

# What is a graph, really?

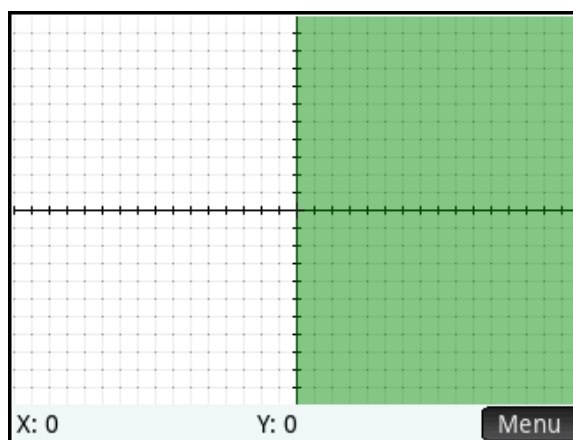
In this short Burst session, we will look at examples of graphs that we typically do not see or use in secondary mathematics education. We will include examples from both the middle and high school curricula. These graphs are all created using the Advanced Graphing App of the HP Prime graphing calculator.

## Example 1: $\sqrt{X^2} = X$

In this example, we use the Advanced Graphing app to visualize common student errors. A graphic representation of the statement  $\sqrt{X^2} = X$  helps students remember that it is only true for non-negative values of X.

Other common errors can be corrected in a similar manner, including

$\sqrt{X^2 + Y^2} = X + Y$  (which is sometimes true).



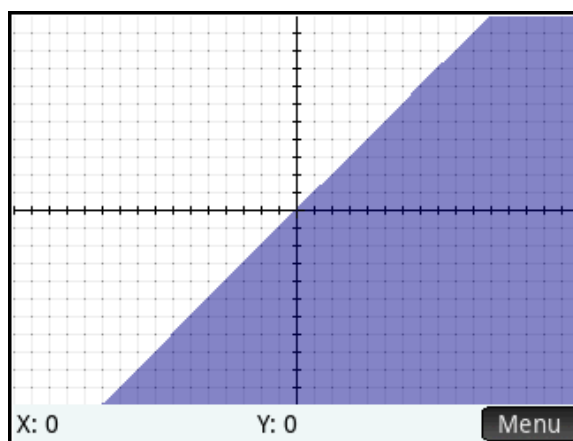
## Example 2: The Trichotomy at Zero

In this example, we help students remember the rules surrounding the multiplication of signed numbers in a fun and visual way.

$Y-X=0$  divides the plane into 3 regions:

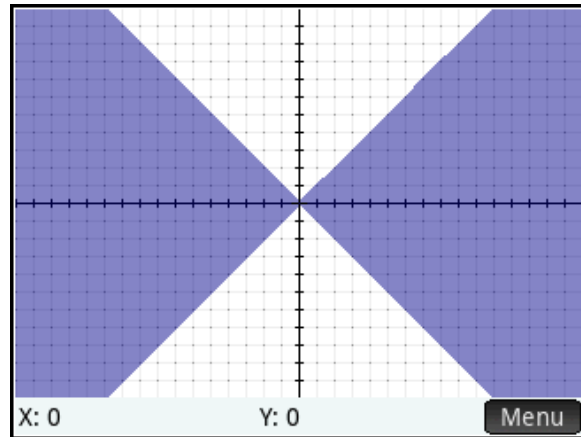
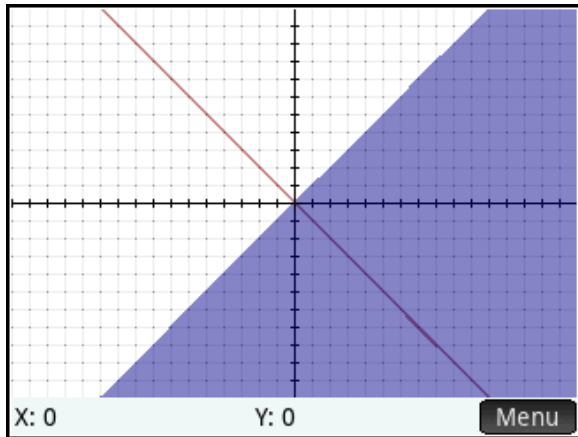
- The line where  $Y-X=0$
- The half-plane where  $Y-X>0$
- The half-plane where  $Y-X<0$

