NCTM Regional Conference Richmond, VA
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Parametric Equations as a Culminating Precalculus Activity

Plan:

- Introduce parametric equations
- Argue that parametric equations are a natural representation for physics and calculus
- Share many examples of parametric equations
- Share my teaching strategies

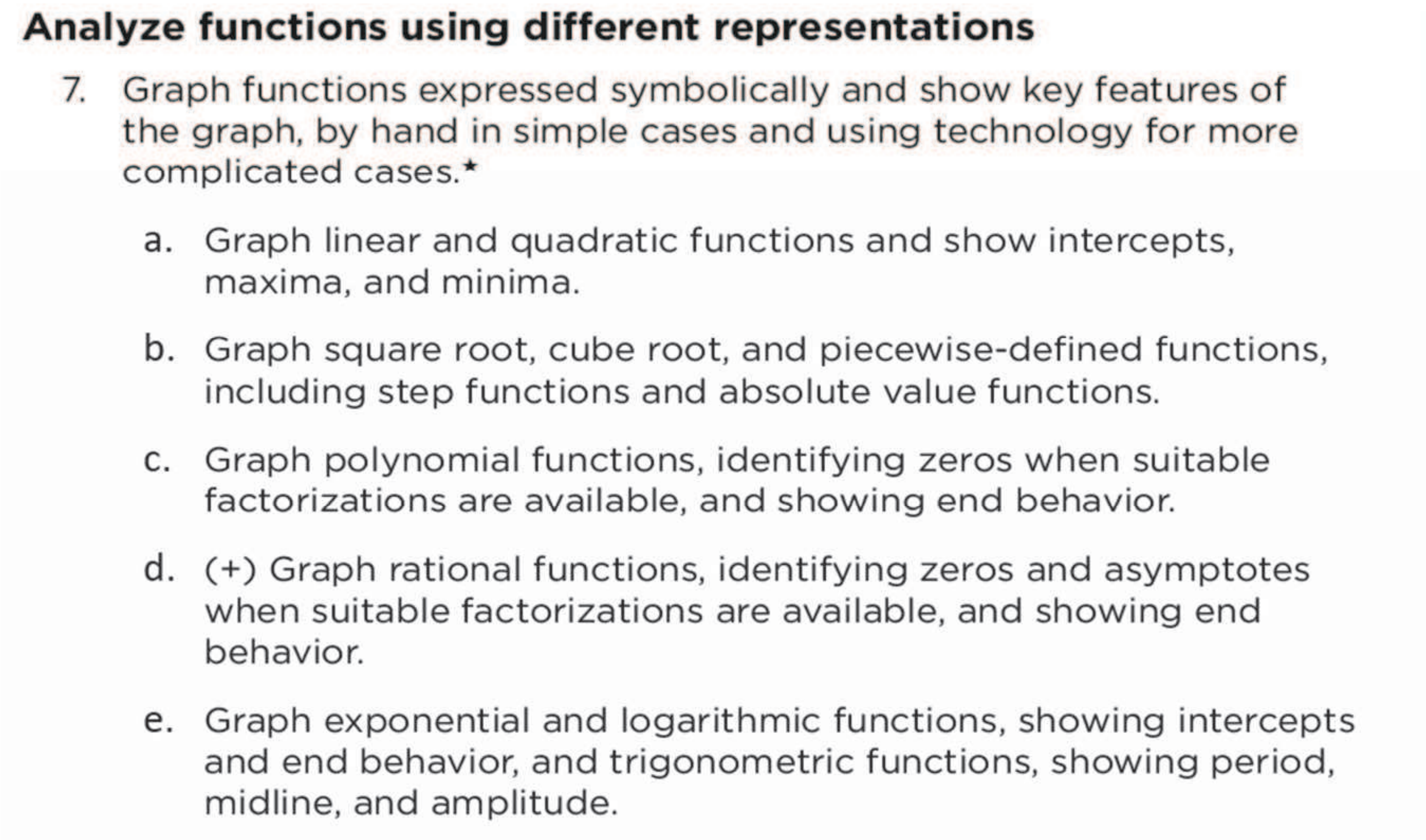
Goal:

- Convince you that this subject belongs in a precalculus curriculum
- Introduce techniques for solving parametric equations
- Suggest that parametric equations are an excellent summary of precalculus and introduction to calculus
- Provide interested parties with many, many interesting examples

Prerequisites:

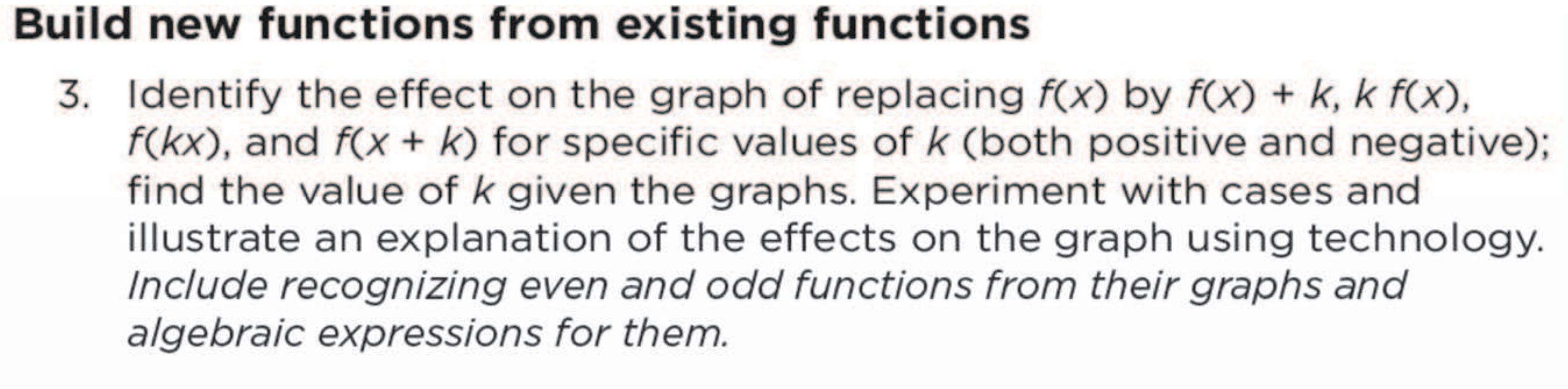
- All of precalculus…
- Algebraic functions, power functions, root functions
- Trigonometric functions, inverse trigonometric functions
- Trigonometric identitities
- Exponential and logarithmic functions
- Rational functions
- Knowledge of function transformations
- In other words, all the most important things for success in calculus

From the Common Core State Standards:



Analyze functions using different representations

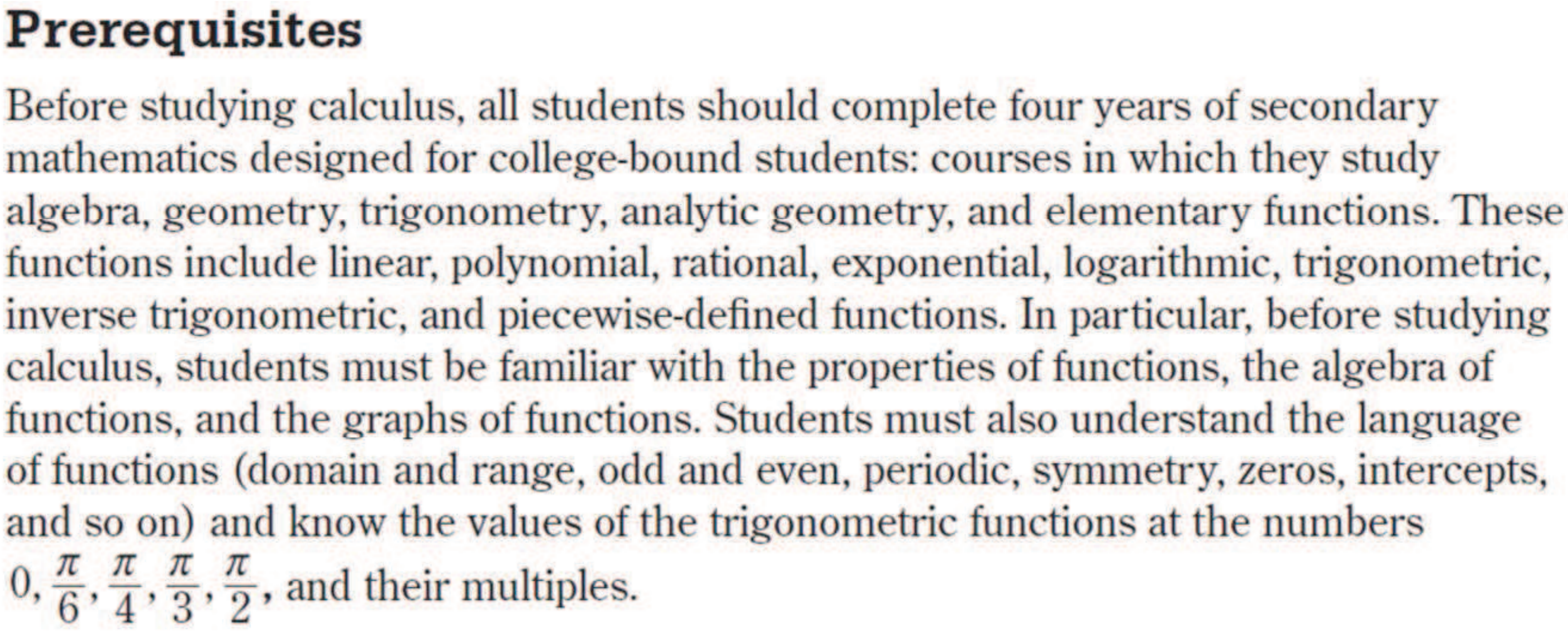
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.



Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

And from the AP Calculus Course Description:



Prerequisites

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at the numbers $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.

Parametric equations are useful when you want to describe how an object moves in a plane when the x and y coordinates of the object behave differently.

