

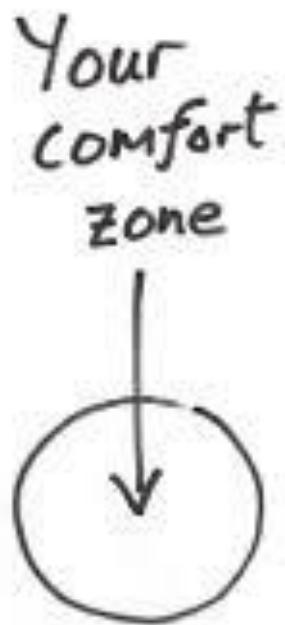


## Bring History to Life in Your Math Class

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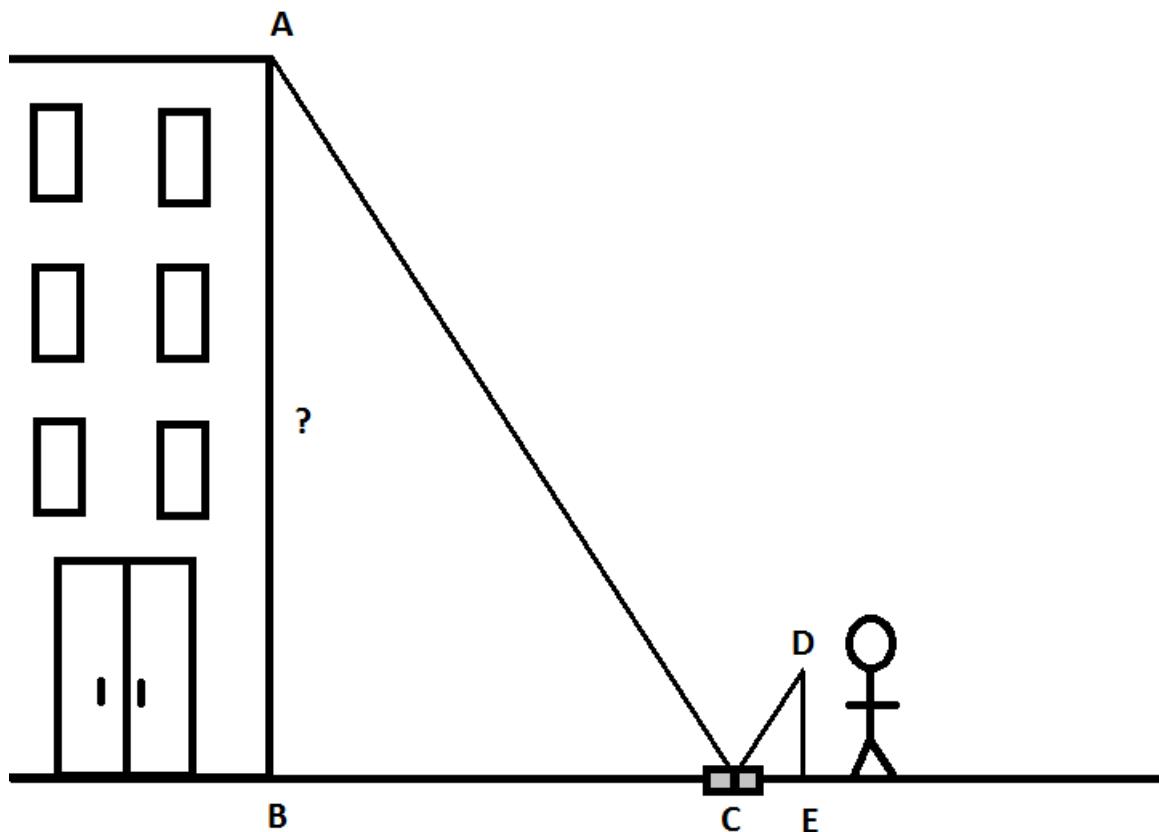




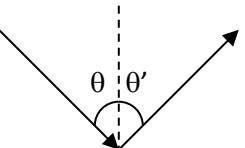
### Description of the staff and mirror method:

