misleading when consumers compare fuel economy improvements, particularly when they use it in place of fuel costs."
- Which is better fuel savings?
  - Increase from 10 to 15 mpg?
  - Increase from 30 to 35 mpg?

MPG illusion



"This 'MPG illusion' demonstrates why it may be more meaningful to express fuel efficiency in terms of consumption (e.g., gallons per mile or per 100 miles) rather than in terms of economy (miles per gallon)."

New Monroney Labels

GPHM (gallons per hundred miles) is now also reported

What about other engine technologies?

Chevy Volt

Nissan Leaf

MPGe, kWh

- MPGe: "Miles per Gallon-equivalent" – "Equivalent" MPG if energy were derived solely from gasoline instead (using 33.7kWh per 1 gallon of gasoline)
- kWh – kilowatt-hour – measure of energy: delivery of 1 kW of power for 1 hour
- Quick review: "Watt" measures power (energy per unit time), so Watt-hour (or KWh) is a measure of energy

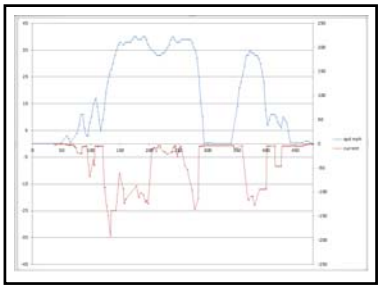
Measuring EV energy usage

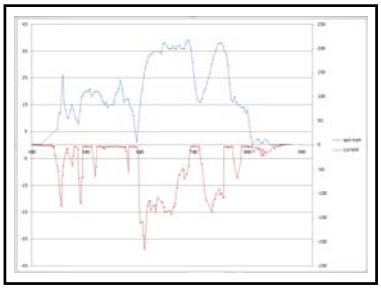
An EV?

Yes!

Collecting Data - Charging

- 5580 Wh delivered from charger
- After 13 mile trip, 430 Wh/mile
- (Does not account for charger efficiency, probably about 80%)





Collecting Data - Driving

- 4.5 mile trip
- Total Ah delivered from batteries: 10.7
- Assuming 145V average, 1550 Wh
- So, average 345 Wh/mile

Additional Calculations

- Rounding to 500Wh/mile:
$$\frac{33.7\text{kWh}}{1\text{gal}} \cdot \frac{1\text{mile}}{0.5\text{kWh}} = 67.4\text{MPGe}$$
- At 12 cents per kWh of electricity:
$$\frac{12\text{cents}}{\text{kWh}} \cdot \frac{0.5\text{kWh}}{\text{mile}} = 6\text{cents/mile}$$
- Contrast with a typical car:
$$\frac{\$3.50}{\text{gallon}} \cdot \frac{\text{gallon}}{30\text{miles}} = 12\text{cents/mile}$$

Thanks!

http://courses.ncssm.edu/math/talks/conferences/
http://eahart.com/prius/psd/

Contact info:
Philip Rash
NC School of Science & Math
rash@ncssm.edu