The graph of  $Y-X<0$  is shown to the right. What happens to the graph when we change the statement to  $(Y+X)*(Y-X)<0$ ?

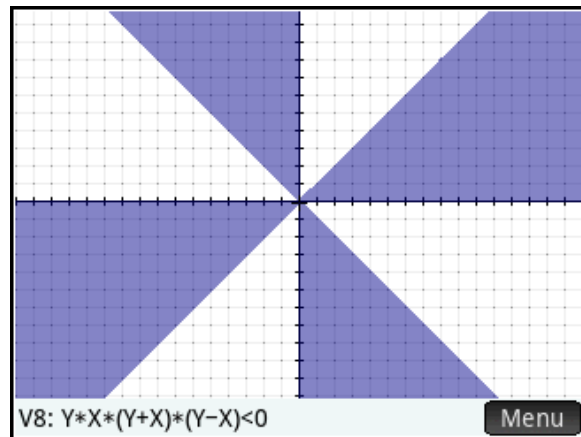
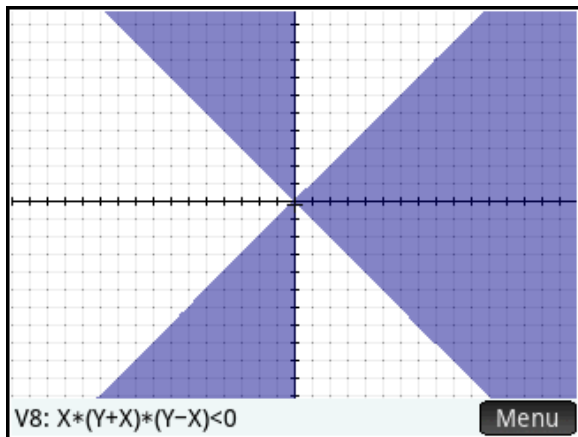


Whenever  $Y+X$  is negative, it will change the sign of the left side of the original statement; wherever  $Y+X$  is positive, it will not change the sign of the left side of the original. So we need only look at the region(s) where  $Y+X < 0$  and switch the "on" parts "off" and the "off" parts "on". Throughout this example, we will use this notion of multiplication by a negative as switching the sign of an expression to predict the appearance of a graph.

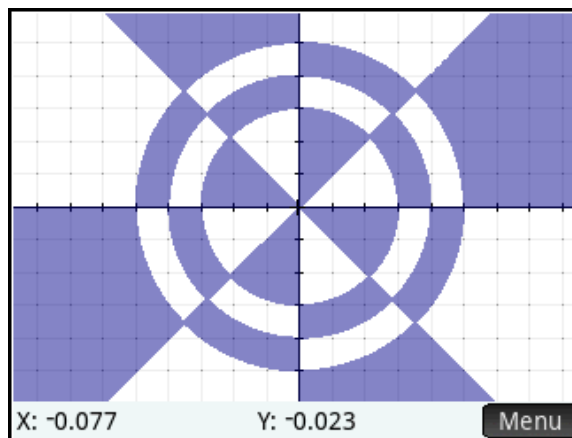
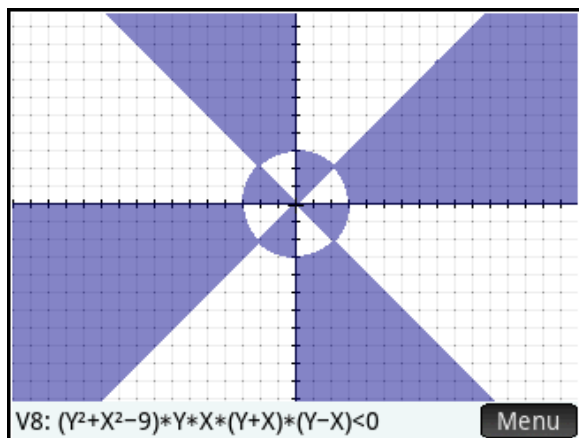
The line  $Y+X=0$  is superimposed on our original graph in the figure to the left below. The region where  $Y+X < 0$  is below that line. The sub-region below that line that is currently blue (true) will become white (false) and vice versa. The graph of  $(Y+X)*(Y-X) < 0$  is shown to the right below.



