Mathematical Discourse

From Question Asking to Question Answering

CORE MATH®

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Strategies to Initiate, Manage, and Connect & Conclude Discourse

I. Initiate Discourse: Formation and Foundation

1. Assign "rich tasks," and/or ask engaging and challenging questions. For example,

   - Simply ask "why?"

   - Ask "compare-and-contrast" questions.

   - Ask "agree/disagree and why" questions.

   - Ask, "How does this connect to previous work?" questions.

   - Ask questions that have students build mathematical concepts one upon another. For example, after reviewing subtraction of whole numbers on a number line, ask, "What do you think it means if we subtract 5 from 3?"

2. Provide think and discussion time. Use *turn-and-talk, think-pair-share,* and *think-write-pair-share.* Keep in mind that not all discourse is student-to-student in pairs or small groups. Discourse may also be whole-class and student-to-teacher.

3. Let students know what is expected—to think about (possibly write about) and then discuss a math idea. Let them know this may be different from prior experiences and expectations in math classes.

II. Manage Discourse: Know when to hold them and Know when to scaffold them

1. Begin with small expectations in terms of discussion time and provide time frames for students. For example, *"Class, you have 30 seconds to decide if you agree or disagree with Jerry's answer and why. Then you will have two minutes to discuss this with your partner and be ready to explain your answer to the class."*

2. Circulate and listen in on conversations while students talk.

3. While circulating during student-to-student discussions, be prepared to guide, refocus, and move past blockages in discussions as needed by asking questions to help scaffold ideas, or making suggestions to discussions. Just be sure to leave room for students to think. Don't do all the thinking for them.

III. Connect & Conclude Discourse: Selection and Connection

1. Choose students to share using both selective methods to have specific ideas shared that you overheard, or using "cold call" to encourage all students to be ready to share and to prevent bias in selecting who shares.

2. Have students share various strategies to a solution.

3. Correct misconceptions as needed. Invite students into this process.

4. Build on student ideas whenever possible. Connect student ideas or ask students to connect ideas that are shared. (*Repeat, revise, restate,* and *add-on* strategies.)

5. Clarify and validate correct mathematical thinking. Connect ideas, methods, thinking, and/or conclusions to the day's learning objective(s).

Always, Never, or Sometimes Questions

A. For 'A' and 'B' in the problem below, when will the sum *always*, *never*, or *sometimes* be a three digit number?

$$\begin{array}{r} A \\ + B \\ \hline \_\ \_\ \_ \end{array}$$

B. When is the following statement *always* true, *never* true, and *sometimes* true?

If a + b = c, then c is a multiple of 5.

C. When is the following statement *always* true, *sometimes* true, and *never* true?

"Multiplication results in a greater value."

In other words, If a > b, and a × b = c, then c > a.

Hess' Cognitive Rigor Matrix

- **Bloom:** What <u>type of thinking</u> (verbs) is needed to complete a task?
- **Webb:** <u>How deeply</u> do you have to understand the content to successfully interact with it? How complex is the content?

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DOK is about depth & complexity—not difficulty!

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Hess' Cognitive Rigor Matrices
Applying Webb's Depth-of-Knowledge Levels to Bloom's Cognitive Process Dimensions

REVISED BLOOM'S TAXONOMY	WEBB'S DEPTH OF KNOWLEDGE LEVELS			
	1 RECALL & REPRODUCTION	2 SKILLS & CONCEPTS	3 STRATEGIC THINKING/ REASONING	4 EXTENDED THINKING
REMEMBER Retrieve knowledge from long-term memory, recognize, recall, locate, identify	Recall a fact, term, principle, or concept; perform a routine procedure			
UNDERSTAND Construct meaning, clarify, paraphrase, represent, translate, illustrate, give examples, classify, categorize, summarize, generalize, infer a logical conclusion, predict, compare/ contrast, match similar ideas, explain, construct models		Use information, conceptual knowledge; select appropriate procedures for a task; perform two or more steps with decision points along the way; solve routine problems; organize or display data; interpret or use simple graphs.	Reason or develop a plan to approach a problem; employ some decision making and justification; solve abstract, complex, or nonroutine problems (DOK 3 problems often allow more than one possible answer.)	Perform investigations or apply concepts and skills to the real world that require time to research, problem solve, and process multiple conditions of the problem or task; perform nonroutine manipulations across disciplines, content areas, or multiple sources
APPLY Carry out or use a procedure in a given situation; carry out or use/apply to an unfamiliar task				
ANALYZE Break into constituent parts, determine how parts relate, differentiate between relevant-irrelevant, distinguish, focus, select, organize, outline, find coherence, deconstruct (e.g., for bias or point of view)				
EVALUATE Make judgments based on criteria, check, detect inconsistencies/fallacies, judge, critique				
CREATE Reorganize elements into new patterns/structures, generate, hypothesize, design, plan, produce				