An example:

$$\begin{cases} x = 5t \\ y = -t(t - 20) \end{cases}$$

Page 1, #2

$$x = 2 - 3t$$

$$y = 4 + t$$

Page 1, #3

$$x = 1 + t^2$$

$$y = t + 2$$

Page 2, #6

$$x = t^2$$

$$y = t^4$$

Page 3, #9 and #10

$$x = 6\cos t$$
$$y = 6\sin t$$

$$x = 3\cos t$$
$$y = 5\sin t$$

Page 5, #16

$$x = 6\cos(2t)$$

$$y = 2\sin t$$

Page 9, #27

$$x = -e^{t}$$

$$y = e^{-2t}$$

Page 10, #30

$$x = \frac{-3}{t-2}$$

$$y = \frac{6}{t-5}$$

2. For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

   (a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

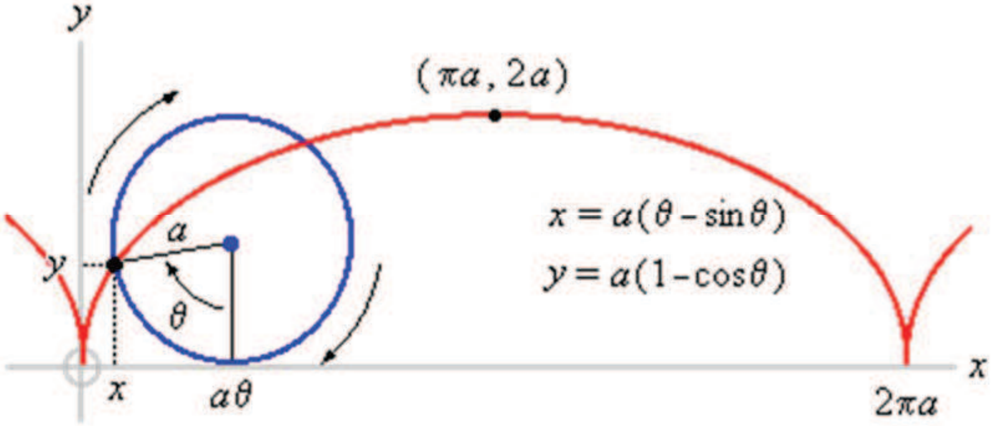
   (b) Find the x-coordinate of the particle's position at time $t = 4$.

   (c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

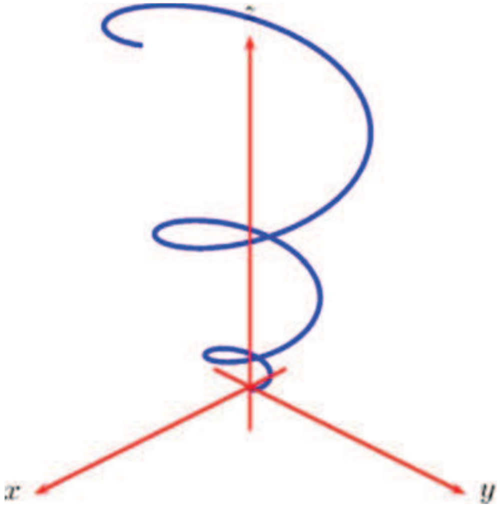
   (d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

Extensions:

- The cycloid, epicycloid, hypocycloid, and conchoids



- Three dimensional parameterizations



Conclusions:

- These problems can be very difficult!
- Each problem is totally unique and interesting, requiring a lot of independent student effort
- Provides an excellent opportunity to review precalculus knowledge
- Serves as an introduction to the parametric equations students encounter in calculus

Questions?

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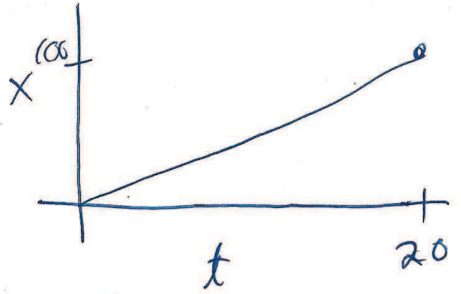
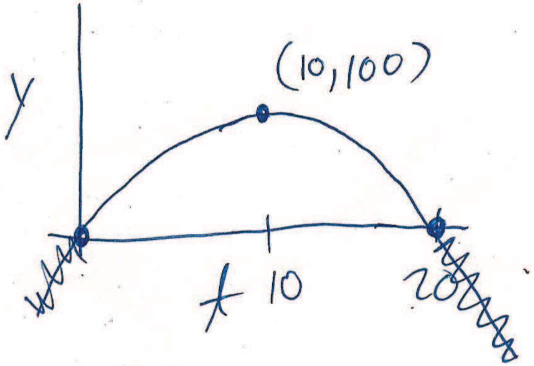
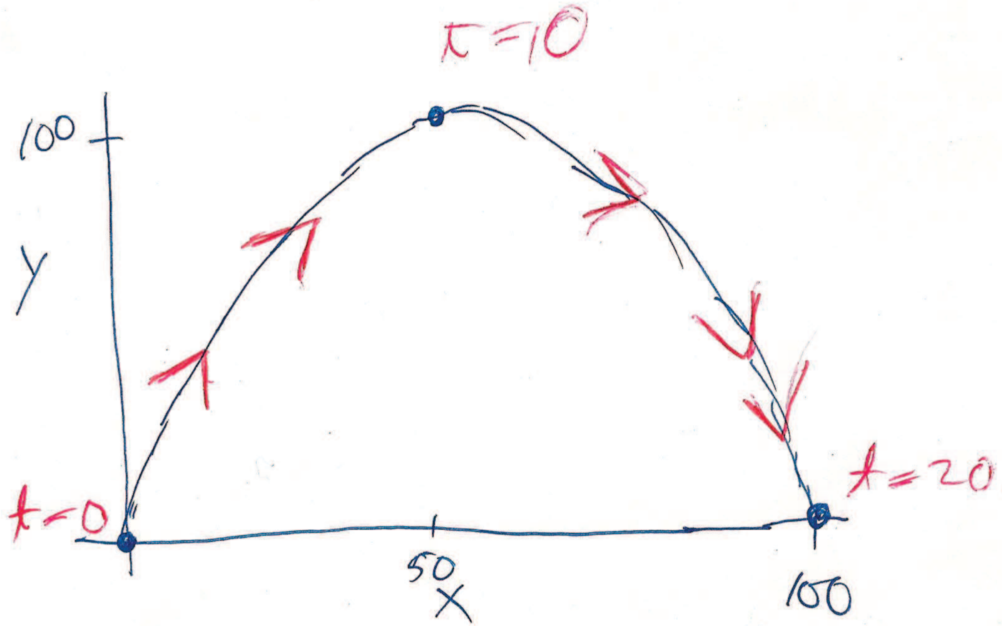
$x = 5t$

$y = -t(t-20)$







Eliminating the Parameter

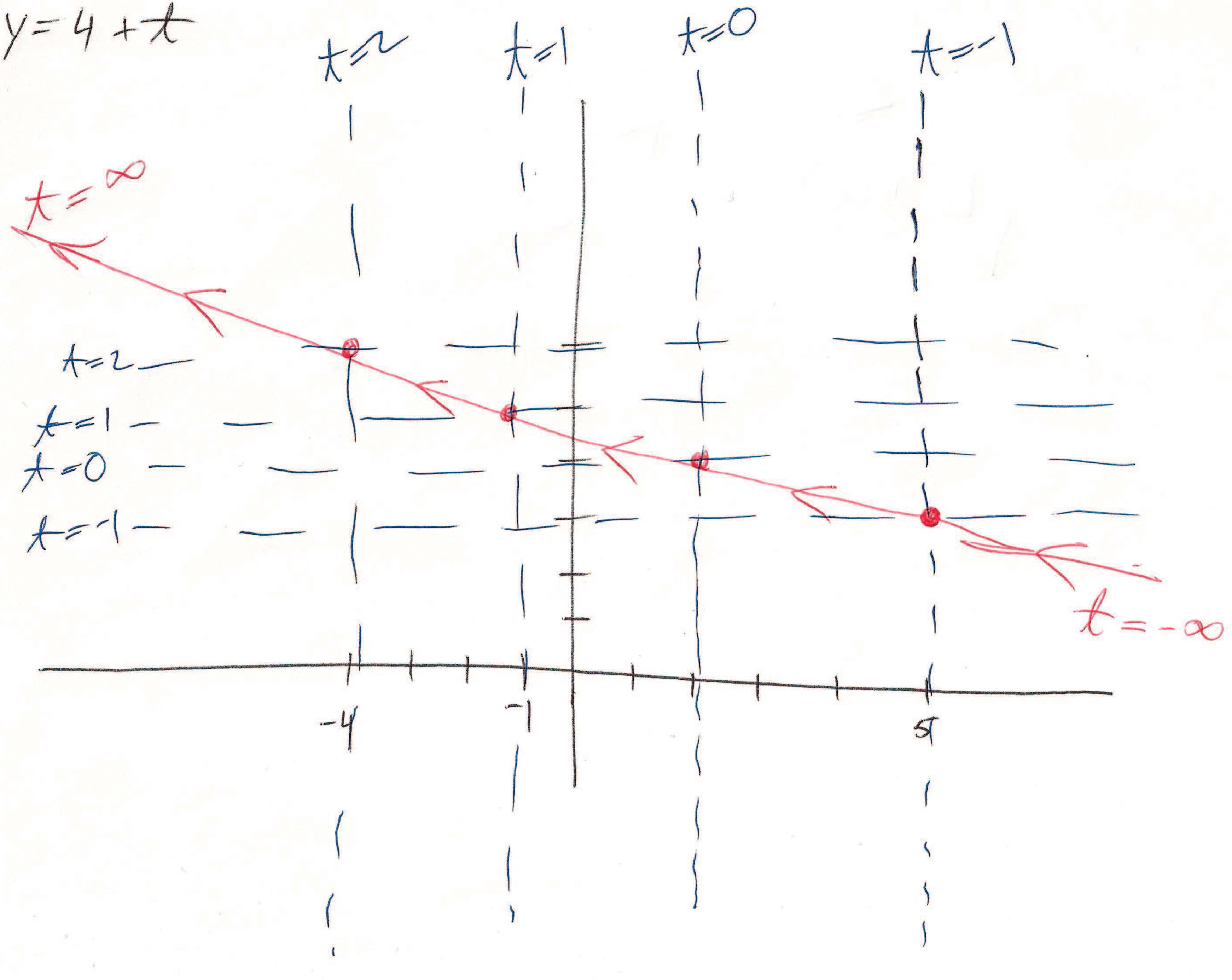
$t = \frac{x}{5}$

$y = -\frac{x}{5}\left(\frac{x}{5} - 20\right)$

$= -\frac{x}{5} \cdot \frac{1}{5}(x-100)$

$$\boxed{y = \frac{-1}{25}x(x-100)}$$

$x = 2 - 3t$
$y = 4 + t$



$$\begin{cases} x = 1 + t^2 \\ y = t + 2 \end{cases}$$

$\boxed{x \geq 1}$

$t \to \infty$	$t \to -\infty$
$x \to \infty$	$x \to \infty$
$y \to \infty$	$y \to -\infty$

