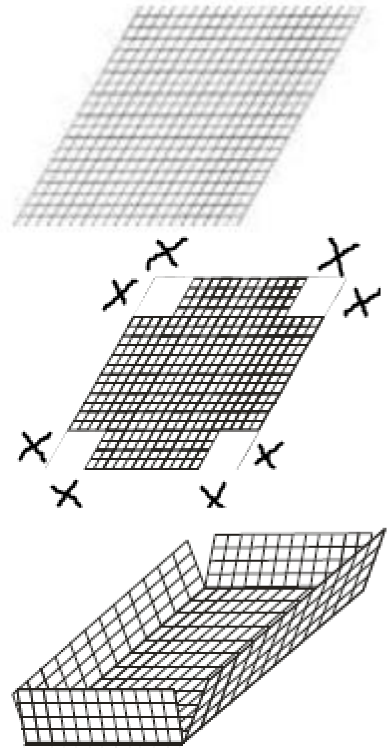
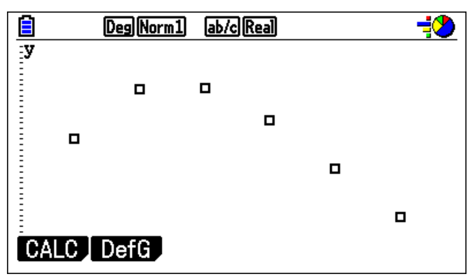
Modeling Functions: From Grid Paper to Graphing Calculator





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Common Core State Standards Addressed

CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

CCSS.Math.Practice.MP4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. They are able to identify important quantities in a practical situation.

CCSS.Math.Practice.MP5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.

CCSS.Math.Practice.MP7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. They notice if calculations are repeated, and look both for general methods and for shortcuts. Mathematically proficient students continually evaluate the reasonableness of their intermediate results.

Creating Equations

Create equations that describe numbers or relationships

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Interpreting Functions

1. Interpret functions that arise in applications in terms of the context: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities.

2. Analyze functions using different representation: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Building Functions

1. Build a function that models a relationship between two quantities: Write a function that describes a relationship between two quantities.

Modeling (*Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards.)*

1. Modeling links classroom mathematics and statistics to everyday life, work, and decision-making.

2. In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

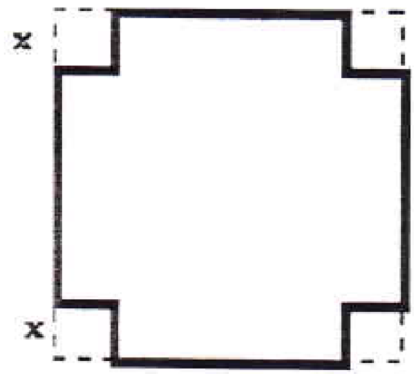
3. Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that is empirically based. Functions are an important tool for analyzing such problems.

Statistics and Probability

1. Connections to Functions and Modeling: Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

The Box Problem

You have been given a 7- inch square piece of card stock to use in constructing an open-topped box to store your electronic gadgets. You are to cut squares from the corners of the cardboard piece, then fold up the sides to make a box. What size squares should you cut out so that the box has maximum volume?



Materials

7.0 inch square piece of 0.5 inch grid card stock, scissors, tape

Investigation

1. Using the 0.5 inch grid, cut out squares from the corners starting with a 0.5 inch square, and increasing the size of the square in 0.5 inch increments. Let x represent the length of each side of the corner cutout.
2. After cutting out the corner square, fold up the sides to form a box. Tape the sides together.
3. Find the length of each side of the box. Fill in the Lengths that correspond with the corner cutouts in the table at the right.
4. Find the volume of each box. Write it in the table and in your box.
5. Share the data with the other members of your group.

X, height in inches	length in inches	width in inches	volume (inches^3)
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			

Analysis

1. Enter the data into your calculator. Store your data in the designated Lists:

   List #1: the x values, the height of the box
   List #2: the length of the box
   List #3: the width of the box
   List #4: the volume of the box

2. Make a scatter plot of the data using List #1 as your independent variable and List #4 as your dependent variable.

3. Let x represent the height of the box, and let y represent the volume of the box. Use an appropriate viewing window and make a scatter plot of the data in List #1 vs List #4. From the scatter plot, can you estimate the height of the cut out square that will determine the maximum volume of the box?

