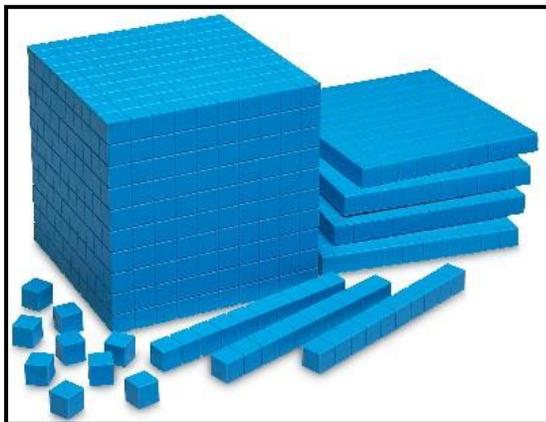


Making Our Base-Ten Number System Concrete & Comprehensible

- Why do students struggle with Base-Ten number sense?
- What are the best instructional models to use when teaching Base-Ten number sense?
- Can working in another Base help teachers better understand their students' challenges?

Flats	Rods	Units
		



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Table 1: Excerpted Common Core State Standards in Mathematics Number and Operations in Base-Ten focusing on place value and its relationship to counting, addition, subtraction, multiplication, & division concepts.

Grade	Excerpted CCSSM NBT Standards
K	<p align="center">Work with numbers 11-19 to gain foundations for place value.</p> <p>CCSS.Math.Content.K.NBT.A.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.</p>
1	<p align="center">Understand place value.</p> <p>CCSS.Math.Content.1.NBT.B.2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: CCSS.Math.Content.1.NBT.B.2.a 10 can be thought of as a bundle of ten ones — called a "ten." CCSS.Math.Content.1.NBT.B.2.b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. CCSS.Math.Content.1.NBT.B.2.c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).</p> <p align="center">Use place value understanding and properties of operations to add and subtract.</p> <p>CCSS.Math.Content.1.NBT.C.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten. CCSS.Math.Content.1.NBT.C.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. CCSS.Math.Content.1.NBT.C.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>

Grade	Excerpted CCSSM NBT Standards
2	<p style="text-align: center;">Understand place value.</p> <p>CCSS.Math.Content.2.NBT.A.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:</p> <p style="padding-left: 40px;">CCSS.Math.Content.2.NBT.A.1.a 100 can be thought of as a bundle of ten tens — called a "hundred."</p> <p style="padding-left: 40px;">CCSS.Math.Content.2.NBT.A.1.b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p> <p>CCSS.Math.Content.2.NBT.A.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.</p> <p style="text-align: center;">Use place value understanding and properties of operations to add and subtract.</p> <p>CCSS.Math.Content.2.NBT.B.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>CCSS.Math.Content.2.NBT.B.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.</p> <p>CCSS.Math.Content.2.NBT.B.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.</p>
3	<p style="text-align: center;">Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>CCSS.Math.Content.3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>
4	<p style="text-align: center;">Generalize place value understanding for multi-digit whole numbers.</p> <p>CCSS.Math.Content.4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.</p> <p>CCSS.Math.Content.4.NBT.A.2 (First part) Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form.</p> <p style="text-align: center;">Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>CCSS.Math.Content.4.NBT.B.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>CCSS.Math.Content.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the</p>

