Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students

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NCTM
What Works Clearinghouse

Overview

- The What Works Clearinghouse (WWC)
- Algebra practice guide
  - Scope
  - Motivation
  - Recommendations, examples, and solutions to common roadblocks
- Staying connected to the WWC
- Q & A

The WWC mission and approach

- Educators are encouraged to use **scientifically-based** research (e.g., IDEA, NCLB)
- Identifying and sifting through education research takes time and often requires specialized skills
- To facilitate research-based decision making, the WWC **summarizes the existing evidence** on a topic in accessible products
  - WWC provides **impartial summaries**
  - WWC uses **rigorous and transparent standards** to assess evidence and only summarizes high quality studies
- **ALL WWC RESOURCES AND PRODUCTS ARE FREE**

WWC: Who, What, and Why

Who we are:

- An initiative of the **Institute of Education Sciences (IES)** at the U.S. Department of Education.

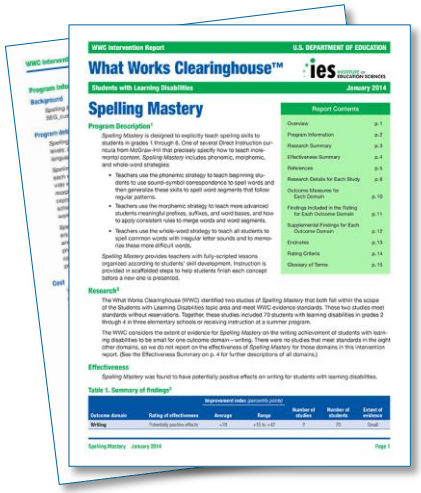
What we do:

- **Evaluate and summarize the research** on educational interventions

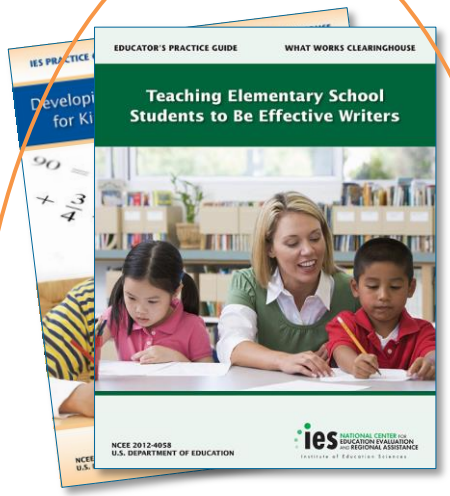
Why we do it:

- Provide educators with the information to **make research-based decisions**.

WWC Products



Intervention reports summarize all the rigorous evidence on a specific intervention to support evidence-based decision making.



Practice guides help educators address classroom challenges by offering concrete suggestions and summarizing the evidence associated with the suggested practices.

Single study reviews assess the quality and summarize the research of a single study that is of interest to the education community.

Quick reviews are the WWC's initial, and immediate, assessment of a single publication that is receiving media attention.

WWC Practice Guides

- Practice guides are:
  - Geared toward helping **educators** and **administrators** address challenges in **classrooms** and **schools**
  - Guided by an **expert panel**
  - Based on **rigorous evidence**
- The WWC has produced 19 practice guides.

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Home > Topics > English Language Learners

Teaching Academic Content and Literacy to English Learners in Elementary and Middle School

Summary

This practice guide provides four recommendations that address what works for English learners during reading and content area instruction. Each recommendation includes extensive examples of activities that can be used to support students as they build the language and literacy skills needed to be successful in school, including examples of how the recommendations align with Common Core and other contemporary state standards. The recommendations also summarize and rate supporting evidence. This guide is geared toward teachers, administrators, and other educators who want to improve instruction in academic content and literacy for English learners in elementary and middle school.

Practice Guide and Related Resources

The practice guide is also available in e-book format. Materials from the webinar with the practice guide panelists are also available. Click below to access any of the available resources.

MOBI (2.8 MB) Download this e-book format to view the practice guide on a smartphone or Kindle.