- 1- Place the mirror flat on the ground, parallel to the wall.  
Be careful not to put it too close to the wall.
- 2- Place yourself behind the mirror, so that the mirror will be between you and the wall.
- 3- Place your eye at the end of the staff and look in the mirror. Move forward or backward until you can see the edge of the roof on the line in the middle of the mirror.  
Make sure that the staff stays very straight.
- 4- Measure the distance between the mirror's line and the wall, the mirror's line and the staff, and the height of the staff.
- 5- Do the math to find the height of the school by using similar triangles.

## Staff and mirror



## Demonstration

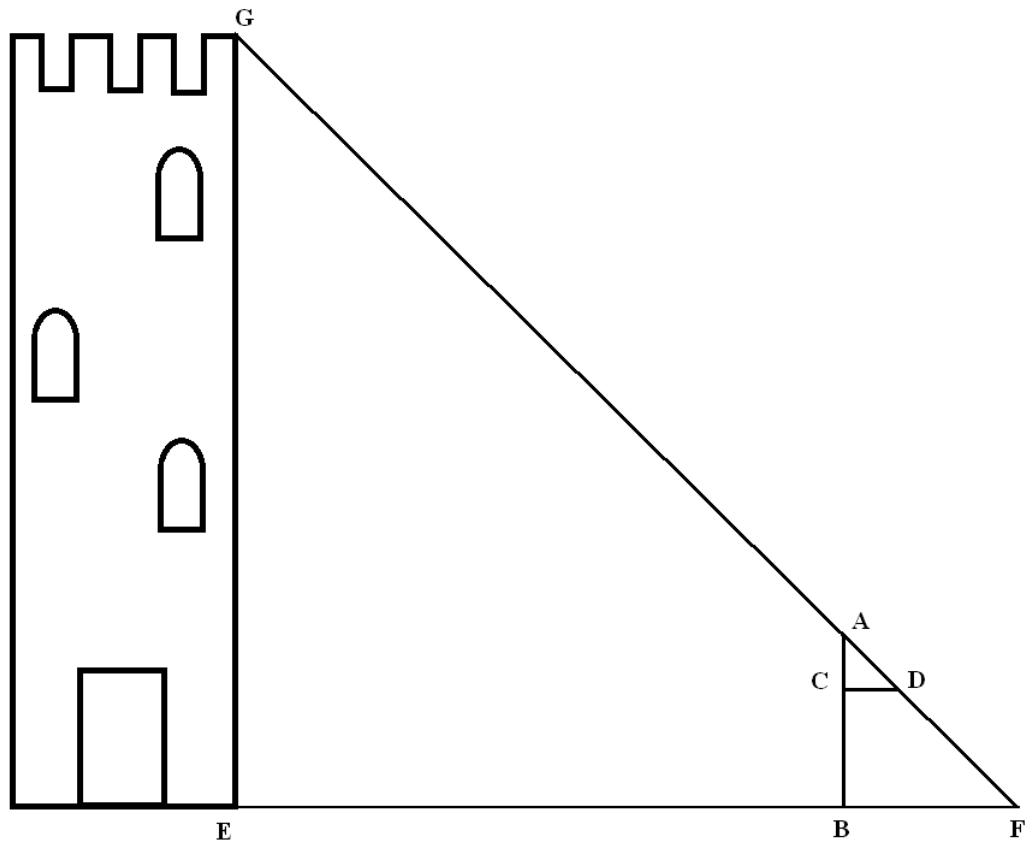
Affirmations	Justifications
$\angle ABC \cong \angle DEC$	$m\angle ABC = \angle DEC = 90^\circ$
$\angle ACB \cong \angle DCE$	By the law of reflection, the angle of incidence $\theta$ equals the angle of reflection $\theta'$ (see image below).   Therefore, $m\angle ACB = m\angle DCE$
$\triangle ABC \sim \triangle DEC$	<i>AA similarity theorem:</i> If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

### Description of the Gerbert stick method:

- 1- Place your eye close to the horizontal part of the stick and look at the top end of the stick.
- 2- Move until you can see the top of the wall come into line with the top end of the stick.
- 3- Measure the distance between the stick and the wall and measure the height of the stick.
- 4- Do the math to find the school's height by using similar triangles.