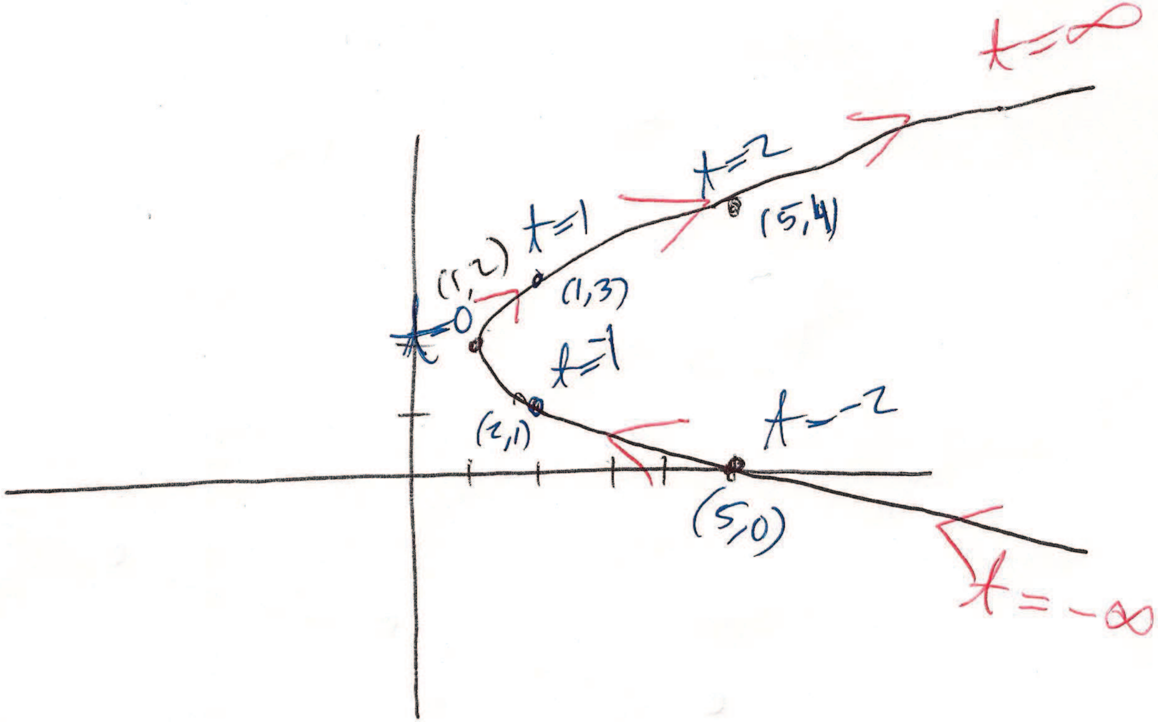
$t = y - 2$

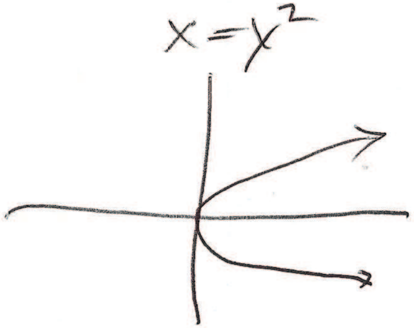
$$\boxed{x = 1 + (y-2)^2}$$

$x - 1 = t^2$

$t = \pm\sqrt{x-1}$

$$\boxed{y = \pm\sqrt{x-1} + 2}$$





$$\begin{cases} x = t^2 & x \geq 0 \\ y = t^4 & y \geq 0 \end{cases}$$

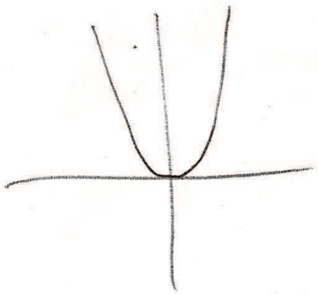
$t \to -\infty \qquad t \to \infty$

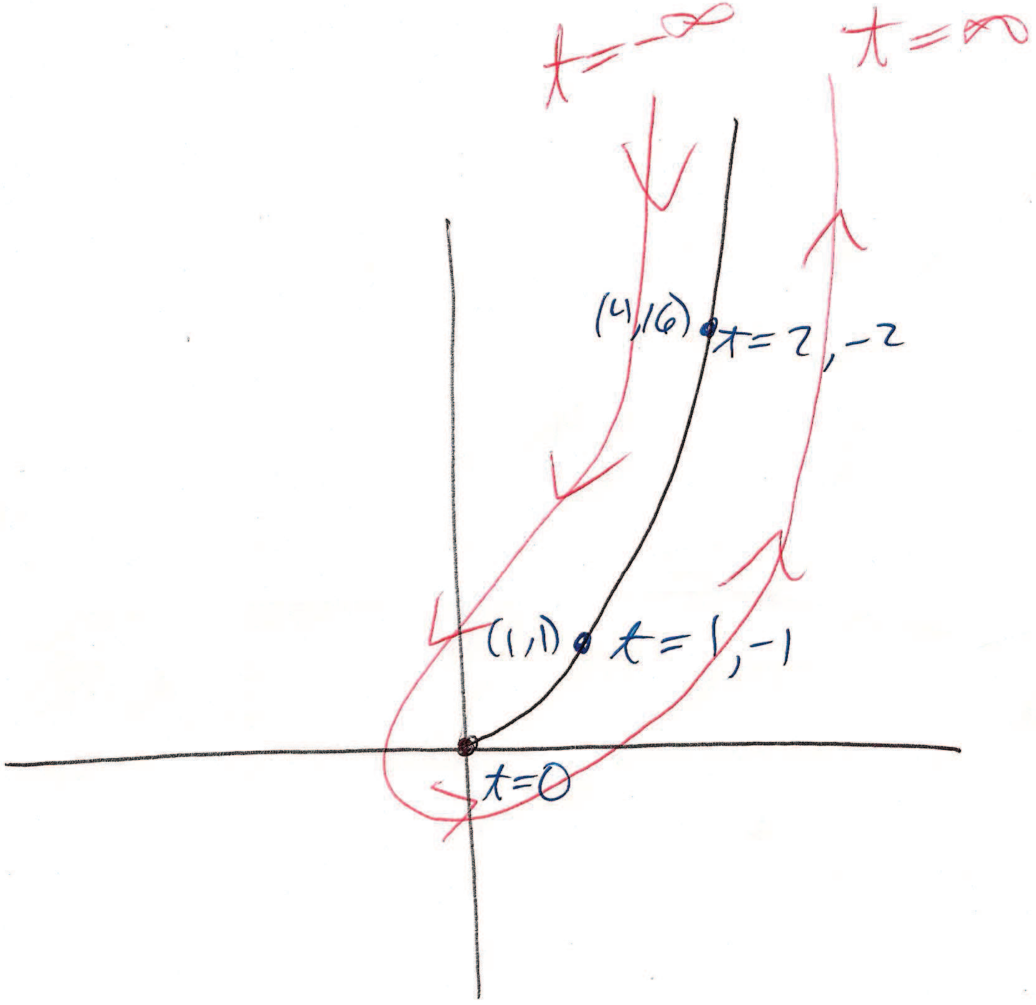
$x \to \infty \qquad x \to \infty$

$y \to \infty \qquad y \to \infty$

$$y = (t^2)^2$$

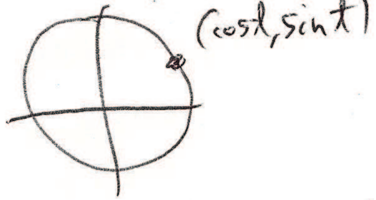
$$y = x^2$$





$$\begin{cases} x = 6\cos t & -6 \le x \le 6 \\ y = 6\sin t & -6 \le y \le 6 \end{cases}$$

?



$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$

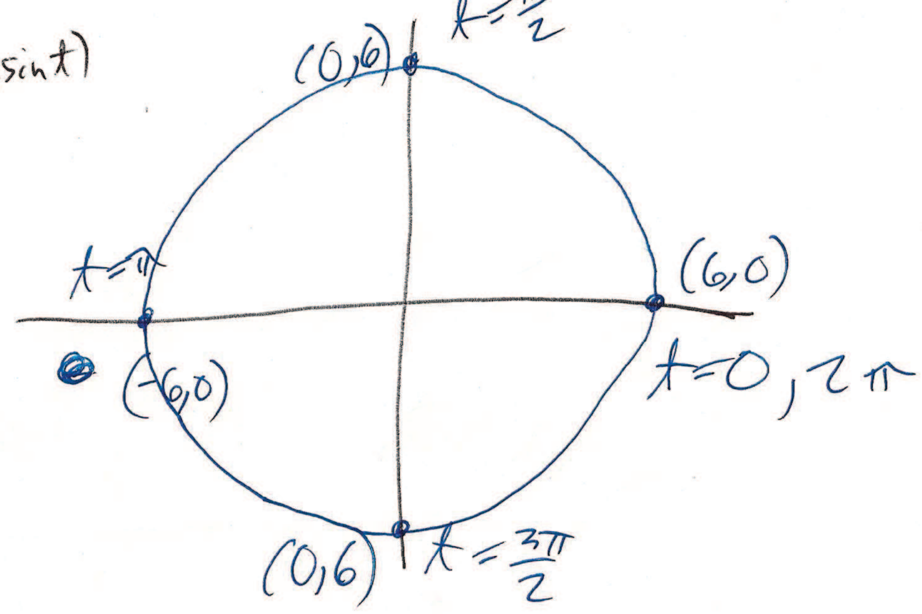
$$\frac{x}{6} = \cos t$$

$$\frac{y}{6} = \sin t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{x^2}{36} + \frac{y^2}{36} = 1$$

$$\boxed{x^2 + y^2 = 36}$$

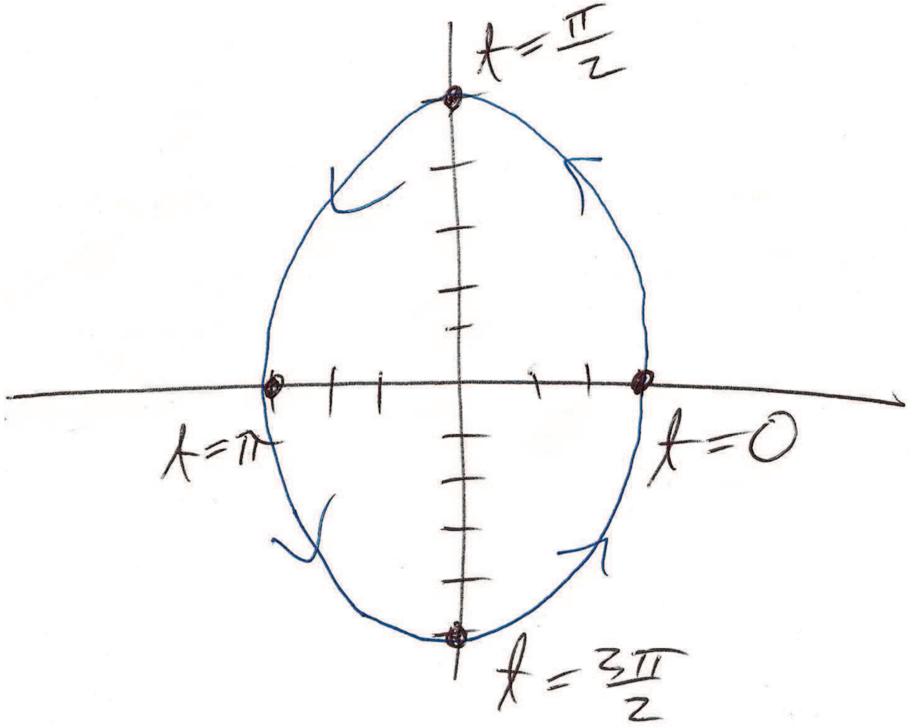


$$\begin{cases} x=3\cos t & -3 \le x \le 3 \\ y=5\sin t & -5 \le y \le 5 \end{cases}$$