We can continue to add factors and predict the appearance of the graphs of  $X*(Y+X)*(Y-X) < 0$  and  $X*Y*(Y+X)*(Y-X) < 0$  (both shown below), based on similar reasoning about the graphs of  $X < 0$  and  $Y < 0$ .

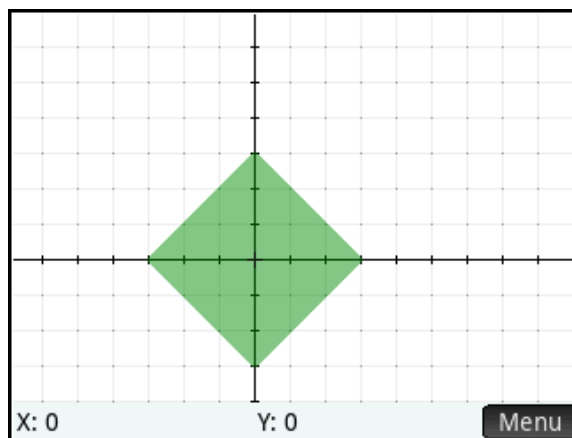
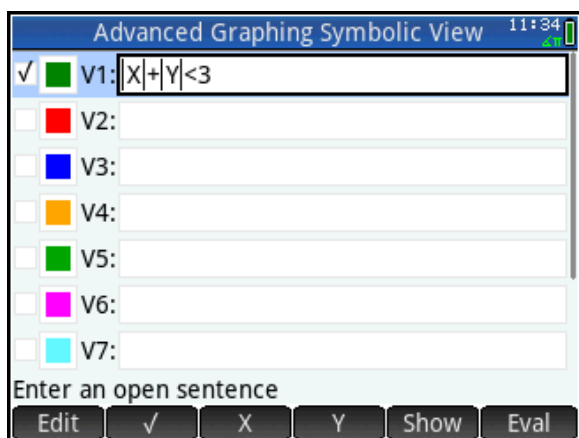


Similarly to  $Y-X=0$ ,  $Y^2+X^2-9=0$  separates the plane into 3 regions. Think about the effect of adding  $(Y^2+X^2-9)$  as a factor to the left side of our statement. Where is the region where  $(Y^2+X^2-9)<0$ ? What statement's graph is shown to the right below?



### Example 3: Argyle designs

In this example, we use the rules for transforming geometric figures to transform graphical representations of algebraic objects. In the Advanced Graphing app, we define V1 to be  $|X|+|Y|<3$ , as shown below. The graph is a diamond shape. We can now use this basic shape to create designs.



How do we define V2 to translate the graph of V1 horizontally or vertically (or both)?