Accessing E-Books (8.49 KB)

Recommendations

Recommendation	Level of Evidence
1. Teach a set of academic vocabulary words intensively across several days using a variety of instructional activities. Source – (6.8 MB)	Strong
2. Integrate oral and written English language instruction into content-area teaching. Source – (6.8 MB)	Strong
3. Provide regular, structured opportunities to develop written language skills.	Minimal

Practice Guide Details

Released: April 2014
Topic: English Language Learners, Literacy
Education Level: Elementary, Middle Grades
Audience: Administrator, Policymaker, Researcher, School Specialist, Teacher
Related Resources: Teaching Academic Content and Literacy to English Learners in Elementary and Middle School Review Protocol

Panel

Scott Baker (Chair)
Center on Research and Evaluation, Southern Methodist University

Esther Geva
University of Toronto

Michael J. Kieffer
New York University

Nonie Lesaux
Harvard University

Sylvia Linan-Thompson
University of Texas at Austin

Joan Morris
Pasadena Unified School District

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Randi Russell
Miami-Dade Public Schools

Handy Definitions

level of evidence rating
minimal evidence

What do practice guides offer?

- **Evidence-based instructional recommendations** that can be used with existing standards or curricula.
  - **Practical action steps** provide specific guidance for implementation
- **Examples** for implementing the recommendations.
- **Solutions for overcoming challenges** to implementing the recommendations**.**

Algebra Practice Guide Overview

- Recommendations are appropriate for **students in grades 6–12** and in **diverse contexts**.
- A **comprehensive search** of the literature between 1993 and 2013 identified 2,800 citations related to algebra instruction.
- Relevant studies were reviewed using **rigorous WWC design standards** (15 studies met standards)
- Review protocol available at www.whatworks.ed.gov.

Panel Member	Affiliation
Jon Star (chair)	Harvard University
Anne Foegen	Iowa State University
Matt Larson	Lincoln Public Schools
William McCallum	University of Arizona
Jane Porath	Traverse City Area Public Schools
Rose Mary Zbiek	Pennsylvania State University

Recommendations

Recommendation	Strong Evidence	Moderate Evidence	Minimal Evidence
1. Use solved problems to engage students in analyzing algebraic reasoning and strategies.			◆
2. Teach students to utilize the structure of algebraic representations.			◆
3. Teach students to intentionally choose from alternative algebraic strategies when solving problems.		◆	

Recommendation 1: Use solved problems to engage students in analyzing algebraic reasoning and strategies.

- **Action Step 1.** Have students discuss solved problem structures and solutions to make connections among strategies and reasoning.

Example 1.2. Questions to facilitate discussion of the structure of problems

- What quantities—including numbers and variables—are present in this problem?
- Are these quantities discrete or continuous?
- What operations and relationships among quantities does the problem involve? Are there multiplicative or additive relationships? Does the problem include equality or inequality?
- How are parentheses used in the problem to indicate the problem's structure?

Solved problem: An example that shows both the problem and the steps used to reach a solution to the problem. A solved problem can be pulled from student work or curricular materials, or it can be generated by the teacher. A solved problem is also referred to as a "worked example."

Sample solved problem:

Solve for x in this equation: $3^{4x+3} = 81$	$3^{4x+3} = 81$ $3^{4x+3} = 3^4$ $4x + 3 = 4$ $4x = 1$ $x = \frac{1}{4}$

Recommendation 1: Use solved problems to engage students in analyzing algebraic reasoning and strategies.

- **Action Step 2.** Select solved problems that reflect the lesson's instructional aim, including problems that illustrate common errors.

Factor: $x^2 - 10x - 24$		
Correct solved problem	**Incorrect solved problem: Sum of the integers does not equal the middle term**	**Incorrect solved problem: Sum of the integers does not equal the middle term**
Student A factored this expression correctly: $x^2 - 10x - 24$ $(x - 12)(x + 2)$	Student B did *not* factor this expression correctly: $x^2 - 10x - 24$ $(x - 4)(x + 6)$	Student C did *not* factor this expression correctly: $x^2 - 10x - 24$ $(x + 12)(x - 2)$

Sample questions to guide discussion of the errors:

1. How can you show that the answers from students B and C are incorrect?
2. What advice would you give to students B and C to help them avoid factoring this type of problem incorrectly in the future?
3. How can you check that student A factored this expression correctly?
4. What strategy would you use to factor this expression and why did you choose that strategy?

Recommendation 1: Use solved problems to engage students in analyzing algebraic reasoning and strategies.

- **Action Step 3.** Use whole-class discussions, small-group work, and independent practice activities to introduce, elaborate on, and practice working with solved problems.

Example 1.10. Incomplete solved problems

Include incomplete solved problems in students' independent practice, asking students to fill in the blank steps of the solved problems.