$$\cos t = \frac{x}{3}$$

$$\sin t = \frac{y}{5}$$

$$\boxed{\frac{x^2}{9} + \frac{y^2}{25} = 1}$$



$$\boxed{\cos 2t = \cos^2 t - \sin^2 t}$$

$$\cos 2t = (1 - \sin^2 t) - \sin^2 t$$

$$\boxed{\cos 2t = 1 - 2\sin^2 t}$$

$$\cos(2t) = \cos^2 t - (1 - \cos^2 t)$$

$$\boxed{\cos 2t = 2\cos^2 t - 1}$$

$$\begin{cases} x = 6\cos 2t \\ y = 2\sin t \end{cases}$$

$-1 \le \cos 2t \le 1$

$-6 \le 6\cos t \le 6$

$-6 \le x \le 6$

$-1 \le \sin t \le 1$

$-2 \le 2\sin t \le 2$

$-2 \le y \le 2$

$\sin t = \frac{y}{2}$

$x = 6\left[\cos 2t\right]$

$x = 6\left[1 - 2\sin^2 t\right]$

$x = 6\left[1 - 2\left(\frac{y^2}{4}\right)\right]$

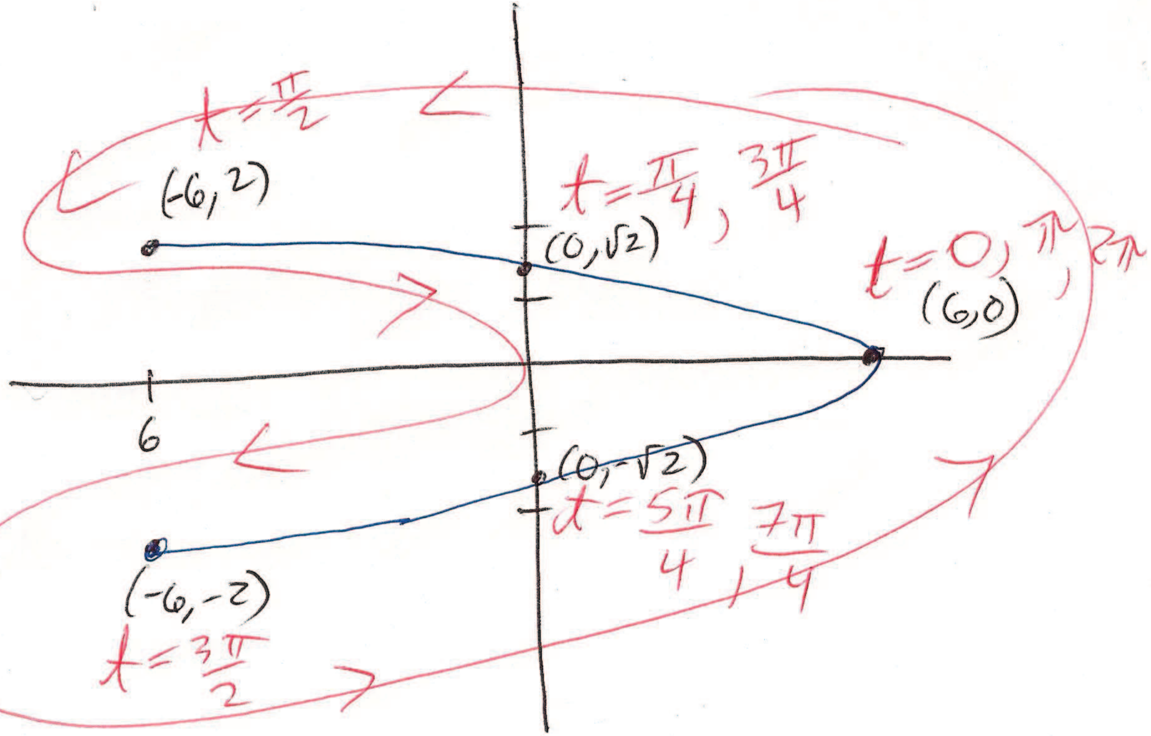
$x = 6\left(1 - \frac{y^2}{2}\right)$

$x = 6 - 3y^2$

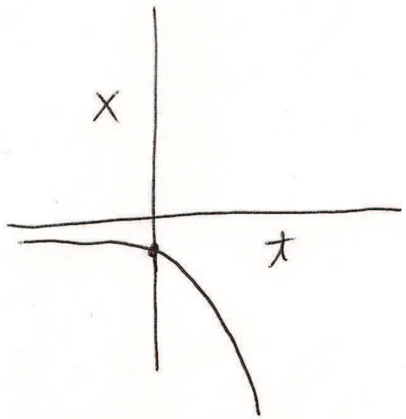
$\boxed{x = -3y^2 + 6}$

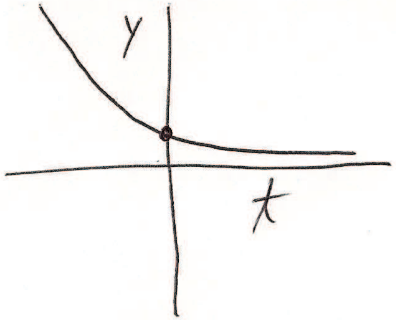
$x = -3(y^2 - 2)$

$\boxed{x = -3(y - \sqrt{2})(y + \sqrt{2})}$



$$\begin{cases} x = -e^{t} \\ y = e^{-2t} \end{cases}$$





$t \to \infty$
$x \to -\infty$
$y \to 0^{+}$

$t \to -\infty$
$x \to 0^{-}$
$y \to \infty$

$x < 0$
$y > 0$

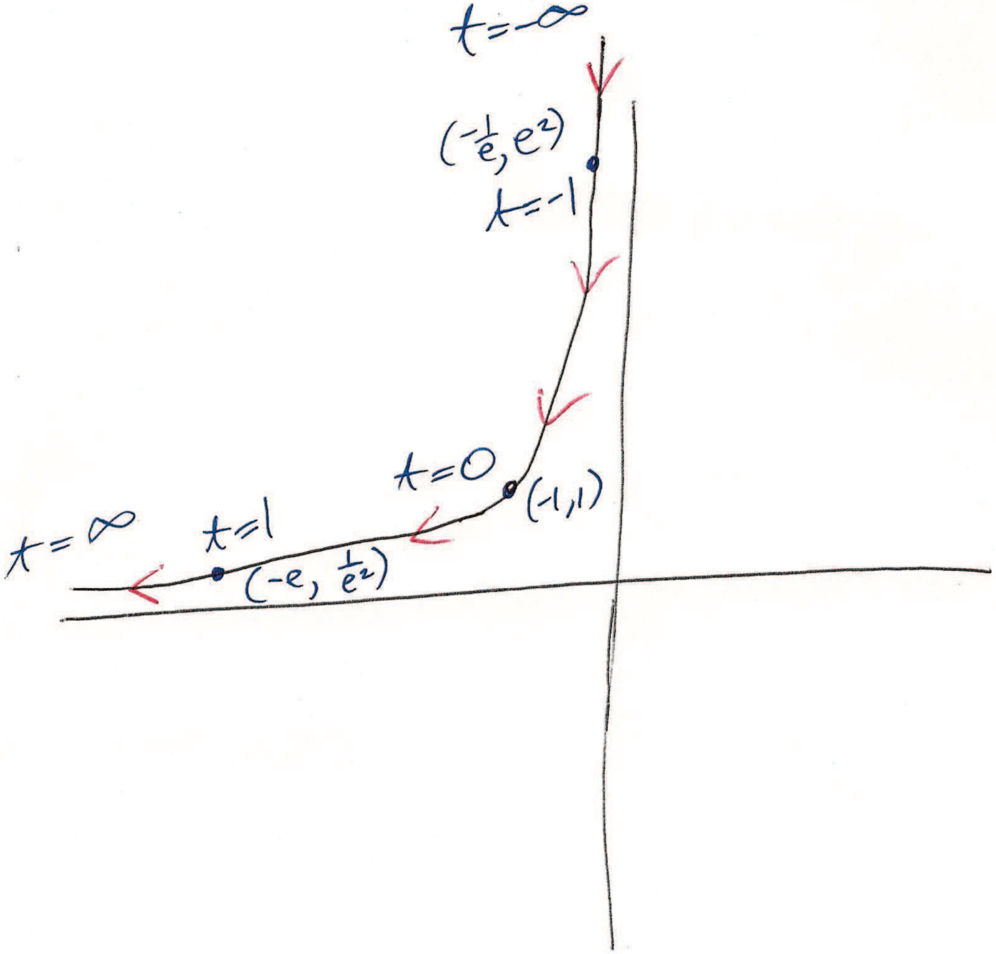
$e^{t} = -x$

$y = \frac{1}{e^{2t}}$

$y = \frac{1}{(e^{t})^{2}}$

$y = \frac{1}{(-x)^{2}}$

$y = \frac{1}{x^{2}}$



$$\begin{cases} x = \frac{-3}{t-2} & t \neq 2,\ x \neq 0,\ \boxed{x \neq -1} \\ y = \frac{6}{t-5} & t \neq 5,\ y \neq 0,\ \boxed{y \neq -2} \end{cases}$$