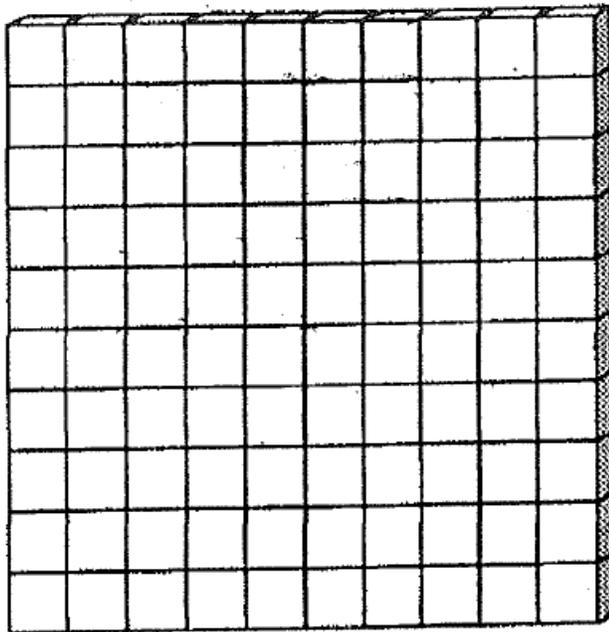
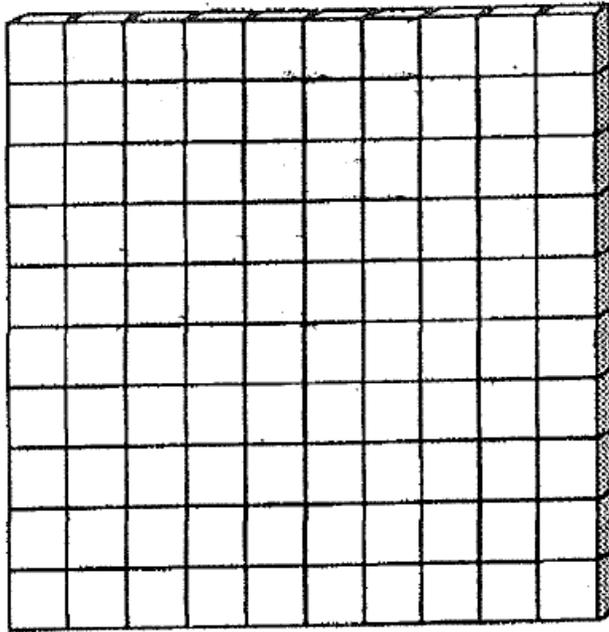
	<p>properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>CCSS.Math.Content.4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>
5	<p style="text-align: center;">Understand the place value system.</p> <p>CCSS.Math.Content.5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</p> <p style="text-align: center;">Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>CCSS.Math.Content.5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.</p> <p>CCSS.Math.Content.5.NBT.B.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>CCSS.Math.Content.5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</p>

Standards for Mathematical Practice

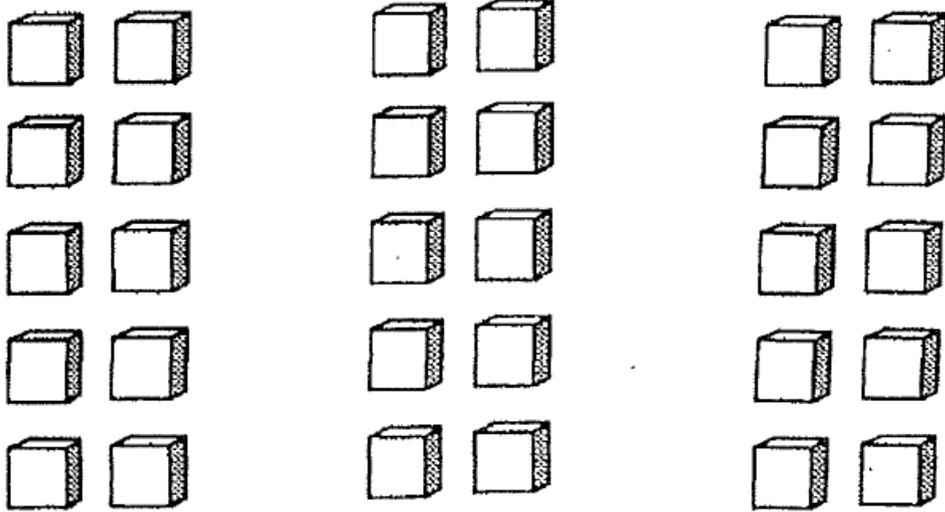
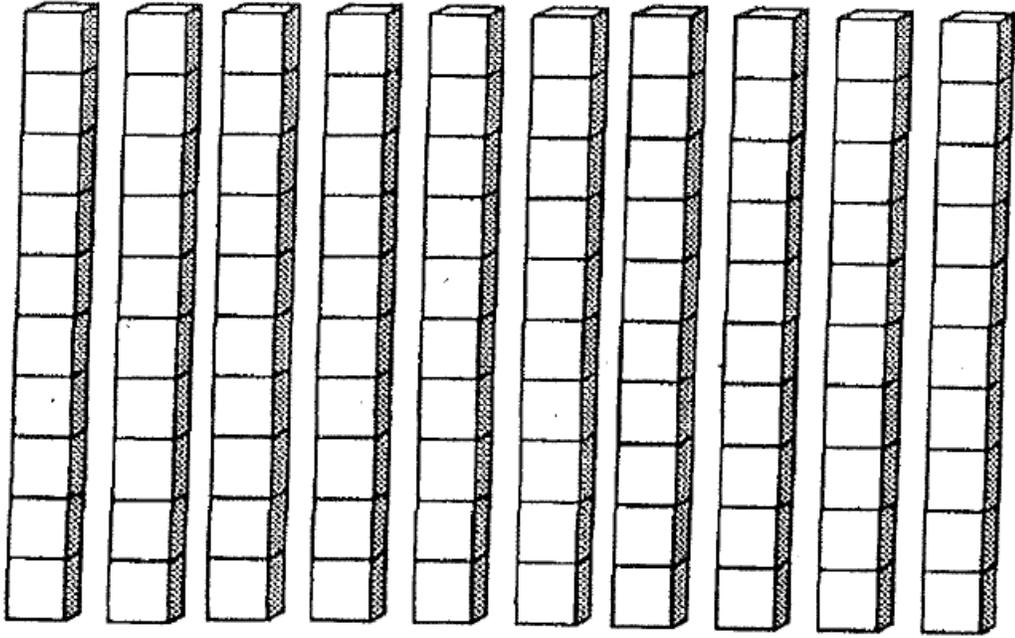
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Base-Ten Blocks