Hess, Carlock, Jones, and Walkup (2009)

Taking a Closer Look: Level 2, Skill/Concept

BLOOM's	DOK Level 2 Skills & Concepts	
	ELA/History Social Studies	Math/Science
REMEMBER		
UNDERSTAND	• Specify, explain, show relationships, explain why, cause-effect • Give examples/nonexamples • Summarize results, concepts, ideas in one text or one data set • Identify main ideas or accurate generalizations of texts or issues • Locate information to support explicit-implicit central ideas	• Specify and explain relationships (e.g., nonexamples/examples, cause-effect) • Make and record observations • Explain steps followed • Summarize results or concepts • Make basic inferences or logical predictions from data/observations • Use models (e.g., diagrams to represent or explain mathematical concepts) • Make or explain estimates
APPLY	• Use context to identify the meaning of words/phrases • Obtain and interpret information using text features • Develop a text that may be limited to one paragraph	• Select a procedure according to criteria and perform it • Solve routine problems, applying multiple concepts or decision points • Retrieve information from a table, graph, or figure and use it to solve a problem • Translate between tables, graphs, words, and symbolic notations (e.g., graph data from a table) • Construct models given criteria
ANALYZE	• Categorize/compare literary elements, terms, facts/details, events • Identify use of literary devices • Analyze format, organization, and internal text structures (e.g., signal words, transitions, semantic clues) of different text • Distinguish relevant-irrelevant information, fact/opinion • Identify characteristic text features; distinguish between texts, genres	• Categorize, classify materials, data, figures based on characteristics • Organize or order data • Compare/contrast figures or data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern
EVALUATE		
CREATE	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	Generate conjectures or hypotheses based on observations or prior knowledge and experience

Key Words

Infer, categorize, organize and display, compare-contrast, modify, predict, interpret, distinguish, estimate, extend patterns, interpret, use context clues, make observations, summarize, translate from table to graph, classify, show cause/effect, relate, edit for clarity

Teacher Role

Questions to differentiate, infer, or check conceptual understanding, models, organizes/reorganizes, explores possible options or connections, provides examples and nonexamples

Student Role

Solves routine problems/tasks involving multiple decision points and concepts, constructs models to show relationships, demonstrates use of conceptual knowledge, compiles and organizes, illustrates/explains with examples or models, examines

Question Stems

- How or why would you use _____?
- Can you explain how _____ affected _____?
- What was the cause of _____?
- How would you compare _____? Contrast_____?
- How would you classify_____? What would you use to classify___?
- How are_____alike? Different?
- How would you classify the type of_____?
- What can you say about_____?
- How would you summarize_____?
- What steps are needed to edit___?
- When would you use an outline to ___?
- How would you estimate___?
- How would you organize to show _____?
- What examples/nonexamples can you find to_____?
- What do you notice about___?