$t \to \infty$ $x \to 0^-$ $y \to 0^+$	$t \to -\infty$ $x \to 0^+$ $y \to 0^-$ ✓
$t \to 2^-$ ✓ $x \to \infty$ $y \to -2^+$	$t \to 2^+$ ✓ $x \to -\infty$ $y \to -2^-$
$t \to 5^-$ ✓ $x \to -1^-$ $y \to -\infty$	$t \to 5^+$ ✓ $x \to -1^+$ $y \to \infty$

$$x = \frac{-3}{t-2}$$

$$t-2 = \frac{-3}{x}$$

$$t = 2 - \frac{3}{x}$$

$$y = \frac{6}{2-\frac{3}{x}-5}$$

$$y = \frac{6}{-3-\frac{3}{x}}$$

$$y = \frac{6}{\frac{-3x-3}{x}}$$

$$y = \frac{6x}{-3x-3}$$

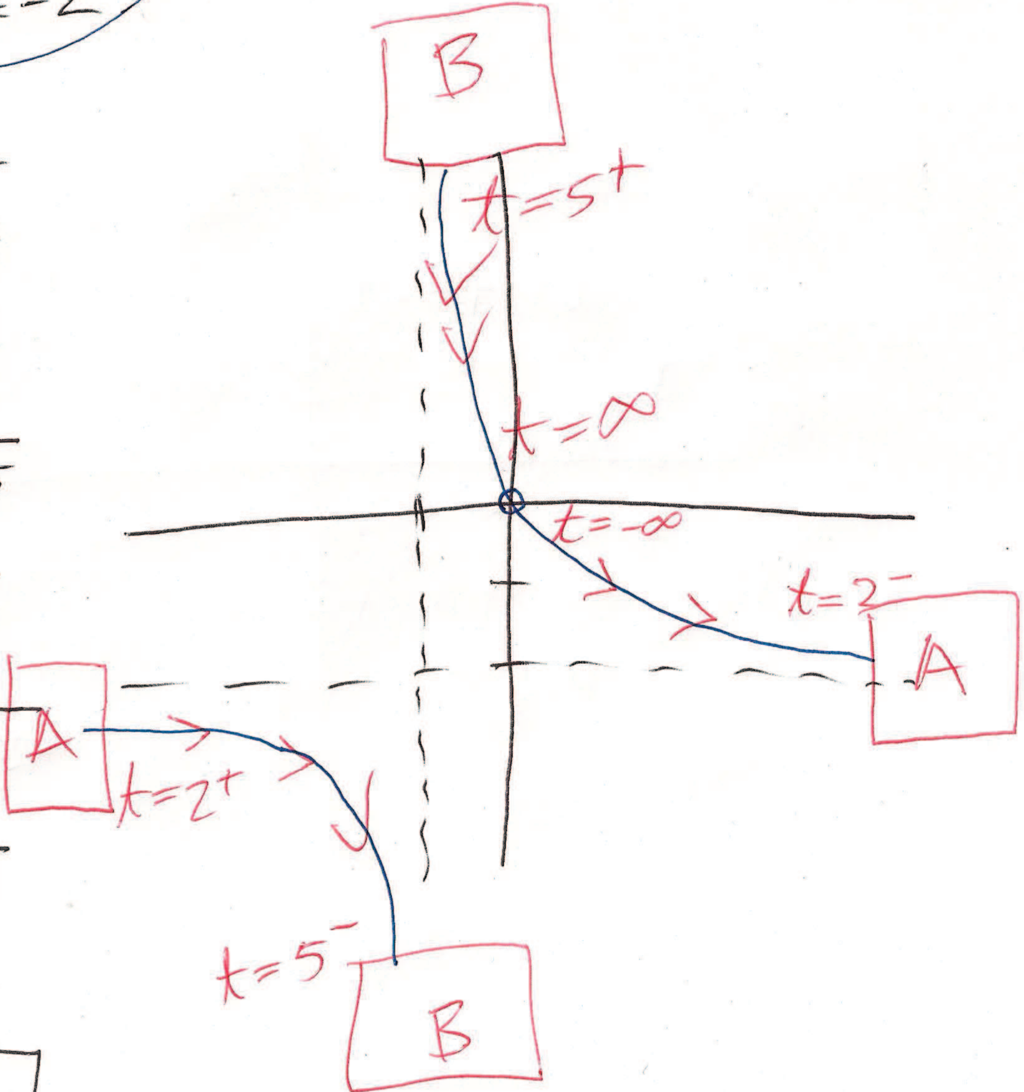
$$y = \frac{-2x}{x+1}$$

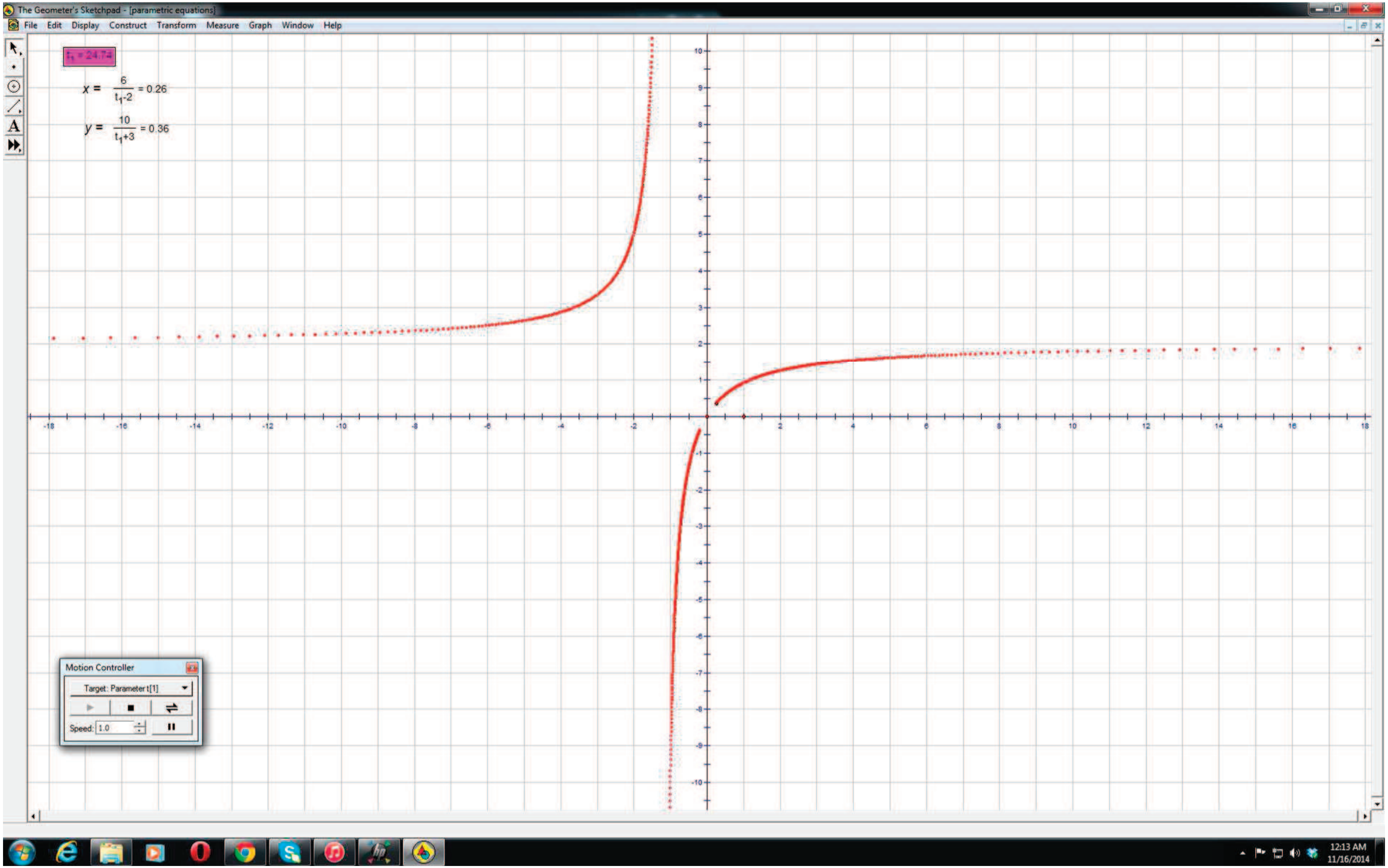
$$\boxed{y = -2 + \frac{2}{x+1}}$$

$$x+1 \overline{)\,-2x\,}\ \ -2$$

$$\underline{-2x-2}$$

$$2$$





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Parametric Equations

Directions: Note restrictions on x, y, and t. Compute end behavior and behavior near undefined points. Then convert each pair of parametric equations to a Cartesian equation by eliminating the parameter. Graph the plane curve traced out by these equations, being careful to consider domain issues. On each graph label several points with " t = " labels, and use arrows to indicate the direction in which the curve is traced out, as t goes from $-\infty$ to ∞. Include the behavior of x and y for values of t near where t is undefined.

1. $x = 3t + 1$
   $y = 2t - 1$

2. $x = 2 - 3t$
   $y = 4 + t$

3. $x = 1 + t^2$
   $y = t + 2$

4. $x = t - 3$

$y = 2 + t^2$

5. $x = t^3$

$y = t^2$

6. $x = t^2$

$y = t^4$

7. $x = \frac{4}{t} + 3$

$y = t + 1$

8. $$x = \frac{6}{t-2}$$
$$y = \frac{10}{t+3}$$

9. $$x = 6\cos t$$
$$y = 6\sin t$$

10. $$x = 3\cos t$$
$$y = 5\sin t$$

11. $\begin{aligned} x &= t^2 - 4 \\ y &= \frac{t}{2} \end{aligned} \qquad -2 \le t \le 3$

12. $\begin{aligned} x &= \frac{1}{\sqrt{t+1}} \\ y &= \frac{t}{t+1} \end{aligned}$

13. $\begin{aligned} x &= 4 \cdot \sqrt[3]{\cos t} \\ y &= 4 \cdot \sqrt[3]{\sin t} \end{aligned}$

14. $\begin{aligned} x &= 2^{-t} \\ y &= 2^{3t} + 1 \end{aligned}$

15. $\begin{aligned} x &= |t-1| \\ y &= t+2 \end{aligned}$

16. $\begin{aligned} x &= 6\cos(2t) \\ y &= 2\sin t \end{aligned}$

17. $\begin{aligned} x &= 1+\frac{1}{t} \\ y &= t-1 \end{aligned}$

18. $\begin{aligned} x &= 5\cos^2 t \\ y &= 3\sin t \end{aligned}$

19. $\begin{aligned} x &= 3\cos t \\ y &= 4\sin^2 t \end{aligned}$

20. $\begin{aligned} x &= 8\cos^2 t \\ y &= 6\sin^2 t \end{aligned}$

21. $\begin{aligned} x &= 8\cos^3 t \\ y &= 8\sin^3 t \end{aligned}$

22. $\begin{aligned} x &= 4\cos t \\ y &= 3\sin^2 t \end{aligned}$

23. $\begin{aligned} x &= 4\cos t \\ y &= 5\sin\left(\frac{t}{2}\right) \end{aligned}$

24. $$\begin{aligned} x &= \sqrt{t} \\ y &= -3\sqrt{16-t} \end{aligned}$$

25. $$\begin{aligned} x &= \cot t \\ y &= 2\sin^2 t \end{aligned}$$

26. $x = e^{-t+3}$
$y = t$

27. $x = -e^{t}$
$y = e^{-2t}$

28. $x = -3\cos t$
$y = 5\sin t$

29. $x = \dfrac{t-3}{t-5}$

$y = t$

30. $x = \dfrac{-3}{t-2}$

$y = \dfrac{6}{t-5}$

Name KEY

Parametric Equations Review

Directions: Convert each pair of parametric equations to a Cartesian equation by eliminating the parameter. Then graph the plane curve traced out by these equations, being careful to consider domain issues. On each graph label several points with " t = " labels, and use arrows to indicate the direction in which the curve is traced out, as t goes from ∞ to -∞.

