Adventures in Probability: From Fraction Rainbows to Striking Umpires

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NCTM Regional Conference
Richmond, Virginia
November 13, 2014
1:30–2:45 pm

Big Ideas in Middle Grades Probability[1]

1. Chance has no memory.
2. The probability of a future event can be described along a continuum, $0 \leq p \leq 1$
3. Experimental outcomes can be used to estimate the probability of an event. The number of trials of a given experiment can affect the quality of the estimate.
4. For some events, the exact probability can be determined using analysis of and reasoning about the event itself.
5. Simulation can be used to gain insight about complex situations in which an element of chance is involved.

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[1] Adapted from Van de Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 6 – 8* (2nd Ed.). Boston, MA: Pearson.

Where can the big ideas listed on the previous page be found in the standards below?

Standards Framework	Relevant Content	Practices and Processes
Common Core State Standards[2]	7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. 7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. 7.SP.7 Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. 7.SP.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.	1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning
Virginia Standards of Learning[3]	6.16 The student will a) compare and contrast dependent and independent events; and b) determine probabilities for dependent and independent events. 7.9 The student will investigate and describe the difference between the experimental probability and theoretical probability of an event. 7.10 The student will determine the probability of compound events, using the Fundamental (Basic) Counting Principle. 8.12 The student will determine the probability of independent and dependent events with and without replacement.	1. Mathematical Problem Solving 2. Mathematical Reasoning 3. Mathematical Communication 4. Mathematical Connections 5. Mathematical Representation

[2] National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Author.

[3] Board of Education, Commonwealth of Virginia. (2009). *Mathematics standards of learning for Virginia public schools.* Richmond, VA: Virginia Board of Education.

The Twelve Circles Game

Overview

Each student gets a game sheet and 10 markers (chips, counters, pennies, M & M's, etc.). The students place their markers in the circles in any arrangement they choose. It is permissible to place multiple markers in one circle. Once the markers are placed, the teacher facilitates ten rolls of two 6-sided dice. For each roll, the student looks to see if they have any markers in the circle corresponding to the sum rolled. The student gets one point for each marker in the designated circle for that roll. The students total their points after ten rolls. The highest total wins.

After the initial game, students will likely want to change the arrangement of their markers. They can refine their strategies over multiple games and make hypotheses about the likelihood of different dice roll outcomes as well as the overall game.

Sample Space

	1	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

Big Ideas Checklist	
	Chance has no memory.
	Probability Continuum [0, 1]
	Experimental Probability
	Theoretical Probability
	Simulation

The Fraction Wall[4]

Overview

Each student gets a game sheet and each group of students gets a set of custom dice, different colored markers, and popsicle sticks (optional). On the handout, one rectangular section of bricks represents a game space, where each row represents one whole. The top row is partitioned into two halves, the second row three thirds, the third row four fourths, etc. The object of the game is to color in the entire brick wall.

The students take turns. Each turn consists of a student rolling the custom dice, and attempting to color in bricks representing the value of the fraction rolled. For example, if a student rolled ¾, she could color in 3 bricks in the fourths row, 6 bricks in the eighths row, 9 bricks in the twelfths row, or bricks from a combination of rows such as one brick from the fourths row and three bricks from the sixths row. If it is not possible to color in the exact quantity rolled, the student loses her turn.

Tips and Strategies for Teachers

- Have the students keep track of their rolls using a log sheet. They can use different colors to differentiate their turns, allowing you to check the accuracy of their work.
- The game does not necessarily need to be competitive. Some of the most productive groups are those who help each other find ways to shade in the bricks.
- Students can use the popsicle sticks (or any straight object) as tools for determining equivalent fractions.
- Sometimes students "get stuck" when they only have one brick remaining. To speed up the game in this situation, you can allow them to "freeze" one die when they roll it. For example, if a student needs to roll ⅛, but they instead roll ⅓, they can freeze the "1" rolled on the numerator die and then focus on simply rolling the "8" on the denominator die.
- For an extra challenge, you can restrict the players from coloring in the exact digits rolled. For example, if a student rolled ½, they could not color in a brick on the top row (halves), but could color in two bricks in the fourths row, three in the sixths row, etc.

Sample Space

	1	**2**	**2**	**3**	**3**	**4**
/ 2						
/3						
/ 4						
/ 6						
/ 8						
/ 12						

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[4] Featured in: Clarke, D. M., Roche, A., & Mitchell, A. (2008). 10 practical tips for making fractions come alive and make sense. *Mathematics Teaching in the Middle School, 13*(7), 372–380.

The World Series Problem (or The Umpires Strike Back)[5]

Overview

The Cardinals and Tigers are playing a best-four-of-seven series, with the winning team to collect a $1,000,000 bonus. When the Cardinals led two games to one, the umpires went on strike due to a labor dispute, leaving nobody to officiate. The strike causes the remainder of the games to be cancelled.

The teams are arguing over how the bonus money should be divided. The Cardinals claim that, because they were ahead, they should get all of the money. The Tigers claim that, because the Series had not ended, they still had a fair shot of winning so the money should be divided evenly. The commissioner believes the settlement should be somewhere in between. You are an arbitrator that has been called in to settle the dispute. If the Cardinals and Tigers are evenly matched and if the money will be divided in proportion to each team's probability of winning the series, how should the money be divided?

Tips and Strategies for Teachers

- Encourage students' use of different representations to model the situation in the task. I have seen students use organized lists, tables, tree diagrams, area models, …
- If the students become stuck or have ventured into an unproductive way of thinking about the task, suggest simulating the situation. A simulation can be conducted by completing a large number of trials using coins or dice (or any device that can model a 50-50 experiment) and recording the results. Use a class composite of the data to discuss whether their hypotheses are valid and to analyze whether the outcomes are equally likely to occur.
- This task can be simplified if needed to differentiate for learners' readiness levels. For example, reducing the series to five games will reduce the number of outcomes and the complexity of the calculations involved. Analogously, a seven-game series could be interrupted with the Cardinals up three games to two.

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5 Adapted from: Kahan, J.A., & Wyberg, T. R. (2003). Problem solving can generate new approaches to mathematics: The case of probability. *Mathematics Teacher, 96*(5), 328–332.