The Gerbert stick was invented by Gerbert d'Aurillac who became pope in 999 and changed his name to Sylvester the Second. He was very interested in philosophy, mathematics, and science. He studied mathematics in Morocco and Spain. Sylvester II was responsible for reintroducing the abacus and the modern system of Arabic digits to Europe.

## Gerbert Stick



## Demonstration

Affirmations	Justifications
$\angle GEF \cong \angle ABF$	Since $m\angle GEF = m\angle ABF = 90^\circ$
$\angle GFE \cong \angle AFB$	Since $\angle F$ is shared by both triangles
$\triangle GEF \sim \triangle ABF$	<i>AA similarity theorem:</i> If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.
$\angle ABF \cong \angle ACD$	Since $m\angle ABF = m\angle ACD = 90^\circ$
$\overline{CD} // \overline{BF}$	Since $\overline{CD} \perp \overline{AB}$ and $\overline{BF} \perp \overline{AB}$
$\angle ADC \cong \angle AFB$	$\angle ADC$ and $\angle AFB$ are corresponding angles with parallel lines, therefore they are congruent.
$\triangle ABF \sim \triangle ACD$	<i>AA similarity theorem:</i> If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.
<b>Conclusion :</b> If $\triangle GEF \sim \triangle ABF$ and $\triangle ABF \sim \triangle ACD$ then $\triangle GEF \sim \triangle ACD$	

Since  $\triangle ACD$  is isosceles we can say that  $m\overline{AC} = m\overline{CD}$

If  $m\overline{AC} = m\overline{CD}$  then  $m\overline{AB} = m\overline{BF}$  since  $\triangle ABF \sim \triangle ACD$

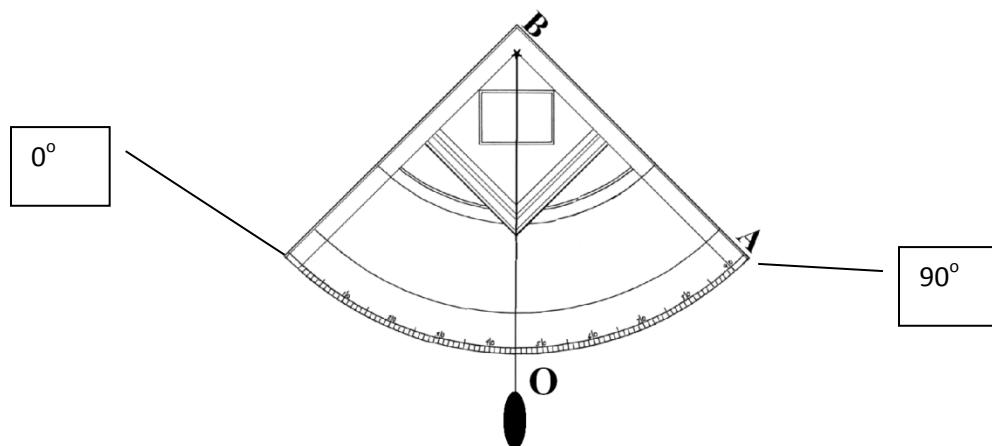
If  $m\overline{AC} = m\overline{CD}$  then  $m\overline{GE} = m\overline{EF}$  since  $\triangle GEF \sim \triangle ACD$

Therefore,  $m\overline{GE} = m\overline{EF} = m\overline{EB} + m\overline{BF} = m\overline{EB} + m\overline{AB}$