$-x + 7 \geq 9$ $-x \geq 2$ _____	$3(x + 2) + 12 \leq 4(1 - x)$ _____ $3x + 18 \leq 4 - 4x$ $7x \leq -14$ $x < -2$	$2(x + 7) - 5(3 - 2x) \geq 7x - 4$ $2x + 14 - 15 + 10x \geq 7x - 4$ _____ $5x \geq -3$ $x \geq -\frac{3}{5}$

Recommendation 1: Potential roadblock

I already use solved problems during whole-class instruction, but I'm not sure students are fully engaged with them.

Suggested Approach. Whole-class discussion and analysis of solved problems can help guide students to notice aspects of the solved problems that are important. Asking questions and fostering discussion encourages students to think critically about solved problems. During whole-class instruction, model engaging with solved problems by using **think-aloud questions**. See Example 1.1 for examples of questions that can foster discussion and analysis, and see Example 1.2 for important aspects of problems to highlight.

Recommendation 2: Teach students to utilize the structure of algebraic representations.

- **Action Step 1.** Promote the use of language that reflects mathematical structure.

Example 2.1. Seeing structure in algebraic representations

Consider these three equations:

$$2x + 8 = 14$$

$$2(x + 1) + 8 = 14$$

$$2(3x + 4) + 8 = 14$$

Though the equations appear to differ, they have similar structures: in all three equations,
2
multiplied by a quantity,
plus 8,
equals 14.

Imprecise language	Precise language
Take out the *x*.	Factor *x* from the expression. Divide both sides of the equation by *x*, with a caution about the possibility of dividing by 0.
Move the 5 over.	Subtract 5 from both sides of the equation.
Use the rainbow method. Use FOIL.	Use the distributive property.
Plug in the 2.	Substitute 2 for x.
Do the opposite to each side.	Use inverse operations. Add the opposite to each side.

Recommendation 2: Teach students to utilize the structure of algebraic representations.

- **Action Step 2.** Encourage students to use reflective questioning to notice structure as they solve problems.

Example 2.5. Reflective questions for noticing structure

- What am I being asked to do in this problem?
- How would I describe this problem using precise mathematical language?
- Is this problem structured similarly to another problem I've seen before?
- How many variables are there?
- What am I trying to solve for?
- What are the relationships between the quantities in this expression or equation?
- How will the placement of the quantities and the operations impact what I do first?

Recommendation 2: Teach students to utilize the structure of algebraic representations.

- **Action Step 3.** Teach students that different algebraic representations can convey different information about an algebra problem.

Example 2.6. Equations of the same line in different forms

Compare different forms of equations for the same line.

	Similarities	Differences
Slope-intercept form $y = mx + b$ $y = 2x - 3$	Both are equations of straight lines It is easy to see that the slope is 2. It is hard to see what the x-intercept is.	**Slope-intercept form** makes it easy to see what the y-intercept is.
Point-slope form $y - y_1 = m(x - x_1)$ $y - 5 = 2(x - 4)$		**Point-slope form** makes it easy to see that the point (4, 5) is on the line.

Recommendation 2: Potential Roadblock

Diagrams don't seem to be very useful to some of my students.

Suggested Approach. Some students will find the correct answer to a problem without the need for a diagram. However, sometimes a student can work toward the right answer without noticing or attending to the problem's structure or even without understanding what the problem is about. Diagrams can illuminate the mathematical structure of a problem and facilitate understanding of the mathematics behind the problem. Thus, even when a diagram is not perceived to be needed for getting the right answer, teachers can encourage students to recognize that diagrams continue to be essential tools in highlighting structure and fostering deeper understanding.

Recommendation 3: Teach students to intentionally choose from alternative algebraic strategies when solving problems.

- **Action Step 1.** Teach students to recognize and generate strategies for solving problems.

Example 3.2. The same problem solved using two different solution strategies[40]

Strategy 1: Devon's solution—apply distributive property first	
Solution steps	**Labeled steps**
$10(y + 2) = 6(y + 2) + 16$	Distribute
$10y + 20 = 6y + 12 + 16$	Combine like terms
$10y + 20 = 6y + 28$	Subtract $6y$ from both sides
$4y + 20 = 28$	Subtract 20 from both sides
$4y = 8$	Divide by 4 on both sides
$y = 2$	

Strategy 2: Elena's solution—collect like terms first	
Solution steps	**Labeled steps**
$10(y + 2) = 6(y + 2) + 16$	Subtract $6(y + 2)$ on both sides
$4(y + 2) = 16$	Divide by 4 on both sides
$y + 2 = 4$	Subtract 2 from both sides
$y = 2$	

Recommendation 3: Teach students to intentionally choose from alternative algebraic strategies when solving problems.

