What's the Difference? Exploring Subtraction Deeply in K-2
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Separate Subtraction

- **How much is left?**
- **Take away**
- **Most intuitive and action involved**

Comparative Subtraction

- **How many more? fewer?**
- **Difference between quantities**
- **More complex because there is no action**

Sense-Making Tools for Subtraction

- **Part-Part-Whole Mats**
- **Tape Diagrams**
- **Open Number Lines**

Table 1: Classification of Addition and Subtraction Word Problems
The table shows the problem types in order of complexity from 1(easiest) to 11 (most complex). Students first use the process of directly modeling the problems (using objects to perform the operation described), they then move on to counting on/back (with counting on from the largest being the most sophisticated). After much exposure they will begin doing these abstractly by using known facts, often using known facts to derive other facts (3 + 3 is 6 so 3 + 4 must be 7.).

Darker shading indicates Kindergarten subtypes. Grade 1 and 2 students will work on the light shading and white subtypes. Students in grades 1 and 2 will work with unshaded subtypes, but these subtypes do not need to be mastered until Grade 2.

Problem Type			
Join (add to)	1. **(Result Unknown)** Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether? 5 + 8 = ☐	5. **(Change Unknown)** Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether? 5 + ☐ = 13	9. **(Start Unknown)** Connie has some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with? ☐ + 5 = 13
Separate (take from)	2. **(Result Unknown)** Connie had 13 marbles. She gave 5 marbles to Juan. How many marbles does she have left? 13 – 5 =☐	6. (**Change Unknown**) Connie has 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give Juan? 13 – ☐ = 5	10. **(Start Unknown)** Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? ☐ – 5 = 8
Part-Part-Whole (put together/ take apart)	3a. **(Whole Unknown) (Total Unknown)** Connie has 5 red marbles and 8 blue marbles. How many marbles does Connie have? 5 + 8 = ☐	3b. **(Both Addends Unknown)** Connie has 13 marbles. How many possible red marbles and blue marbles does she have? ☐ + ☐ = 13	8. **(Part Unknown) (Addend Unknown)** Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have? 5 + ☐ = 13 13 – 5 = ☐
Compare	4. **(Difference Unknown)** Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan? 5 + ☐ = 13 13 – 5 = ☐	7. **(Compare Quantity Unknown) (Bigger Unknown)** Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? 5 + 8 = ☐ $8 + 5 = \square$ "Fewer" version: Connie has 5 fewer marbles than Juan. Connie has 8 marbles. How many marbles does Juan have? 8 + 5 = ☐	11. **(Referent Unknown)/(Smaller Unknown)** "Fewer" version: Juan has 5 fewer marbles than Connie. Connie has 13 marbles. How many does Juan have? 13 – 5 = ☐ "More" version: Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have? 13 – 5 = ☐ ☐ + 5 = 13

Table information adapted from and reprinted with permission from the author. Children's Mathematics: Cognitively Guided Instruction. By Carpenter, T.P.; Fennema, E.; Franke, M.L.; Levi, L.; and Empson, S.B. Shading is a result of adapted information from CCSS, p. 88.