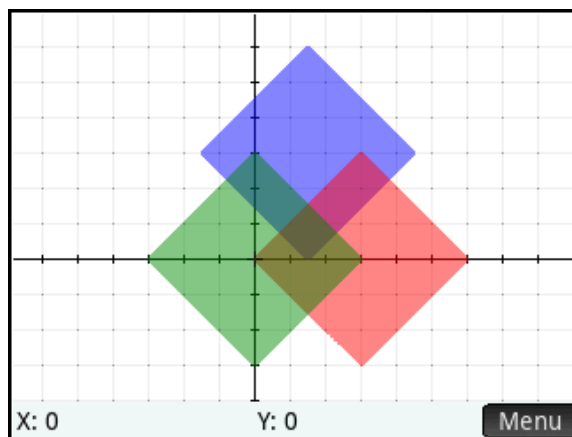
In the figure to the left below, both V2 and V3 have been defined as translations of V1, each with a different color. The graphs are shown to the right below.

Advanced Graphing Symbolic View 10:54

- V1:  $|x|+|y|<3$
- V2:  $V1(x-3,y)$
- V3:  $V1(x-1.5,y-3)$
- V4:
- V5:
- V6:
- V7:

Enter an open sentence

Edit ✓ X Y Show Eval



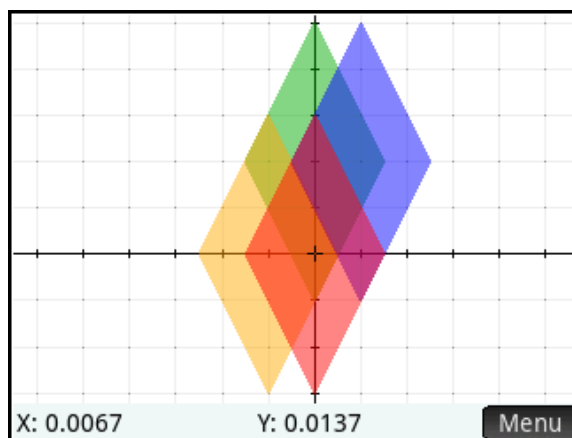
The original figure can be dilated as well. In the figure to the left below, V1 is not checked so it will not be graphed; however, V2-V5 are dilations defined in terms of V1 (some directly, some indirectly). The graphs are shown in the figure to the right below.

Advanced Graphing Symbolic View 12:01

- V1:  $|x|+|y|<3$
- V2:  $V1(2*x,y)$
- V3:  $V2(x-1,y-2)$
- V4:  $V2(x+1,y)$
- V5:  $V2(x,y-2)$
- V6:
- V7:

Enter an open sentence

Edit ✓ X Y Show Eval



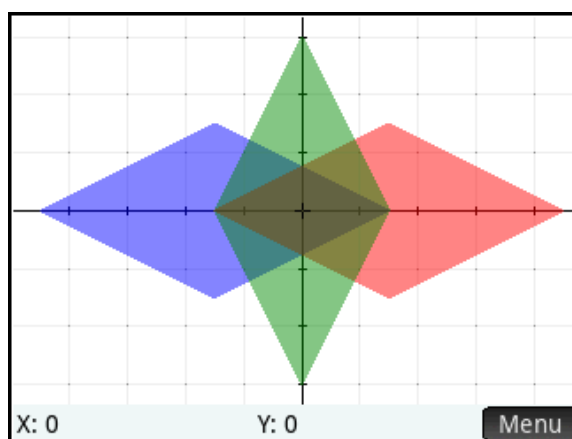
Finally, the original figure can be reflected horizontally, vertically, or inverted through  $y=x$ .

Advanced Graphing Symbolic View 11:48

- V1:  $2*|x|+|y|<3$
- V2:  $V1(y,x-1.5)$
- V3:  $V2(-x,y)$
- V4:
- V5:
- V6:
- V7:

