Developing Mathematical Thinkers

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DISCUSSION

- Write 3-5 words or phrases that describe the kind of mathematical thinking you want students to be able to do.

From a presentation by Cathy Seeley, 2014

In this session...

- Thinking like a mathematician
- Nurturing talent
- Mathematical Habits of Instruction . . .

From a presentation by Cathy Seeley, 2014

Being a 'Doer' of Mathematics

"The only way to know mathematics
is to ***do*** mathematics."

Paul Halmos, mathematician

"It is pretty hard to ***understand*** mathematics
without ***doing*** some mathematics."

Jordan Ellenberg, mathematician and writer

From a presentation by Cathy Seeley, 2014

Thinking Like a Mathematician

- Doing real mathematics--figuring out hard problems
- Exploration that's sometimes messy;
  trying things that sometimes work, sometimes don't
- Reasoning, explaining and arguing
- Zooming in and zooming out
- Reflecting, considering, analyzing

From a presentation by Cathy Seeley, 2014

What math do all students need?

- The Big Three:
  - **Understand mathematics** (make sense of it)
  - **Do the arithmetic** (skills, facts, procedures)
  - **Use mathematics** (thinking, reasoning, applying, solving a range of problems)
- The New Basics: deep transferable skills for *versatilizing*:
  - Problem solving, reasoning, research, communication, creativity

From a presentation by Cathy Seeley, 2014

Mathematical Thinking

Overarching goal:

Students who can think mathematically . . .

From a presentation by Cathy Seeley, 2014

Premise:

What all students need for their future is as much about how they ***think*** as about what they ***know*** . . .

and helping every student succeed is as much about ***how*** we teach as about ***what*** we teach.

From a presentation by Cathy Seeley, 2014

Math Reasoning Inventory™

mathreasoninginventory.com

Marilyn Burns, PI

Funded by Gates Foundation

https://mathreasoninginventory.com/Home/AssessmentsOverview

DISCUSSION

- How did the teacher find out what Marisa was thinking?
- Had Marisa likely had experience developing mathematical habits of mind?

From a presentation by Cathy Seeley, 2014

The difference between...

- Learning clues, keys, and tricks vs. constructively struggling with good problems
- Learning how to do mathematical procedures vs. learning mathematical habits of mind

From a presentation by Cathy Seeley, 2014

Mathematical Habits of Mind

- **Performing thought experiments**
- **Finding, articulating, and explaining patterns**
- Generalizing from examples; articulating generality in precise language
- Creating and using representations
- **Expecting mathematics to make sense**

From a presentation by Cathy Seeley, 2014

Al Cuoco, E. Paul Goldenberg, June Mark.
"Organizing a Curriculum around Mathematical Habits of Mind."
Mathematics Teacher May 2010

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly **and quantitatively**.
3. **Construct viable arguments; critique others' reasoning.**
4. **Model with mathematics.**
5. Use appropriate tools **strategically**.
6. **Attend to precision.**
7. Look for and make use of **structure**.
8. Look for and make use of **regularity** in reasoning.

From a presentation by Cathy Seeley, 2014

Virginia's Mathematical Processes

1. Mathematical Problem Solving

2. Mathematical Reasoning

3. Mathematical Communication

4. Mathematical Connections

5. Mathematical Representations

Virginia Standards of Learning, Introduction and Goals, 2009

Problem Solving, Applications, Mathematical Modeling

- **Persevere in solving problems**
- Explain to selves; analyze givens, constraints; consider analogous problems; represent the situation in different ways; search for regularity / trends; plan solution path
- **Solve problems that apply the mathematics being learned**
- **Solve problems that may involve mathematics not yet learned**
- Use mathematical modeling to solve problems that may not be well defined; Interpret, reflect, fine-tune the model
- When solving problems, keep an eye on the big picture while also attending to the details (zooming in and out); shift perspective
- **Evaluate the reasonableness of results.**

From a presentation by Cathy Seeley, 2014

Reasoning, Explaining, Justifying

- **Includes quantitative reasoning**; make sense of quantities and relationships in problems
- Make conjectures and explore the truth of conjectures.
- Analyze situations; recognize and use counterexamples.
- **Justify conclusions and communicate/respond to arguments.**
- **Decide whether arguments of others make sense; ask useful questions to clarify or improve the arguments.**

From a presentation by Cathy Seeley, 2014

Representing, Connecting, Communicating Precisely

- Look for commonalities/relationships in similar problems or in mathematical ideas

- Coherently represent a problem; decontextualize and contextualize

- **Communicate precisely; make clear, effective arguments; use clear definitions**

- **Put precision in perspective (finding balance . . .)**

- **Does this make sense?**

From a presentation by Cathy Seeley, 2014

Tools

- Pencil/paper, manipulatives/concrete models, compass, protractor, calculator, spreadsheet, CAS, software, . . .
- Find and use external information (Internet, research, etc.)
- **Mental math**
- **Make decisions and understand limitations**

From a presentation by Cathy Seeley, 2014

Mathematical Habits of Mind--Common Themes

- Solving problems
- Thinking, Reasoning, and Reflecting
- Discussing and Communicating
- Justifying and Explaining
- Generalizing from Patterns
- Connecting
- Making sense
- Patience, persistence

From a presentation by Cathy Seeley, 2014

DISCUSSION: Potential

- How smart do you think Marisa is?
- Why don't some students reach their mathematical potential?