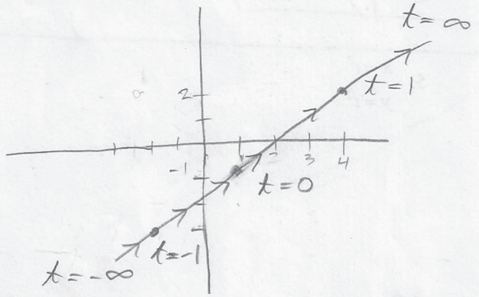
1. $x = 3t + 1$
$y = 2t - 1$

$t \to -\infty$	$t \to \infty$
$x \to -\infty$	$x \to \infty$
$y \to -\infty$	$y \to \infty$

$t = \frac{x-1}{3}$

$y = \frac{2}{3}(x-1) - 1$

$y = \frac{2}{3}x - \frac{5}{3}$



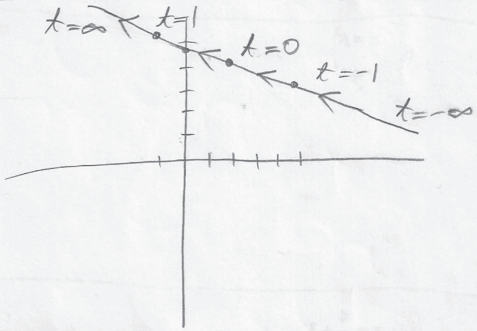
2. $x = 2 - 3t$
$y = 4 + t$

$t \to -\infty$	$t \to \infty$
$x \to \infty$	$x \to -\infty$
$y \to -\infty$	$y \to \infty$

$t = \frac{2-x}{3}$

$y = \frac{-(x-2)}{3} + 4$

$y = \frac{-x}{3} + \frac{14}{3}$

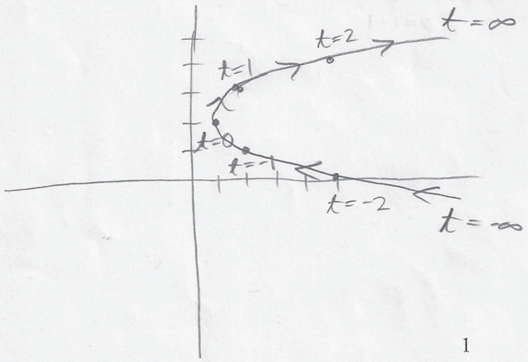


3. $x = 1 + t^2$ $x \geq 1$
$y = t + 2$

$t = y - 2$

$x = (y-2)^2 + 1$

$t \to -\infty$	$t \to \infty$
$x \to \infty$	$x \to \infty$
$y \to -\infty$	$y \to \infty$



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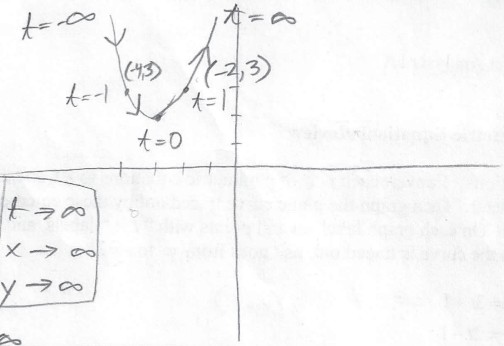
4. $x = t - 3$ $y \geq 2$
$y = 2 + t^2$

$t = x + 3$

$y = t^2 + 2$

$y = (x+3)^2 + 2$

$t \to -\infty$	$t \to \infty$
$x \to -\infty$	$x \to \infty$
$y \to \infty$	$y \to \infty$

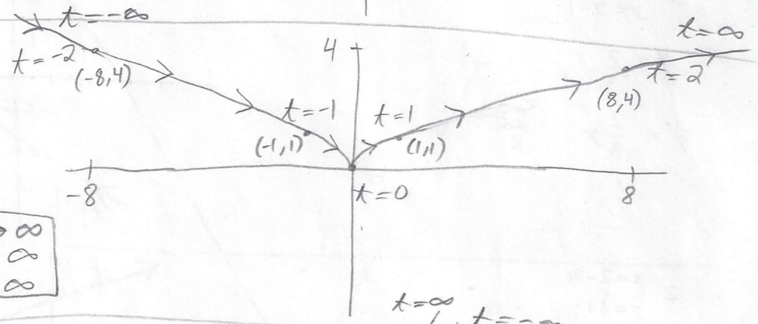


5. $x = t^3$
$y = t^2$ $y \geq 0$

$t = \sqrt[3]{x}$

$y = x^{2/3}$

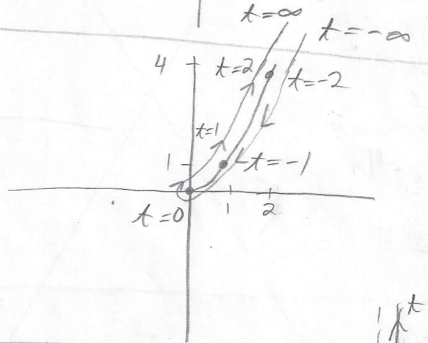
$t \to -\infty$	$t \to \infty$
$x \to -\infty$	$x \to \infty$
$y \to \infty$	$y \to \infty$



6. $x = t^2$ $x \geq 0$
$y = t^4$ $y \geq 0$

$y = x^2$

$t \to -\infty$	$t \to \infty$
$x \to \infty$	$x \to \infty$
$y \to \infty$	$y \to \infty$



$t \neq 0$
$x \neq 3$
$y \neq 1$

7. $x = \frac{4}{t} + 3$
$y = t + 1$

as $t \to \infty$
$x \to 3^+$
$y \to \infty$

as $t \to -\infty$
$x \to 3^-$
$y \to -\infty$

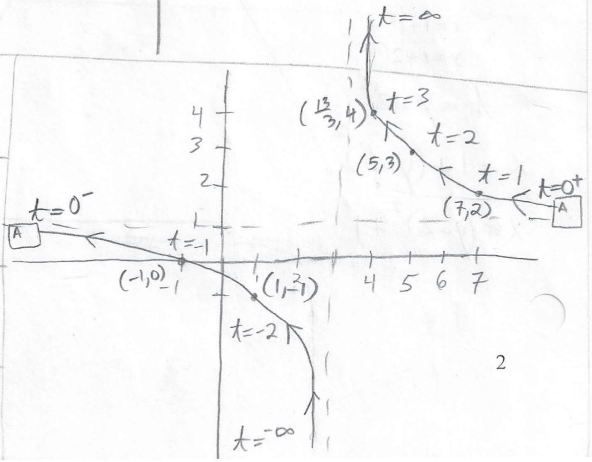
as $t \to 0^-$
$x \to -\infty$
$y \to 1$

as $t \to 0^+$
$x \to \infty$
$y \to 1$

$x - 3 = \frac{4}{t}$

$t = \frac{4}{x-3}$

$y = \frac{4}{x-3} + 1$



8. $x = \frac{6}{t-2}$ $\quad t \neq 2 \quad x \neq 0$

$y = \frac{10}{t+3}$ $\quad x \neq -\frac{6}{5}, y \neq 2$ $\quad t \neq -3, y \neq 0$

$t \to -\infty$ $x \to 0^-$ $y \to 0^-$	$t \to \infty$ $x \to 0^+$ $y \to 0^+$
$t \to 2^-$ $x \to -\infty$ $y \to 2^+$	$t \to 2^+$ $x \to \infty$ $y \to 2^-$
$t \to -3^-$ $x \to -\frac{6}{5}^+$ $y \to -\infty$	$t \to -3^+$ $x \to -\frac{6}{5}^-$ $y \to \infty$

$t - 2 = \frac{6}{x}$

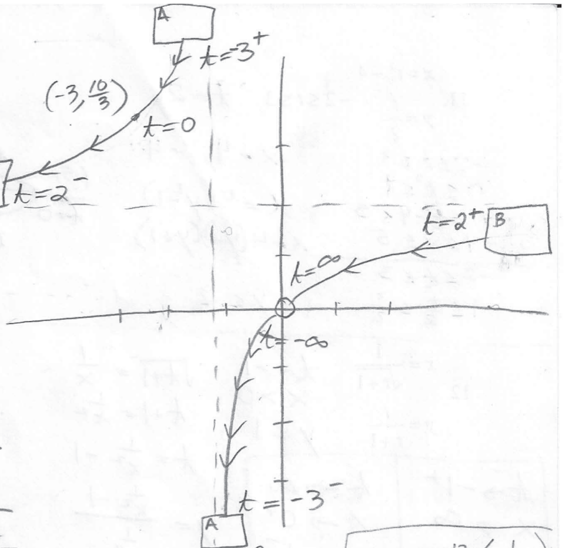
$t = 2 + \frac{6}{x}$

$y = \frac{10}{\left(2+\frac{6}{x}\right)+3}$

$y = \frac{10}{5+\frac{6}{x}}$

$y = \frac{10x}{5x+6}$

$$5x+6 \overline{\smash{)}\,10x} \quad 2$$
$$\underline{10x+12}$$
$$-12$$



$y = 2 - \frac{12}{5x+6}$; $\boxed{y = 2 - \frac{12}{5}\left(\frac{1}{x+\frac{6}{5}}\right)}$

---

9. $x = 6\cos t$ $\quad -6 \le x \le 6$

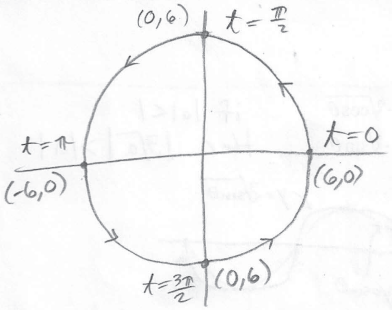
$y = 6\sin t$ $\quad -6 \le y \le 6$

$\cos t = \frac{x}{6} \quad \sin t = \frac{y}{6}$

$\cos^2 t + \sin^2 t = 1$

$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{6}\right)^2 = 1$

$x^2 + y^2 = 36$



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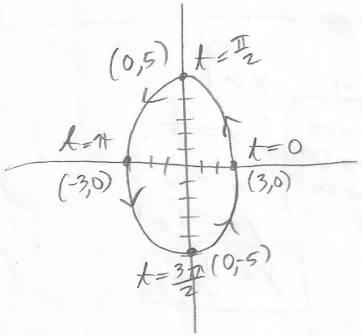
10. $x = 3\cos t$ $\quad -3 \le x \le 3$

$y = 5\sin t$ $\quad -5 \le y \le 5$

$\underbrace{\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1}$

unit circle stretched by 3 in x direction & stretched by 5 in the y direction



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11. $x = t^2 - 4$

$y = \frac{t}{2}$ $\quad -2 \le t \le 3$ $\quad t = 2y$

$-2 \le t \le 3$

$0 \le t^2 \le 9$

$-4 \le t^2 - 4 \le 5$

$-4 \le x \le 5$

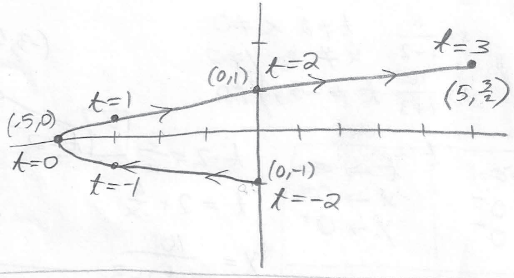
$x = 4y^2 - 4$

$x = 4(y^2 - 1)$

$x = 4(y-1)(y+1)$

$-2 \le t \le 3$

$-1 \le \frac{t}{2} \le \frac{3}{2}$ $\quad -1 \le y \le \frac{3}{2}$



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12. $x = \frac{1}{\sqrt{t+1}}$ $\quad t > -1$