- **Action Step 2.** Encourage students to articulate the reasoning behind their choice of strategy and the mathematical validity of their strategy when solving problems.

Example 3.8. Sample student dialogue discussing two different solution strategies[42]

Divide students into small groups to discuss two solved problems. Instruct students to discuss each solution, check each answer, and explain which solution strategy they would select and why. In the following sample dialogue, two students, Ben and Krista, discuss Devon and Elena's solution strategies from Example 3.2.

Krista: What did Elena do?

Ben: She subtracted $6(y + 2)$ as one whole term, so she was left with $4(y + 2)$ on one side of the equation. She subtracted $6(y + 2)$ from $10(y + 2)$ to get $4(y + 2)$.

Krista: Yeah, Elena didn't distribute.

Ben: Then Elena divided by 4 on both sides and then subtracted 2 on both sides.

Krista: It looks like Devon distributed first and then combined like terms before subtracting on both sides.

Ben: They both got the same answer. Devon just did a few extra steps, I believe.

Krista: They both did the problem correctly, but they just did it in different ways.

Ben: Devon distributed…

Krista: …and combined, while Elena subtracted like terms. Both Elena and Devon basically did the same steps after that, but just in a different order.

Ben: Elena's strategy is quicker and more efficient.

Teacher: What relationship between the steps of Devon's strategy and those of Elena's strategy helps explain why Elena's strategy is more efficient?

Krista: Devon's first step of distributing the 10 and 6 made the expressions look less complicated, but it didn't eliminate the need to subtract twice and divide once.

Recommendation 3: Teach students to intentionally choose from alternative algebraic strategies when solving problems.

- **Action Step 3.** Have students evaluate and compare different strategies for solving problems.

Example 3.9. Small-group comparison and discussion activity

Objectives:

- ☑ Share and compare multiple solution strategies
- ☑ Use precise mathematical language to describe solution steps
- ☑ Explain reasoning and mathematical validity

Directions: Pair students off to work on algebra problems so that students with different strategies have the opportunity to talk with each other. For example, if two strategies are prevalent and approximately half of the students use each, students may be put into groups A and B based on like strategies and then each paired with a student from the other group. Partners can discuss the strategies they used to solve the first problem (e.g., What strategy did each person use? How did the strategies differ from one another? What was the partner's rationale for using a different strategy? Did both strategies produce the same answer?). Challenge students to use their partner's strategy when solving the next problem. Conclude the activity by asking students to reflect on what they discussed with their partners, explaining the most important ways in which the two strategies differ. Have students record the strategies discussed by the class.

Recommendation 3: Potential Roadblock

My special-education students need a very structured process for solving algebra problems. Introducing multiple strategies and asking students to choose among strategies might be hard on them.

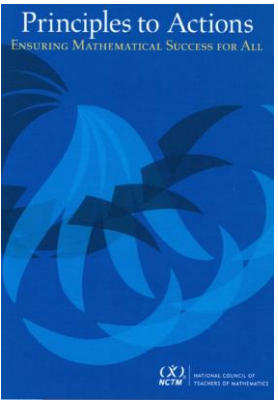
Suggested Approach. Special-education students may require explicit instruction in how to solve problems, but it is important to distinguish between providing explicit instruction and teaching only a single solution strategy and asking students to memorize the steps of that strategy. Students are better served if they come to view mathematics *not* as a game like "Memory" in which they associate a problem with a specific method. This view of mathematics places even greater demands on students' memory—given the number of problem types and methods to be memorized—and thus presents particular challenges for students in special education. Teachers can help special-education students understand alternative strategies by being explicit not only about the steps of a strategy but also about its underlying rationale, including how, when, and why it is applicable or useful for particular problems.

Prominent themes

- The practice guide highlights three general and interrelated themes:
  - Developing a **deeper understanding** of algebra.
  - Promoting **process-oriented thinking**.
  - Encouraging **precise communication**.

Applications for the Guide

The overarching message is that effective teaching is the non-negotiable core necessary to ensure that all students learn mathematics. The six guiding principles constitute the foundation of PtA that describe high-quality mathematics education.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Applications for the Guide

Teaching and Learning. An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

Applications for the Guide

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning and problem solving.
- Use and connect mathematical **representations**.
- Facilitate meaningful mathematical **discourse**. Pose purposeful **questions**.
- Build **procedural fluency** from conceptual understanding.
- Support **productive struggle** in learning mathematics.
- **Elicit and use evidence** of student thinking.

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Applications for the Guide

Professionalism. In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.

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