

# Learning Math by Making Mistakes



Note: This version does not include the pictures of math mistakes in advertisements – it was the only way to make it fit on the nctm website. You can find these pictures on Pinterest.

National Council of Teachers of Mathematics  
Annual Meeting  
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- Presenter:
  - Karen G. Gartland, M.Ed.
  - Middle School Mathematics Educator/Curriculum Specialist
  - Adjunct Professor, Lesley University School of Education

# What's this presentation about?



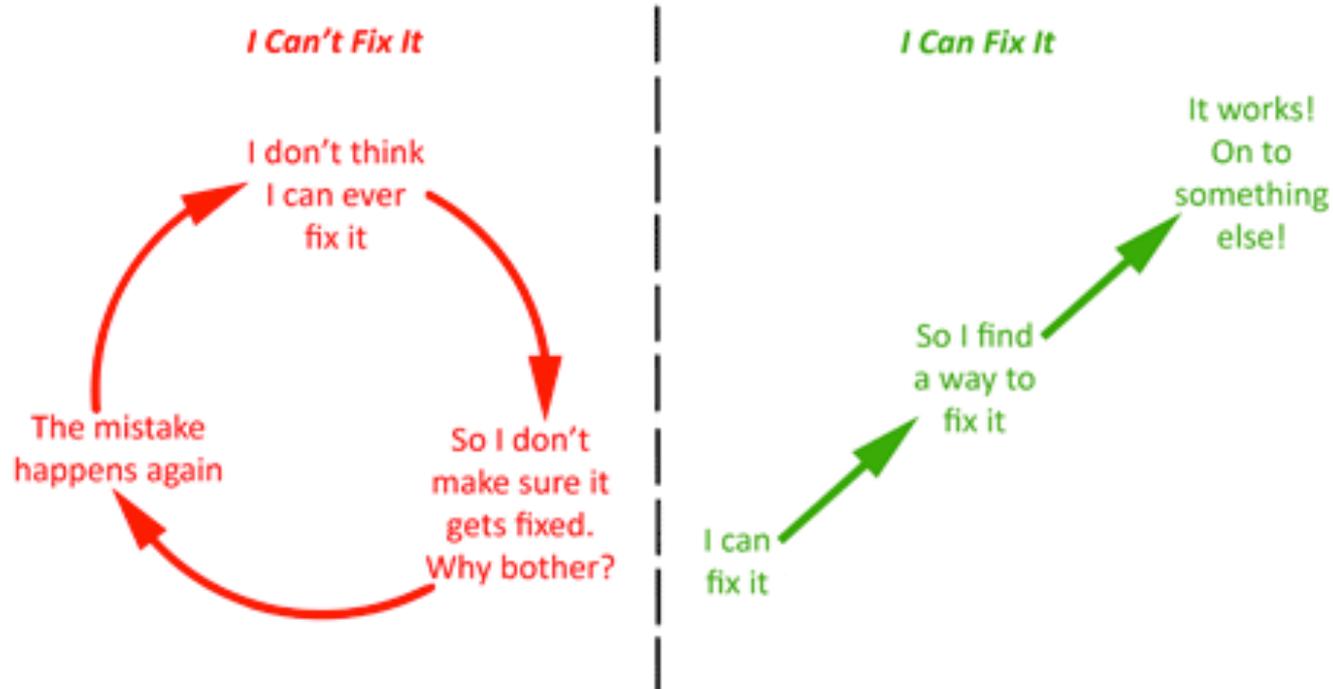
- Why not?
- Kind of Mistakes
- The research says....
- Embracing mistakes

# Why students don't like to make mistakes



- How do you view your mistakes?

## How Do You View Your Mistakes?



Credit: Hunter Maats and Katie O'Brien

Edutopia, Teaching Students to Make Mistakes  
Maats/O'Brien

# Why students don't like to make mistakes



- How do you view your mistakes?
- How do your students view their mistakes?

# Why students don't like to make mistakes

- How do you view your mistakes?
- How do your students view their mistakes?
- Many students don't like to make mistakes
  - The risk of embarrassment
  - Feel "stupid" when they make a mistake
  - It feels uncomfortable



# On making mistakes (early Fall).....

HEY,  
IT'S  
OK.

I don't like making mistakes.  
Then Mrs. Gartland said you can  
learn from your mistakes. I still  
don't like making mistakes.