Taking a Closer Look:
Level 3, Strategic Thinking & Reasoning

BLOOM'S	DOK LEVEL 3 STRATEGIC THINKING/ REASONING	
	ELA/HISTORY SOCIAL STUDIES	MATH/SCIENCE
REMEMBER		
UNDERSTAND	• Explain, generalize, or connect ideas using supporting evidence (e.g., quote, example, text reference) • Identify/make inferences about explicit or implicit themes • Describe how word choice, point of view, or bias may affect the readers' interpretation of a text • Write multi-paragraph composition for specific purpose, focus, voice, tone, & audience	• Use concepts to solve nonroutine problems • Explain, generalize, or connect ideas using supporting evidence • Make and justify conjectures • Explain thinking when more than one response/solution is possible • Explain phenomena in terms of concept
APPLY	• Apply concepts to a new concept • Revise final draft for meaning or progression of ideas • Apply internal consistency of text organization and structure to composing a full composition • Apply word choice, point of view, style, to impact readers'/viewers' interpretation of a text	• Design an investigation for a specific purpose or research question • Conduct a designed investigation • Use concepts to solve nonroutine problems • Use and show reasoning, planning, and evidence • Translate between problem & symbol notation when not a direct translation
ANALYZE	• Analyze information within a text or source • Analyze interrelationships among concepts, issues, problems • Analyze or interpret author's craft (literary devices, viewpoint, potential bias) to create or critique a text or support text interpretation • Use reasoning and evidence to generate criteria for making and supporting an argument or judgment (e.g., Was FDR a great president? Who was the greatest ballplayer?)	• Compare information data sets or texts or across related data sets • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph • Analyze similarities/differences between research procedures or solutions
EVALUATE	• Cite evidence and develop a logical argument for conjectures • Describe, compare, and contrast solution methods • Verify reasonableness of results • Justify or critique conclusions drawn	• Cite evidence and develop a logical argument for concepts or solutions • Describe, compare, and contrast solution methods • Verify reasonableness of results
CREATE	• Synthesize information within one source or text • Develop a complex model for a given situation • Develop an alternative solution	• Synthesize information within one data set, source, or text • Formulate an original problem given a solution • Develop a scientific/ mathematical model for a complex situation

Key Words

Critique, appraise, revise for meaning, assess, investigate, cite evidence, test hypothesis, develop a logical argument, use concepts to solve nonroutine problems, explain phenomena in terms of concepts, draw conclusions based on data

Teacher Role

Questions to probe reasoning and underlying thinking, asks open-ended questions, acts as a resource and coach, provides criteria and examples for making judgments and supporting claims, encourages multiple approaches and solutions; determines when/where (text, concept) depth and exploration is most appropriate

Student Role

Uncovers and selects relevant and credible supporting evidence for analyses, critiques, debates, claims, and judgments; plans, initiates questions, disputes, argues, tests ideas/solutions, sustains inquiry into topics or deeper problems, applies to the real world

Question Stems

- How is _____ related to _____?
- What conclusions can you draw _____?
- How would you adapt _____ to create a different_____?
- How would you test_____?
- Can you predict the outcome if_____?
- What is the best answer? Why?
- What conclusion can be drawn from these three texts?
- What is your interpretation of this text?
- Support your rationale.
- How would you describe the sequence of_____?
- What facts would you select to support_____?
- What is the impact on the reader for use of this (rhetorical device, analogy, figurative language use, visual image, etc.)?
- What underlying bias is there_____?
- Can you elaborate on the reason_____?
- What would happen if___?
- Can you formulate a theory for___?
- How would you test___?
- Can you elaborate on the reason?
- What is the author's chain of reasoning or point of view for _____?

Taking a Closer Look:
Level 4, Extended Thinking

BLOOM's	DOK LEVEL 4 EXTENDED THINKING	
	ELA/HISTORY SOCIAL STUDIES	MATH/SCIENCE
REMEMBER		
UNDERSTAND	• Use multiple sources to elaborate on how concepts or ideas specifically draw from other content domains or differing concepts (e.g., research paper, arguments of policy: should this law be passed? What will be the impact of this change?) • Develop generalizations about the results obtained or strategies used and apply them to a new problem or contextual scenario.	• Relate mathematical or scientific concepts to other content areas, other domains, or other concepts • Develop generalizations of the results obtained and the strategies used (from investigation or readings) and apply them to new problem situations
APPLY	• Select or devise an approach among many alternatives to research and present a novel problem or issue • Illustrate how multiple themes (e.g., historical, geographic, social) may be interrelated within a text or topic	• Select or devise an approach among many alternatives to solve a problem • Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
ANALYZE	• Analyze multiple sources of evidence, or multiple works by the same author, or across genres or time periods • Analyze complex/abstract themes, perspectives, concepts • Gather, analyze, and organize multiple information sources • Compare and contrast conflicting judgments or policies (e.g., Supreme Court decisions)	• Analyze multiple sources of evidence • Analyze complex/abstract themes • Gather, analyze, and evaluate information
EVALUATE	• Evaluate relevancy, accuracy, and completeness of information across multiple sources • Apply understanding in a novel way, provide argument or justification for the application • Critique the historical impact (e.g., policy, writings, discoveries, etc.)	• Gather, analyze, and evaluate information in order to draw conclusions • Apply understanding in a novel way, provide argument or justification for the application
CREATE	• Synthesize information across multiple sources or texts in order to articulate a new voice, alternate theme, new knowledge, or nuanced perspective	• Synthesize information across multiple sources or text • Design a mathematical model to inform and solve a practical or abstract situation