From a presentation by Cathy Seeley, 2014

Factors to consider

- Student factors: Motivation, intelligence, beliefs
- Teacher factors: Knowledge, expectations, beliefs
- Instructional factors
  - Nature of the task
  - Opportunities to struggle, think, figure things out, discuss
  - A classroom environment of trust, collaboration, respect, and (eventual) success, where perseverance and constructive feedback of each other are valued

From a presentation by Cathy Seeley, 2014

Intelligence

- Fixed vs. growth mindset . . .
  (Carol Dweck, *Mindset*, 1999)

- Your mindset influences confidence, perseverance, and your willingness to take risks

- From brain research:
  The activities a person engages in can change their intelligence.

- Who determines the activities a student engages in?

From a presentation by Cathy Seeley, 2014

Targeting beliefs with action

- Students' beliefs matter.
- Teachers' beliefs and actions matter.
- Modest interventions make a difference.

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High Expectations means...

- Challenging our habits and beliefs
- Setting challenging standards for all students
- Doing whatever it takes for students to achieve the standards
- Never thinking in advance that you know where they're headed or what they need
- Making sure they all get to struggle and succeed

From a presentation by Cathy Seeley, 2014

Answer-getting vs. learning mathematics

- USA:
  How can I teach my kids to get the answer to this problem?

- Japanese:
  How can I use this problem to teach the mathematics of this unit?

  – Devised methods for slowing down, postponing answer-getting

Phil Daro, 2012

From a presentation by Cathy Seeley, 2014

The difference between Japan and the US

- "You quit teaching too soon and go on to the next thing."
- "We finish."
- Finishing happens when students have learned.
- And learning is incomplete if students aren't developing mathematical thinking.

From a presentation by Cathy Seeley, 2014, Thanks to Phil Daro

Marisa didn't get to finish...

From a presentation by Cathy Seeley, 2014

Upside-down teaching

- From: *"I - We - You"*
- To: *"You - We - I"*
- *Or: "You - Y'all - We - I"*

Thanks to Phil Daro, Deborah Ball, Magdalene Lampert, and Cathy Seeley

From a presentation by Cathy Seeley, 2014

Upside-down teaching

- Start with a rich problem
- Engage students in dealing with the problem, constructively struggling with the problem and the mathematics
- Students discuss, compare, interact, question
- Teacher helps students connect and notice what they've learned

From a presentation by Cathy Seeley, 2014

Mathematical Habits of Instruction

- Use a problem-centered, upside-down teaching model
- Use appropriate technology appropriately
- Learn (and help students learn) to zoom out, zoom in, and go back and forth
- Help students learn to notice and use patterns, connections, and properties within and across mathematical topics and problems (mathematical structure)
- Use formative assessment to pay attention to learning

From a presentation by Cathy Seeley, 2014

Achievement Gap

From a presentation by Cathy Seeley, 2014

Untapped Potential

From a presentation by Cathy Seeley, 2014

What if we raise the floor AND the ceiling?

From a presentation by Cathy Seeley, 2014

Two Sides of Untapped Potential

- Bringing up all students to achieve their highest levels of mathematics and science--raising the floor
- Identifying the stars
- Raising the ceiling and letting them soar
- Untapped potential within each student, within groups of students, and at the school, district, state and national level--potential we haven't reached . . . YET.

From a presentation by Cathy Seeley, 2014

Untapped Potential

From a presentation by Cathy Seeley, 2014

Unlimited Potential

From a presentation by Cathy Seeley, 2014

Even our *best* students...

...will benefit from a strong, diverse, engaging, relevant classroom.

From a presentation by Cathy Seeley, 2014

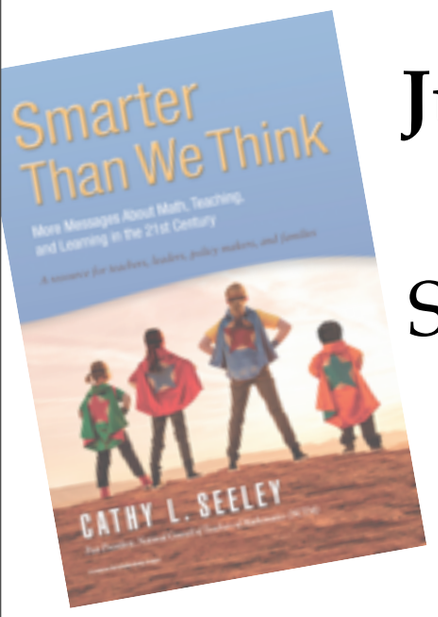
Their future is in our hands

...and ours is in theirs

From a presentation by Cathy Seeley, 2014

E-mail for a copy of the slides: cseeley@utexas.edu

(also uploaded with a handout on NCTM site)



Just published April 2014: *Smarter Than We Think*

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Smarter Than We Think*, Upside-Down Teaching*, Clueless, Mathematical Habits of Mind*, Mathematical Habits of Instruction

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Special conference sale price on *Smarter Than We Think*, Math Solutions

Faster Isn't Smarter--Messages About Math, Teaching, and Learning in the 21st Century, Seeley 2009

http://mathsolutions.com/fasterisntsmarter *(Download 5 messages)*

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