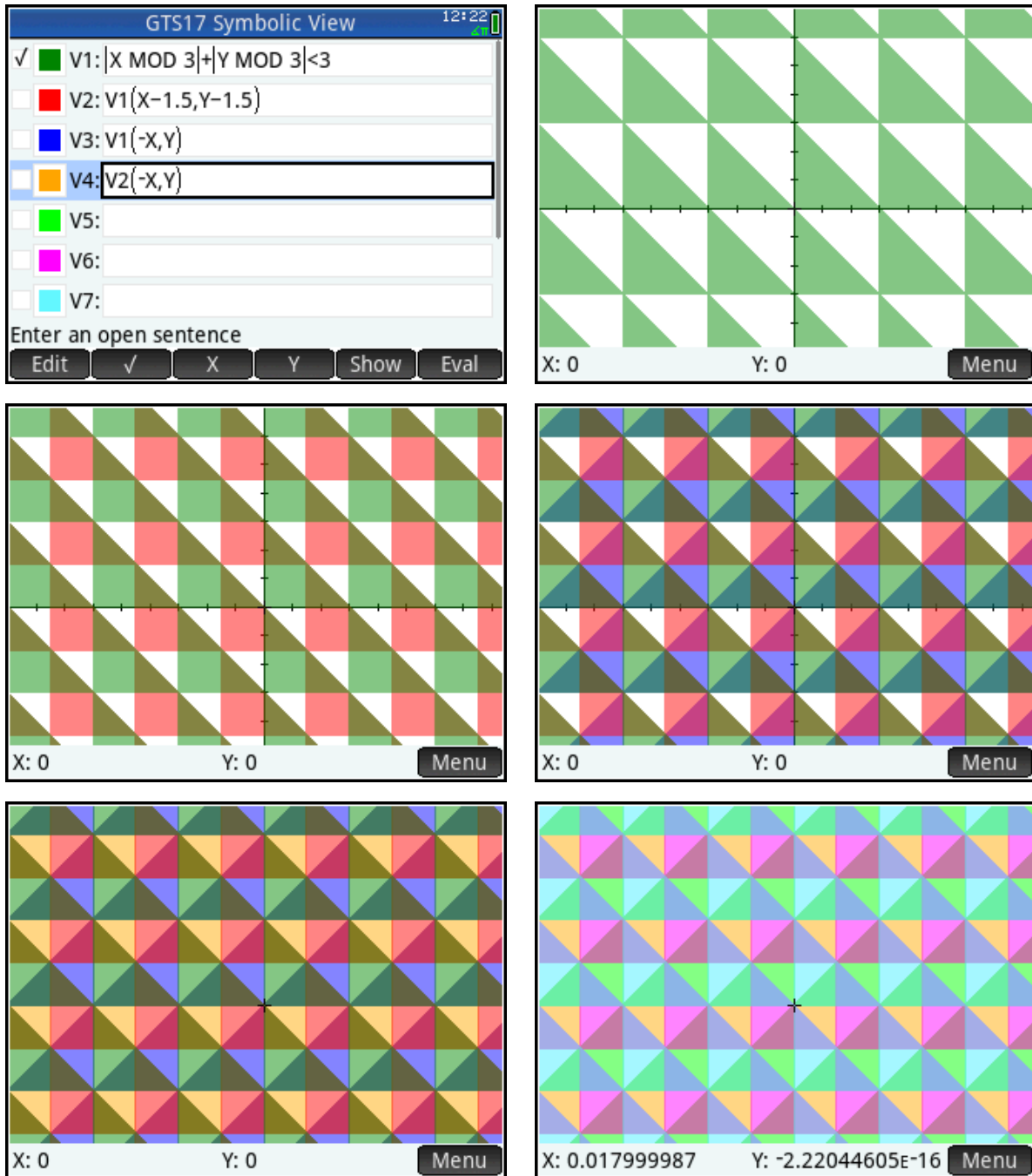
Enter an open sentence

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### Example 4: It's a MOD, MOD, MOD, MOD world

In this example, we extend the previous example by replacing  $|X|$  with  $|X \text{ MOD } 3|$  and  $|Y|$  with  $|Y \text{ MOD } 3|$ . The first figure below shows the Symbolic view, with all the statements defined. The rest of the figures show each of the 4 statements activated in turn.



In this short Burst session, we have presented a number of examples which expand the power of visualization in pedagogically useful ways.