# What is a mistake?

mistake 

[mi-**steyk**]

Spell

Syllables

Examples

Word Origin

Less negative

Negative connotation

...ion, calculation, opinion, or judgment caused by poor reasoning, carelessness, insufficient knowledge, etc.

2. a misunderstanding or misconception.

**verb (used with object), mistook, mistaken, mistaking.**

3. to regard or identify wrongly as something or someone else:  
*I mistook him for the mayor.*

4. to understand, interpret, or evaluate wrongly; misunderstand; misinterpret.



# Kinds of Mistakes



## ■ “Sloppy” Mistakes

- Most often do not happen because we don't understand
- Usually happen without thought
- Often happen because we're not concentrating
- More “circumstantial”

## ■ Misconceptions

- Misunderstanding the content
- Misunderstanding the problem
- Mistakes made on “purpose”

## ■ Learning New Material Mistakes

When we try to learn something new and we have a lot to learn about the material

# The Research Says....

## ■ Derek Muller

- Doctoral Candidate at University of Sydney
- Article titled “Confuse Students to Help them Learn”
- Two Videos
  - One clear and concise
  - One less clear
- Learned more from the confusing videos
- “Confusion is likely to promote learning at deeper levels of comprehension under appropriate conditions.”  
– Research team



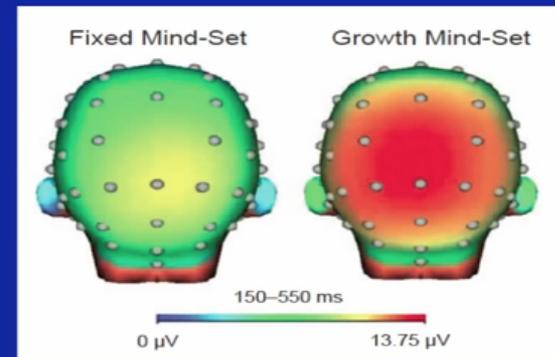
# The Research Says....

“The brain is like a muscle; that the more you use it the more it grows. They’ve found that neural connections form and deepen most when we make mistakes doing difficult tasks rather than repeatedly having success with easy ones.”

- **Dr. Carol Dweck**
  - Professor at Stanford Univ.
  - Author of: Mindset: The New Psychology of Success

## The Brain's Response to Errors

Moser, Schroder, Heeter, Moran, & Lee, 2011



# The Research Says....

- Alina Tugend

- On Perfectionism
- Japanese Classrooms

- Paul Shoemaker

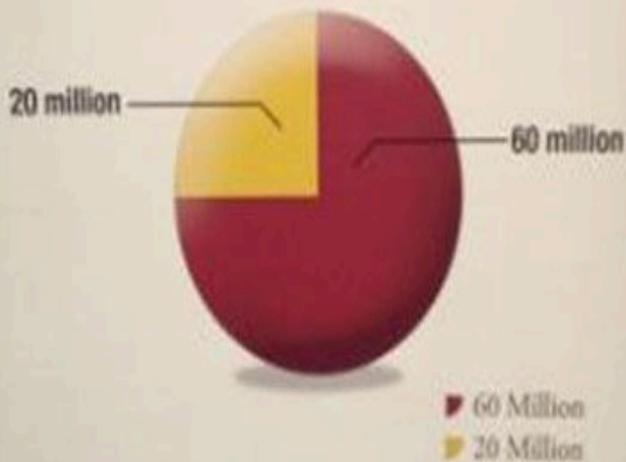
- “The wisdom of profiting from making mistakes, deliberate or otherwise, is not lost on savvy executives.”

Harvard Business Review



I don't think that's right...

$\frac{1}{3}$  of our operating  
budget  
goes towards  
financial aid.



Is the  
mistake  
in the data  
or in the  
visual?

# Changing the Perspective About Mistakes



MAKING  
MISTAKES is OK

- Highlighting the value in making mistakes....
  - Consider mistakes as valuable learning experiences
  - Learn to appreciate the mistakes we make
  - Students become less afraid of making them
  - More willing to take risks

# Creating a Classroom Culture

- Recognizing and praising:
  - Risk-taking
  - Persistence
  - Accepting challenges
  - Staying on task
  - Kindness toward helping others
  - Process
  - Asking questions



# Explaining the mistake

Learning Math by Mistake – Student Handout

Directions:

1. If the answer is correct, explain why.
2. If the answer is incorrect, identify what the mistake is, and show math reasoning as to how to correct the error.