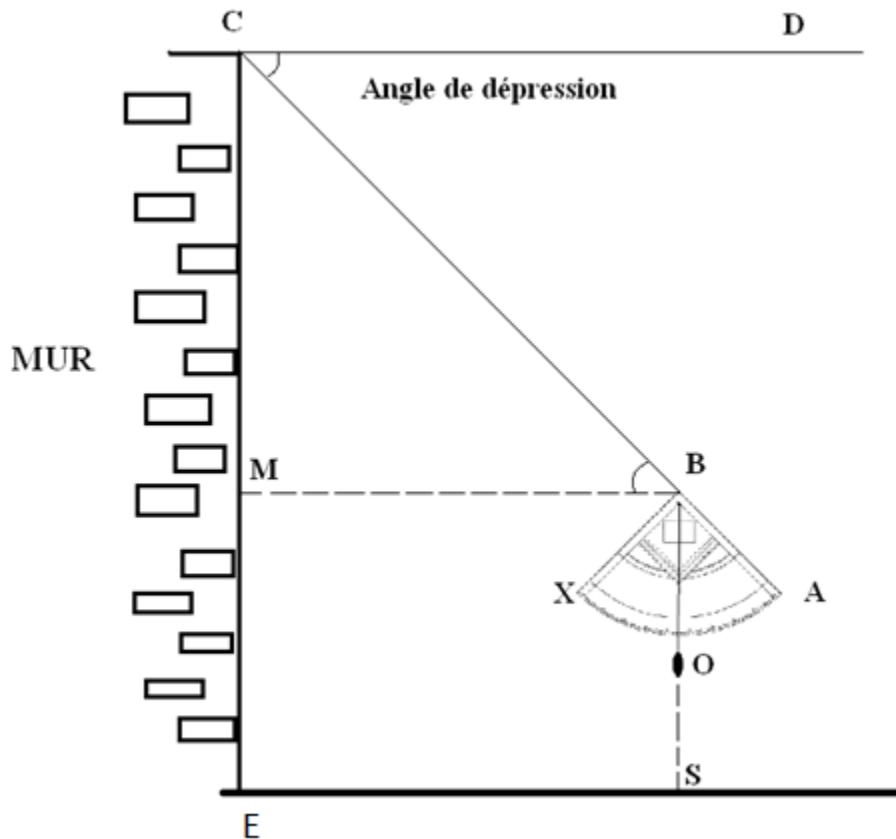
In conclusion,  $m\overline{GE} = m\overline{EB} + m\overline{AB}$

### Description of the quadrant method:

- 1- Position yourself at a certain distance from the wall, so you can easily see the top of it. (Don't go too far because you will have to measure the distance between you and the wall.)
- 2- Place your eye next to extremity A of the quadrant and look at the top of the wall through the straw.
- 3- Make the quadrant pivot so that the segment AB of the quadrant makes a line with the top of the wall. (The quadrant must be straight)
- 4- Note the measure of the angle formed by the rope on the quadrant.  
NB: This angle ( $\angle XBO$ ) is congruent to the angle of depression formed by the horizon on top of the wall and the line connecting the top of the wall and the quadrant. (See the sketch on next page.)
- 5- Take the following measurements: Angle XBO, distance of the extremity B of the quadrant to the ground (Segment  $\overline{BS}$  on the sketch, next page), distance between the extremity B of the quadrant and the wall (Segment  $\overline{BM}$  on the sketch, next page).



# Quadrant



## Demonstration and calculations

$\angle DCB \cong \angle XBO$  by definition (you can try to prove it with your students if you want).

$\angle CBM \cong \angle DCB$  since  $\angle CBM$  and  $\angle DCB$  are alternate angles with parallel lines.  
Therefore, they are congruent.

Therefore,  $\angle XBO \cong \angle CBM$

$$\tan \angle XBO = \tan \angle CBM = m\overline{CM} / m\overline{MB}$$
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}.$$

$$\text{Therefore, } m\overline{CM} = \tan \angle XBO \bullet m\overline{MB}$$

$$\text{In conclusion, } m\overline{CE} = m\overline{CM} + m\overline{BS} = \tan \angle XBO \bullet m\overline{MB} + m\overline{BS}$$