$y = \frac{t}{t+1}$ $\quad x > 0$

$y < 1$

$t \to -1^+$ $x \to \infty$ $y \to -\infty$	$t \to \infty$ $x \to 0^+$ $y \to 1^-$

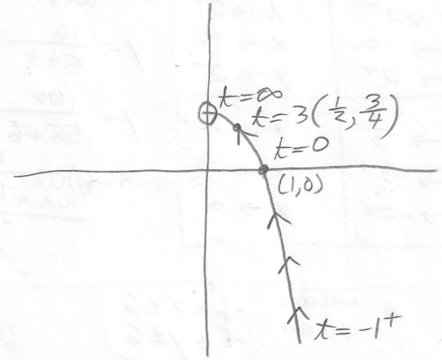
$\sqrt{t+1} = \frac{1}{x}$

$t + 1 = \frac{1}{x^2}$

$t = \frac{1}{x^2} - 1$

$y = \frac{\frac{1}{x^2} - 1}{\frac{1}{x^2}}$

$y = 1 - x^2$



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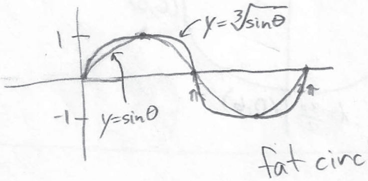
13. $x = 4 \cdot \sqrt[3]{\cos\theta}$

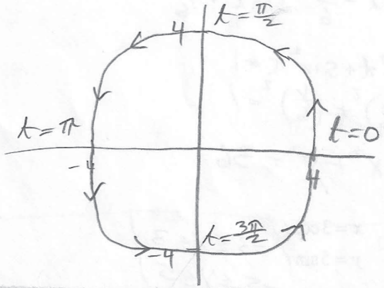
$y = 4 \cdot \sqrt[3]{\sin\theta}$

if $|a| < 1$

then $|\sqrt[3]{a}| > |a|$



fat circle!



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14. $x = 2^{-t}$ $\quad x > 0$

$y = 2^{3t} + 1$ $\quad y > 1$

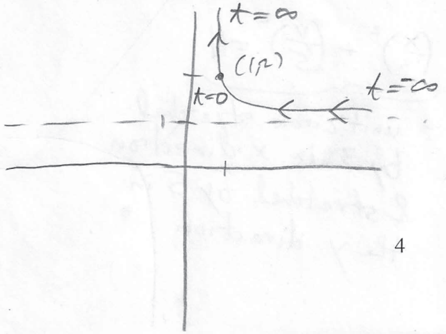
$t \to -\infty$ $x \to \infty$ $y \to 1^+$	$t \to \infty$ $x \to 0^+$ $y \to \infty$

$x = \frac{1}{2^t}$

$2^t = \frac{1}{x}$

$y = (2^t)^3 + 1$

$\boxed{y = \frac{1}{x^3} + 1}$



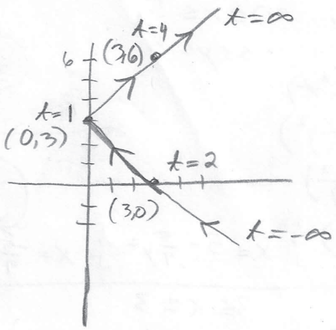
15. $x = |t-1|$
$y = t+2$

$t = y-2$

$x = |y-3|$

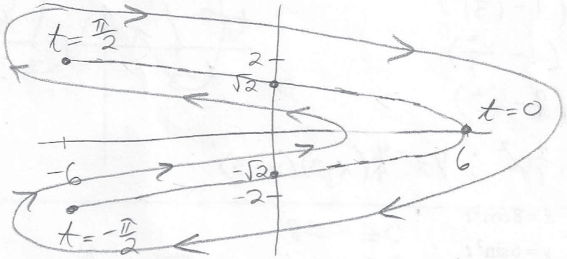
$x \geq 0$

$t \to -\infty$ $x \to \infty$ $y \to -\infty$	$t \to \infty$ $x \to \infty$ $y \to \infty$



16. $x = 6\cos(2t)$
$y = 2\sin t$

$x = 6(1 - 2\sin^2 t)$
$x = 6 - 12\sin^2 t$
$x = 6 - 12\left(\frac{y}{2}\right)^2$
$x = 6 - 12\left(\frac{y^2}{4}\right)$
$x = 6 - 3y^2$
$x = -3(y^2 - 2)$



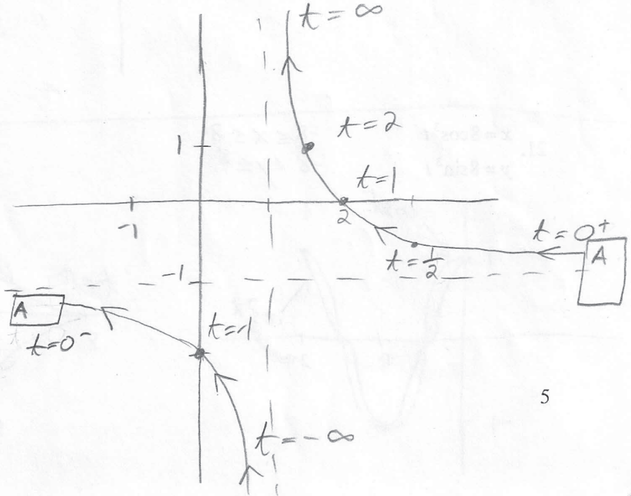
$-6 \leq x \leq 6$
$-2 \leq y \leq 2$

$t \neq 0$ $x \neq 1$ $y \neq -1$

17. $x = 1 + \frac{1}{t}$
$y = t-1$

$x = 1 + \frac{1}{t}$
$x - 1 = \frac{1}{t}$
$t = \frac{1}{x-1}$
$y = \frac{1}{x-1} - 1$

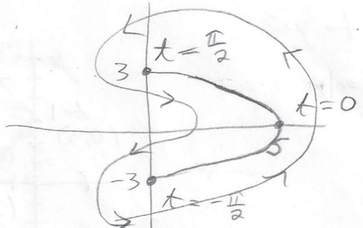
as $t \to 0^-$ $x \to -\infty$ $y \to -1$
as $t \to 0^+$ $x \to \infty$ $y \to -1$
as $t \to -\infty$ $x \to 1^-$ $y \to -\infty$
as $t \to \infty$ $x \to 1^+$ $y \to \infty$



18. $x = 5\cos^2 t$
$y = 3\sin t$

$0 \leq x \leq 5$
$-3 \leq y \leq 3$

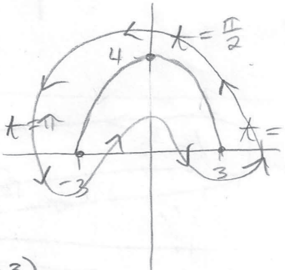
$x = 5(1 - \sin^2 t)$
$x = 5\left(1 - \left(\frac{y}{3}\right)^2\right)$
$x = 5\left(1 - \frac{y^2}{9}\right)$
$x = \frac{5}{9}(9 - y^2)$; $x = 5 - \frac{5}{9}y^2$; $x = \frac{-5}{9}(y-3)(y+3)$



19. $x = 3\cos t$
$y = 4\sin^2 t$

$-3 \leq x \leq 3$
$0 \leq y \leq 4$

$y = 4(1 - \cos^2 t)$
$y = 4\left(1 - \left(\frac{x}{3}\right)^2\right)$
$y = 4\left(1 - \frac{x^2}{9}\right)$
$y = \frac{4}{9}(9 - x^2)$
$y = 4 - \frac{4}{9}x^2$; $y = \frac{-4}{9}(x-3)(x+3)$

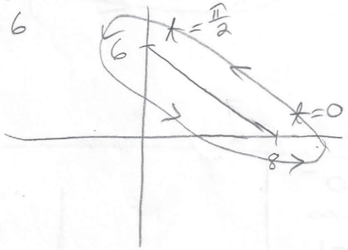


20. $x = 8\cos^2 t$
$y = 6\sin^2 t$

$0 \leq x \leq 8$
$0 \leq y \leq 6$

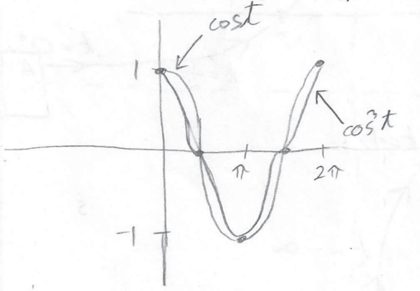
$\frac{x}{8} + \frac{y}{6} = 1$

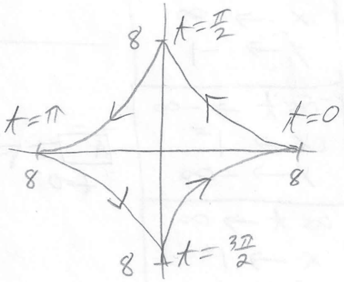
$y = 6 - \frac{3}{4}x$



21. $x = 8\cos^3 t$
$y = 8\sin^3 t$

$-8 \leq x \leq 8$
$-8 \leq y \leq 8$





~~More~~ Parametric Equations ~~Practice~~

#2

Directions: Convert each pair of parametric equations to a Cartesian equation by eliminating the parameter. Then graph the plane curve traced out by these equations, being careful to consider domain issues. On each graph label several points with " $t =$ " labels, and use arrows to indicate the direction in which the curve is traced out, as t goes from $-\infty$ to ∞.