**Change the percent to  
a decimal:      $5\% = \underline{0.5}$**

**FOR BIG MISTAKES**

Are there any incorrect answers?

If any of the answers are incorrect:

1. Explain why
2. Correct the error

$$3.625 > 11$$

$$3.51 > 2.3$$

Which is greater?

$$6.5 < 11.34$$

$$14.2 < 5.623$$

What error is being made?  
Which one is different?

**What's the error?**

**How would you explain to another student what they are doing incorrectly?**

$$3(x+2) = 3x + 2$$

$$-4(x - 5) = -4x - 5$$

Simplify the expression

$$-2(2x + 3) = -4x + 3$$

$$-(x-3) = -x - 3$$

# What are the errors?

Are the same errors being made in each example?

If the errors are different, are they related errors?

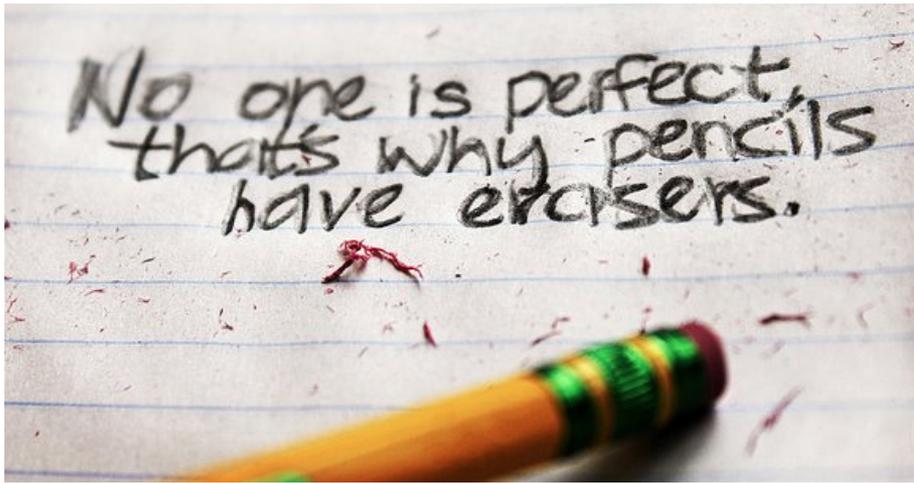
$$2x^2 * x^3 = 3x^5$$

$$x^3 * x^5 = x^{15}$$

Simplify

$$x^3 + x^3 = 2x^6$$

$$x^3 * x^{-7} = x^{10}$$



## Students Analyzing Their Own Mistakes

- Is there a mistake?
- Can you explain what the mistake is?
- Is there any part of the process that's correct?
- Justify your reasoning about the mistake.
- Explain how to fix the mistake.

# Reflecting on your mistakes....



Unit Test on \_\_\_\_\_  
Test Corrections

Name \_\_\_\_\_

Corrections Score \_\_\_\_\_

Directions:

1. Attempt the problem again yourself first!
2. You do have the option to seek out assistance.
3. Attach these corrections to your test when you are ready to turn it in.

Problem #	What <b>kind</b> of mistake did you make?	Rework of the problem	What do you understand differently about the concept now?

Which is the  
greater  
fraction?

$$\frac{1}{8} > \frac{1}{6} \quad \text{because } 8 > 6$$

This is incorrect because if you divide a whole thing into 8 parts and another thing into 6 parts, the one divided into eighths will be smaller.

# Adding Fractions

Solve the following fraction problems:

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{8}$$

$$\begin{array}{l} 2+1=3 \\ 4+4=8 \end{array}$$

$$\frac{4}{5} + \frac{3}{10} = \frac{7}{15}$$

$$\begin{array}{l} 4+3=7 \\ 5+10=15 \end{array}$$

I understand I made a mistake because you're not supposed to add the denominators. You must not do this because it gives you a smaller fraction and you need to have a common denominator to add correctly.