4. Would it be reasonable to make more boxes so that the maximum volume can be determined? Why or why not?

5. Clear the data stored in Lists 1, 2, 3, and 4. Highlight List 1 and use the keystrokes below to **model** the additional boxes that can be made.

OPTN F1 (List) F5 (Seq) X,θ,T , X,θ,T , 0 , 3 . 8 , 0 . 1 EXE

6. Highlight List 2. Use the keystrokes below to determine the length of the side of the box.

   7 − 2 × SHIFT 1 (List) 1 EXE

7. Highlight List 3. Use the keystrokes below to determine the width of the side of the box.

   SHIFT 1 (List) 2 EXE

8. Highlight List 4. Use the keystrokes below to calculate the volume of the box.

   SHIFT 1 (List) 1 × SHIFT 1 (List) 2 × SHIFT 1 (List) 3 EXE

9. Make a scatter plot of the data using List #1 as your independent variable and List #4 as your dependent variable. Do you notice any difference between the first scatter plot you made and this one?

10. From the second scatter plot, could you find the length of the side of the cut out square that will maximize the volume of the box? Could you find the corresponding maximum volume? What are they?

11. Use your calculator to find an appropriate model to represent a relationship for the data. Write it in the space below. Store this equation in Y1.

    Equation: ____

12. Using the equation you obtained in #11, find the **exact** size of the cut out square that will determine the maximum volume of the box. What is the maximum volume **rounded to the nearest hundredth**?

13. What are the dimensions of the box that determines the maximum volume **rounded to the nearest hundredth**? **Label your answers appropriately.**

    Length= _______ Width= _______ Height= _______

14. Refer back to the table and look at the height, length, and width columns. Using the graph and the table, write a function that would represent this situation. Store this equation in Y2.

    Equation: ____

15. Is the equation you wrote in #14 equivalent to the model you found from your data in #11? Explain below!

16. Find the x-intercepts for your equation in #14. What are they?

17. What is the real world meaning of the x-intercept(s)? Explain below!

18. Find the domain and range for the equation in #14. Write them as inequalities, **rounded to the nearest hundredth**.

    Domain: ____________________ Range: ____________________

19. Why do you think the domain and range make sense for this situation? Explain below!

20. Consider a 9 inch square piece of cardboard. Find the value of x, the length, the width, and the height that give the maximum volume **rounded to the nearest hundredth**. What is the corresponding volume? Label your answers appropriately!

    x = _______ Length= _______ Width= _______ Height= _______ Volume = __________

21. Consider a 12 inch square piece of cardboard. Find the value of x, the length, the width, and the height that give the maximum volume **rounded to the nearest hundredth**. What is the corresponding volume? Label your answers appropriately!

    x = _______ Length= _______ Width= _______ Height= _______ Volume = __________

22. Compare the values of x for the three different size square sheets of cardboard you started with to construct your box. Make a conjecture about what fraction of the side length should be cut from each corner of the square sheet to make a box of maximum volume.

23. If you were given a rectangular piece of cardboard that was 8.5 inches by 11 inches and you were to construct a box of maximum volume, what equation would model this situation? What would be the size of the cut out square, the dimensions of the box, and the corresponding volume, **rounded to the nearest hundredth?** Label your answers appropriately!

Equation: ______________________________

x = _______ Length= _______ Width= _______ Height= _______ Volume = __________

24. Find the domain and range for the equation in #23. Write them as inequalities, **rounded to the nearest hundredth**.

Domain: ____________________ Range: ____________________

25. You have 600 square feet of material for building a greenhouse that is shaped like half a cylinder. L represents the length of the greenhouse and r represents the radius of the half cylinder.

A. The surface area S of the greenhouse is given by the formula $S = \pi r^2 + \pi r L$. Substitute 600 for S and then write an expression for L in terms of r.

B. The volume V of the greenhouse is given by the formula $V = \frac{1}{2}\pi r^2 L$. Write an equation that gives V as a polynomial function of r.

C. Graph the volume function you found in part B. What are the dimensions r and L that will maximize the volume of the greenhouse? What is the maximum volume? **Round your answers to the nearest hundredth. Label your answers appropriately.**

Radius= __________ Length= __________ Volume= __________

References

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