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The missing component in probability reasoning: What do you mean by "Random?"

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Randomly doing something

S1: Great! I can do whatever I want.

S2: No. You are doing this randomly. That means you cannot have any preference.

S3: I think it means half half.

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Let's start with a problem:

There are a black bucket and a white bucket. Pam randomly distributed 4 balls into the two buckets. What is the probability that there is exactly 1 ball in the black bucket?

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One student says:

Each ball can be randomly put in either bucket, so there are $2^4 = 16$ possible results:

WWWW	WWWB	WWBW	WWBB
WBWW	WBWB	WBBW	WBBB
BWWW	BWWB	BWBW	BWBB
BBWW	BBWB	BBBW	BBBB

Among all the possible results, 4 satisfies that exactly 1 ball was put in the black bucket, so the probability should be 4/16 = 1/4.

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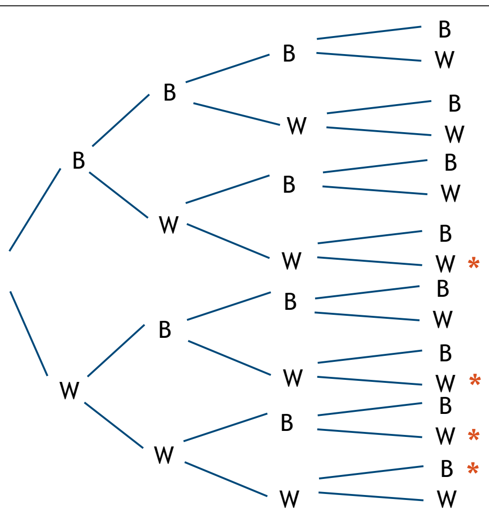

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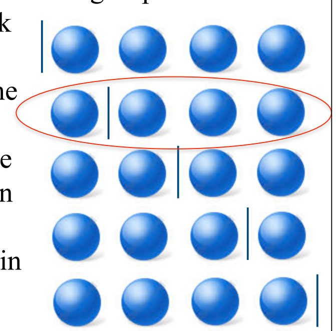
Another student says:

I first put all 4 balls in a line. Then I use a stick to randomly divide them into two groups. The balls on the left of the stick go into the black bucket. The balls on the right of the stick go into the white bucket. Since there are five possible places where I can put the stick and only one case puts exactly one ball in the black bucket, then the probability should be 1/5.



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Who was right?

Who was wrong?

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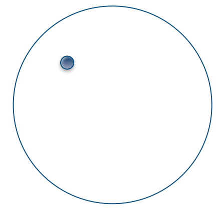
Another problem

The radius of a circle is 1. Kenny randomly selected a point inside the circle. What is the probability that the point is no more than 1/2 from the center of the circle.



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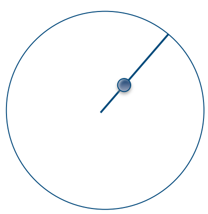
One student says:

A point must be on a radius. So I first randomly select a radius. Then I randomly select a point in the radius. It's obvious that the point have 50% percent of the change to be no more than 1/2 away from the center of the circle.



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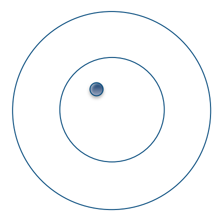
Another student says:

I draw a circle with the same center but a radius of 1/2. Then I know that if the dot is inside or on the smaller circle, its distance from the center is no more than 1/2. Since the area of the smaller circle is 1/4 of the larger circle, then if I randomly place a dot inside the bigger circle, there should be a 25% chance that it is inside the small circle.



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Who was right?

Who was wrong?

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Two problems

- Mary flipped a regular coin 10 times and got 8 Heads and 2 Tails. What is the probability that she gets a Head in her next toss?
  A. > 1/2 B. < 1/2 C. = 1/2
- Melva was fishing in a lake where some fish were tagged in a study. 8 of the first 10 fish she caught were tagged. What is the probability that the next fish she caught is tagged?
  A. > 1/2 B. < 1/2 C. = 1/2

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Why do we ask students to rely on Theoretical Probability in the first problem but Experimental Probability in the second problem?

Why is what we call “the right way” of reasoning in one problem exactly “the wrong way” of reasoning in the other problem?

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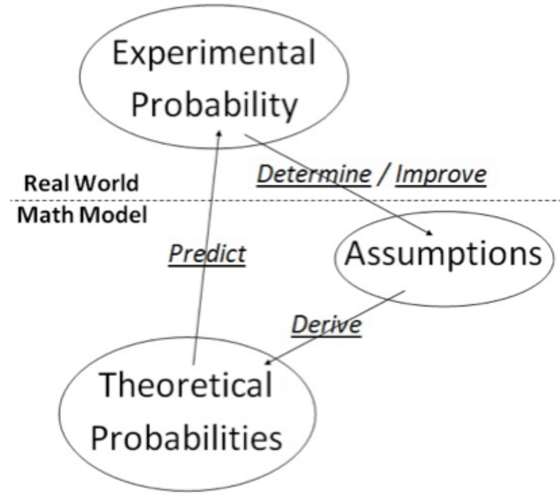

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NCTM Process Standards

Reasoning & Proof:

- Recognize reasoning and proof as fundamental aspects of mathematics
- Develop and evaluate mathematical arguments and proofs

Communication:

- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely.

Connections:

- Recognize and use connections among mathematical ideas

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Transition to more advanced theory

A probability space consists of three parts:

- A sample space, Ω, which is the set of all possible outcomes.
- A set of events , where each event is a set containing zero or more outcomes.
- The assignment of probabilities to the events; that is, a function P from events to probabilities.

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Who was right?
Who was wrong?

Both explanations in both problems are sound. Students just made different assumptions. Or we may say that they had different understanding of what “random” means in the corresponding scenarios.

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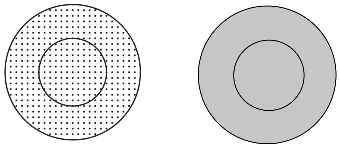
Explanation of Different Assumptions in the Ball Problem

- One (1st) student in fact made the assumption that the destination of each ball is independent from each other.
- The other student in fact didn’t make the assumption of independence. When balls are placed in a line, their destinations are no longer independent from each other.

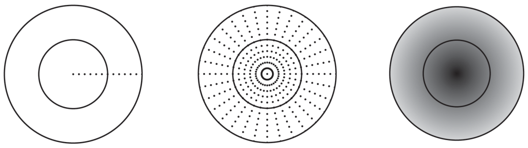
*The assumptions might be made or not made unconsciously.

Illustration of Different Assumptions in the Circle Problem

One (2nd) student:



The other student:

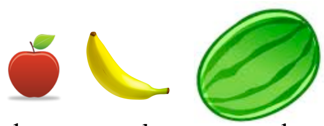


Randomness, in formal probability reasoning, is NOT an intuitional description of an object's behavior or an event's property. Rather, it is a very clear and precise assumption on which all following deductive reasoning will rely.

In coin flipping, when we say the result is random, we mean that there is a 50 to 50 chance to get a Head or a Tail.

In die tossing, when we say the result is random, we mean that there is a chance of 1/6 to get each number.

Last problem



There are an apple, a banana, and a water melon in the fridge. You randomly pick one fruit from the fridge. What is the probability that you pick the apple?

S1: 0%. I must see the water melon first. It's so much bigger.
S2: 100%. I don't like the other two.
S3: 1 in 3. Didn't you hear what the teacher just said?

Thank you!

For additional information,
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