1. $x = 4\cos t$
   $y = 3\sin^2 t$

$\boxed{-4 \le x \le 4 \quad 0 \le y \le 3}$

$y = 3(1-\cos^2 t)$

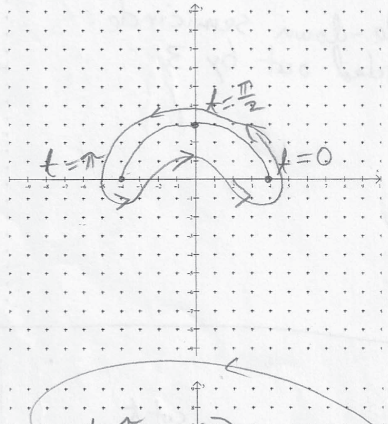
$y = 3\left(1-\left(\frac{x}{4}\right)^2\right)$

$= 3\left(1-\frac{x^2}{16}\right)$

$= -\frac{3}{16}(x^2-16)$

$y = -\frac{3}{16}(x-4)(x+4)$

$y = -\frac{3}{16}x^2 + 3$



2. $x = 4\cos t$
   $y = 5\sin\left(\frac{t}{2}\right)$

$-4 \le x \le 4$
$-5 \le y \le 5$

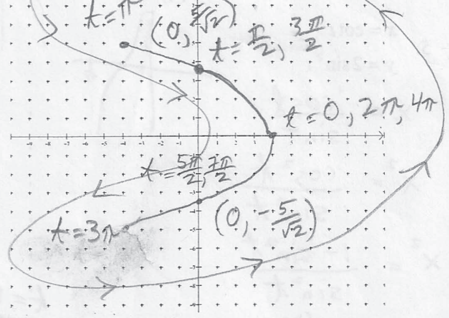
$x = 4\cos(2\cdot\frac{t}{2})$

$= 4(1-2\sin^2(\frac{t}{2}))$

$= 4(1-2[\frac{y^2}{25}])$

$= \frac{-8}{25}(y^2 - \frac{25}{2})$ $\quad x = \frac{-8}{25}y^2 + 4$

$x = \frac{-8}{25}(y - \frac{5}{\sqrt{2}})(y + \frac{5}{\sqrt{2}})$

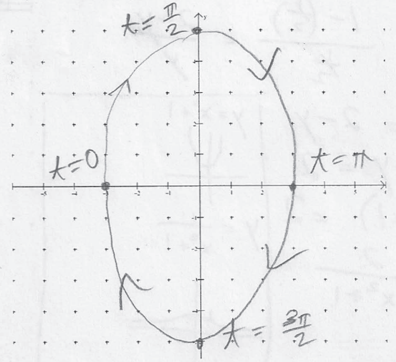


3. $x = -3\cos t$
   $y = 5\sin t$

$-3 \le x \le 3$
$-5 \le y \le 5$

$\cos t = \frac{-x}{3} \quad \sin t = \frac{y}{5}$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$



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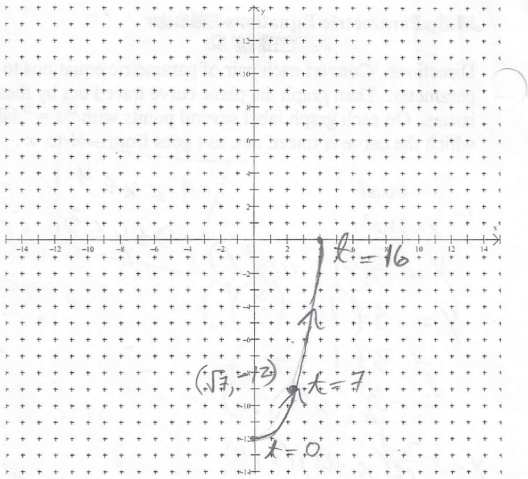
4. $x = \sqrt{t}$
   $y = -3\sqrt{16-t}$

$t \ge 0, \; x \ge 0$
$t \le 16, \; y \le 0$

$t = x^2$

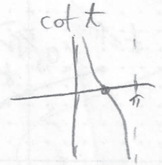
$y = -3\sqrt{16-x^2}$

upside-down semicircle
stretched out by 3



as $t \to 0^+$ $x \to \infty$ $y \to 0^+$	as $t \to \pi^-$ $x \to -\infty$ $y \to 0^+$

5. $x = \cot t$
   $y = 2\sin^2 t$



$x = \frac{\cos t}{\sin t}$

$x^2 = \frac{\cos^2 t}{\sin^2 t}$

$x^2 = \frac{1-\sin^2 t}{\sin^2 t}$

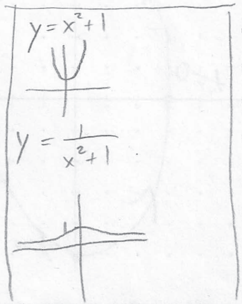
$x^2 = \frac{1-\left(\frac{y}{2}\right)}{\frac{y}{2}} = \frac{2-y}{y}$

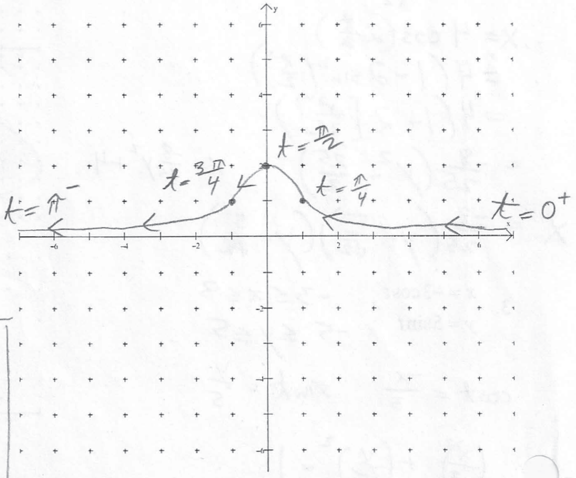
$yx^2 = 2-y$

$yx^2 + y = 2$

$y(x^2+1) = 2$

$y = \frac{2}{x^2+1}$





5. $x = e^{-t+3}$ $x > 0$

$y = t$

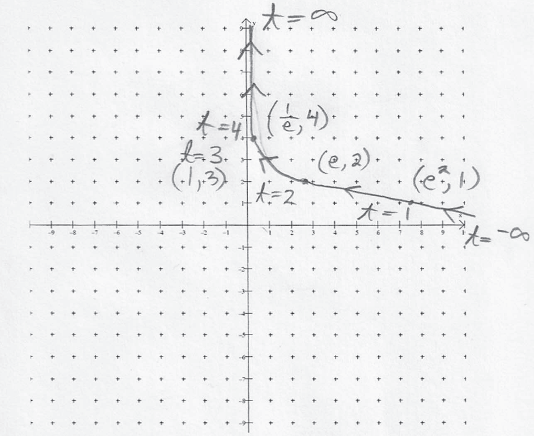
$t \to -\infty$ $x \to \infty$ $y \to -\infty$	$t \to \infty$ $x \to 0^+$ $y \to \infty$

$x = e^{-y+3}$

$\ln x = -y + 3$

$\ln x - 3 = -y$

$y = 3 - \ln x$



6. $x = -e^t$ $x < 0$

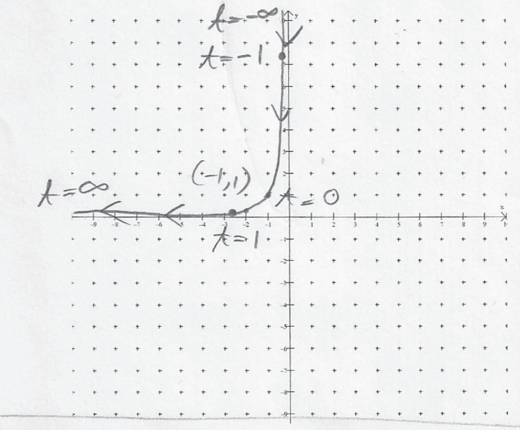
$y = e^{-2t}$ $y > 0$

$y = \frac{1}{e^{2t}}$

$t \to -\infty$ $x \to 0^-$ $y \to \infty$	$t \to \infty$ $x \to -\infty$ $y \to 0^+$

$y = \frac{1}{(e^t)^2} = \frac{1}{(-x)^2} = \frac{1}{x^2}$

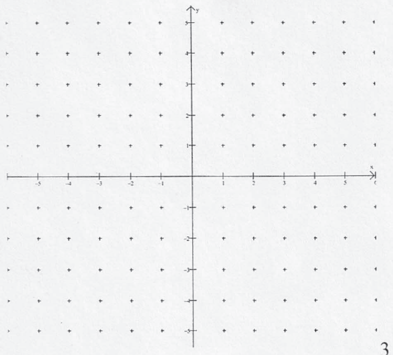
$y = \frac{1}{x^2}$



7. $x = -3\cos t$

$y = 5\sin t$

See Page 1



$t-3 = t-5$
$-3 = -5$

8. $x = \frac{t-3}{t-5}$ $t \neq 5$

$y = t$ $y \neq 5$

$t \to -\infty$ $x \to 1^-$ $y \to -\infty$	$t \to \infty$ $x \to 1^+$ $y \to \infty$
$t \to 5^-$ $x \to -\infty$ $y \to 5^-$	$t \to 5^+$ $x \to \infty$ $y \to 5^+$

$x = \frac{y-3}{y-5}$

$x(y-5) = y-3$

$xy - 5x = y - 3$

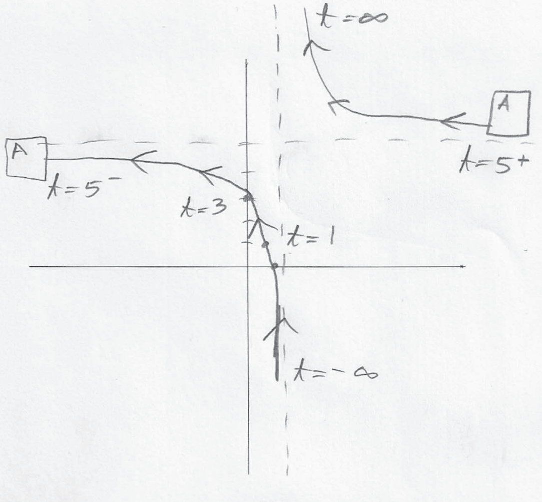
$xy - y = 5x - 3$

$y(x-1) = 5x - 3$

$y = \frac{5x-3}{x-1}$

$$\begin{array}{r} 5 \\ x-1 \overline{)\,5x-3} \\ \underline{5x-5} \\ 2 \end{array}$$

$\boxed{y = 5 + \frac{2}{x-1}}$



9. $x = \frac{-3}{t-2}$ $t \neq 2,\ x \neq 0$

$x \neq -1$

$y = \frac{6}{t-5}$ $t \neq 5,\ y \neq 0,\ y \neq -2$

$t \to -\infty$ $x \to 0^+$ $y \to 0^-$	$t \to 2^-$ $x \to \infty$ $y \to -2^+$	$t \to 5^-$ $x \to -1^-$ $y \to -\infty$
$t \to \infty$ $x \to 0^-$ $y \to 0^+$	$t \to 2^+$ $x \to -\infty$ $y \to -2^-$	$t \to 5^+$ $x \to -1^+$ $y \to \infty$

$x = \frac{-3}{t-2}$

$t - 2 = \frac{-3}{x}$

$t = 2 - \frac{3}{x}$

$y = \frac{6}{2 - \frac{3}{x} - 5} = \frac{6}{-3 - \frac{3}{x}} = \frac{6}{\frac{-3x-3}{x}}$

$y = \frac{6x}{-3x-3} = \frac{-2x}{x+1}$

$$\begin{array}{r} -2 \\ x+1 \overline{)\,-2x} \\ \underline{-2x-2} \\ 2 \end{array}$$

$\boxed{y = -2 + \frac{2}{x+1}}$