(Use when teaching subtraction with the Base-Ten two-color blocks baggie or when teaching multiplication with an area model)



Base-Ten Blocks



Using the Base-Ten Baggie of Two Colors of Blocks for Subtraction

Directions: Make copies of Base-Ten blocks using two different colors of copy paper. Cut out pieces. Use a quart-sized plastic baggie to model appropriate place value exchanges.

- Subtraction is not commutative, but many students resort to subtracting the smaller number from the larger number no matter if it is in the minuend or subtrahend.
- An example of mistakenly seeing subtraction as commutative:

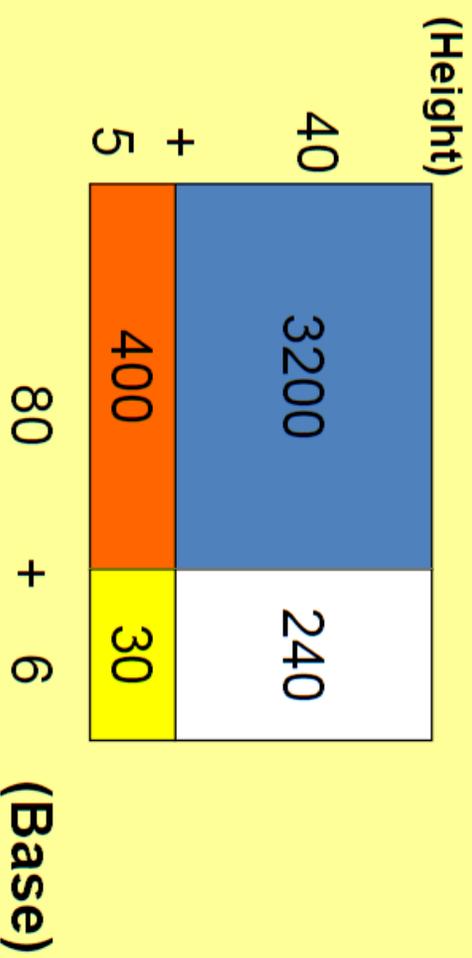
$$\begin{array}{r} 23 \\ - 17 \\ \hline 14 \end{array}$$

- The Base-Ten baggie of two colors of blocks addresses this overgeneralization.
- For subtraction, place the minuend (top number in a subtraction problem) into the baggie. For our example, place 2 rods (2 tens) and 3 units (3 ones) into the baggie.
- Next, students are being asked to “remove” 17 (one rod and 7 units). However, there are not 7 units in the bag to remove. What needs to be done?
- Exchange 1 rod (ten) for 10 units. Use another color to do exchanges in order to keep track which manipulatives represent the original problem and which represent the exchanges. The main point to emphasize during this step is that the total number of blocks has not changed; only the grouping has changed. This is why I call this “*exchanging*” rather than “*borrowing*.”
- Then, remove 7 units (from the subtrahend). Six units will be left in the baggie.
- Next, move to the tens place. One rod is in the baggie and the problem is prompting the student to remove 1 rod. Afterwards, no rods will remain.
- The final answer is what is left in the baggie (i.e., 6 units/ones). Students can check their work by doing the problem’s inverse, $6 + 17$, to make sure it equals 23.
- ✓ To use the Base-Ten baggie with decimals, simply allow the ones place to be the rod making the tenths place the smaller square. The procedures work the same as when working with whole numbers. For both addition and subtraction, the focus is still on the exchanges.
- ✓ With both addition and subtraction, show students how to draw representations of what they are doing with their baggies. This will provide a powerful strategy for them to use on tests when physical manipulatives are not allowed.