# Solving Proportions

Solve for x:

$$\frac{3}{5} = \frac{15}{x} \quad \frac{3}{5} = \frac{15^3}{x} \quad \frac{3}{1} = \frac{3}{x} \quad x=1$$

$$\frac{6}{x} = \frac{25}{10} \quad \frac{18}{x} = \frac{25}{10} \quad \frac{1}{x} = \frac{25}{2} \quad 2=25x \quad \frac{2}{25} = x$$

$$\frac{x}{22} =$$

I made a mistake in solving these proportions. I cross cancelled when I shouldn't have. Cross cancelling changes the ratios in the proportion, which will change the answer. I understand my mistake now.

# Exponents

$$5^{-3} = -125$$

This is incorrect because negative exponents create fractions. In this problem, it would be  $\frac{1}{5^3}$ , or  $\frac{1}{125}$ , not  $-125$ .

Find the slope: (3, -2) (5, 1)

$$m = \frac{1 - (-2)}{3 - 5}$$

This is incorrect because you must subtract the whole coordinate pair from the other coordinate pair and not switch up the x and y values.

# Slope

Rule-based description of his mistake.

What could we ask him to focus more on conceptual understanding rather than just the rule?

I used to hate making mistakes, but then Mrs. Gartland said that you could learn from them. I realized that making mistakes can teach yourself what to do.

Signed,



New thoughts on  
mistakes from middle  
school students

# “Tips to improve”

“Own up to them, don't give them the chance to become worse”

“think of them as tips to improve”



# LIKE!?!?

I like to make mistakes  
So then I can learn from  
them.

From

your favorite student



# "The Record...."

On making mistakes.....

"As a human being I can say, I bet I have the record for the most mistakes. But from these mistakes I have learned a lot."

"If you make a mistake, fess up to it, that way you can get past these issues."

# Don't be afraid....

On making mistakes.....

When you make a mistake you learn much more than when you don't, often times it's better to make a mistake than to not. Nobody is perfect and everyone makes mistakes. It is through mistakes that we evolved and continue to evolve. Therefore don't be afraid to make mistakes and try to learn from them.

# Always try to learn something

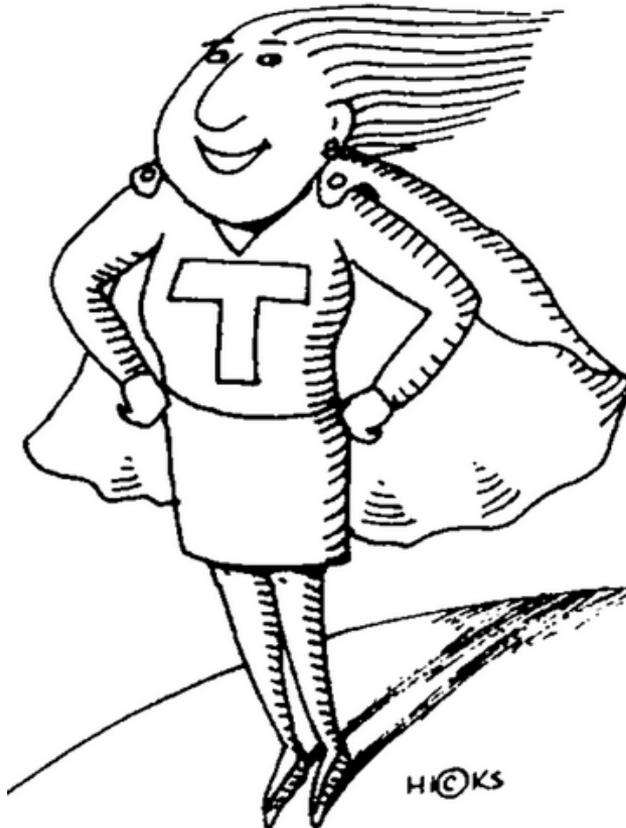


On making mistakes.....

I identify what I did wrong, and try to correct it in the future. I always try to learn something from my mistakes.

# The Teacher's Role

- Dig deep
- Know when to step in
- Facilitating discussions
- Providing feedback
- Assessments
- Categorizing students' mistakes



# “Take Aways”

- Incorporate a growth-mindset in your classroom to focus on the value of recognizing and analyzing mistakes
- Help your students realize that mistakes are a part of learning
- Working through mistakes with hard work, grit, and perseverance
- Learning to analyze mistakes can lead to greater knowledge about the content

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