Key Words
Initiate, design and conduct, collaborate, research, synthesize, self-monitor, critique, produce/present

Teacher Role
Questions to extend thinking and broaden perspectives; facilitates teaming, collaboration, self-evaluation

Student Role
Designs, takes risks, researches synthesizing multiple resources, collaborates, plans, organizes, and modifies, creates concrete tangible products

Question Stems
- What changes would you make to solve or address this major problem or issue?
- How would you improve upon this invention or innovation?
- Can you propose an alternative solution to . . . ?
- What could be done to minimize/maximize __ ?
- In what way would you design or redesign . . . and why?
- What evidence would you cite to defend the actions of __?
- How would you evaluate __?
- How would you prioritize criteria for making this decision . . . and why?
- How would you evaluate the works by this author over time?
- What information would you use to support a differing perspective___?
- Can you formulate and test a conjecture for__?
- Can you predict the potential benefits and drawbacks if this law does/does not pass?
- Can you construct a model that would change __?
- Can you think of an original way to apply__?
- What information can you gather to support your idea about___?

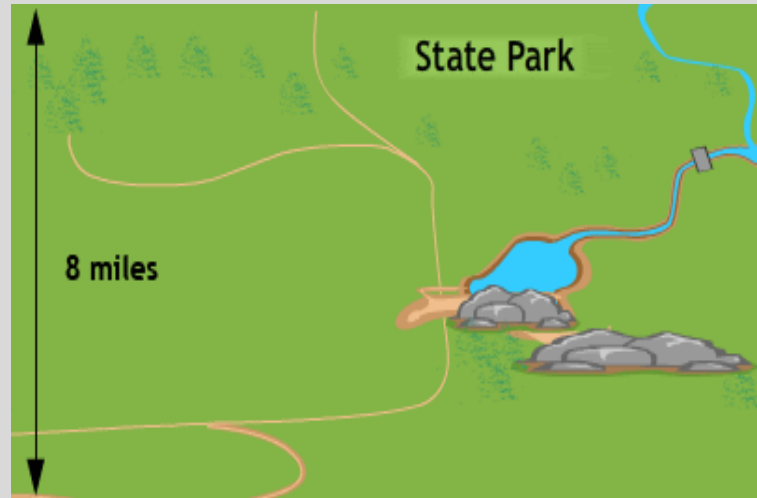## State Park

The perimeter of the rectangular state park shown is 42 miles.

A ranger estimates that there are 9 deer in each square mile of the park.

If this estimate is correct, how many total deer are in the park?

Explain your answer using numbers, symbols, and words.

PARCC Grade 4 Sample
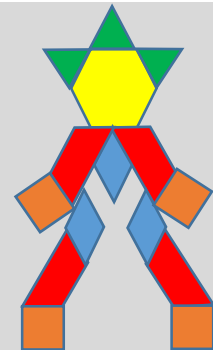(http://www.ccsstoolbox.com/parcc/PARCCPrototype_main.html)

State Park

8 miles

Pattern Block Man

1. If the hexagon represents one whole, what fraction do

    a) All the triangles represent together?
    b) All the trapezoids represent together?
    c) All the rhombi represent together?

    Explain your reasoning.

2. What total value do all four shapes represent together?

What would be a good extension for added discourse that would likely challenge students reasoning and allow you to see how they are understanding fractions?