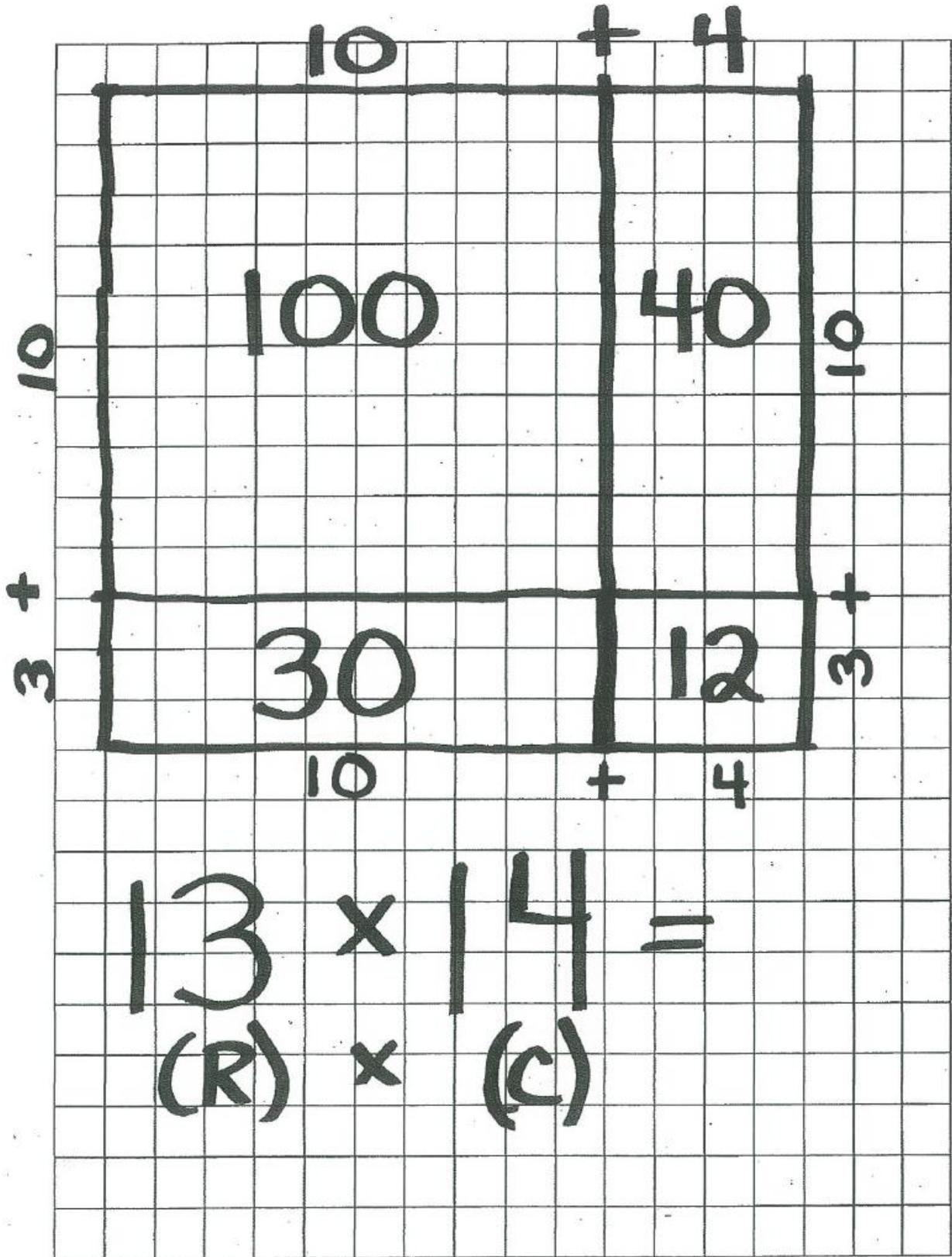
Teaching Multiplication by Emphasizing *Place Value*,
Partial Products, and an *Area Model of Multiplication*

- Area Model of Multiplication
- (Height “rows” x Base “columns”)

$$\begin{array}{r} 45 \\ \times 86 \\ \hline 30 \\ 400 \\ 240 \\ \hline 3200 \\ 3870 \end{array}$$



Area Model of Multiplication using cm Paper



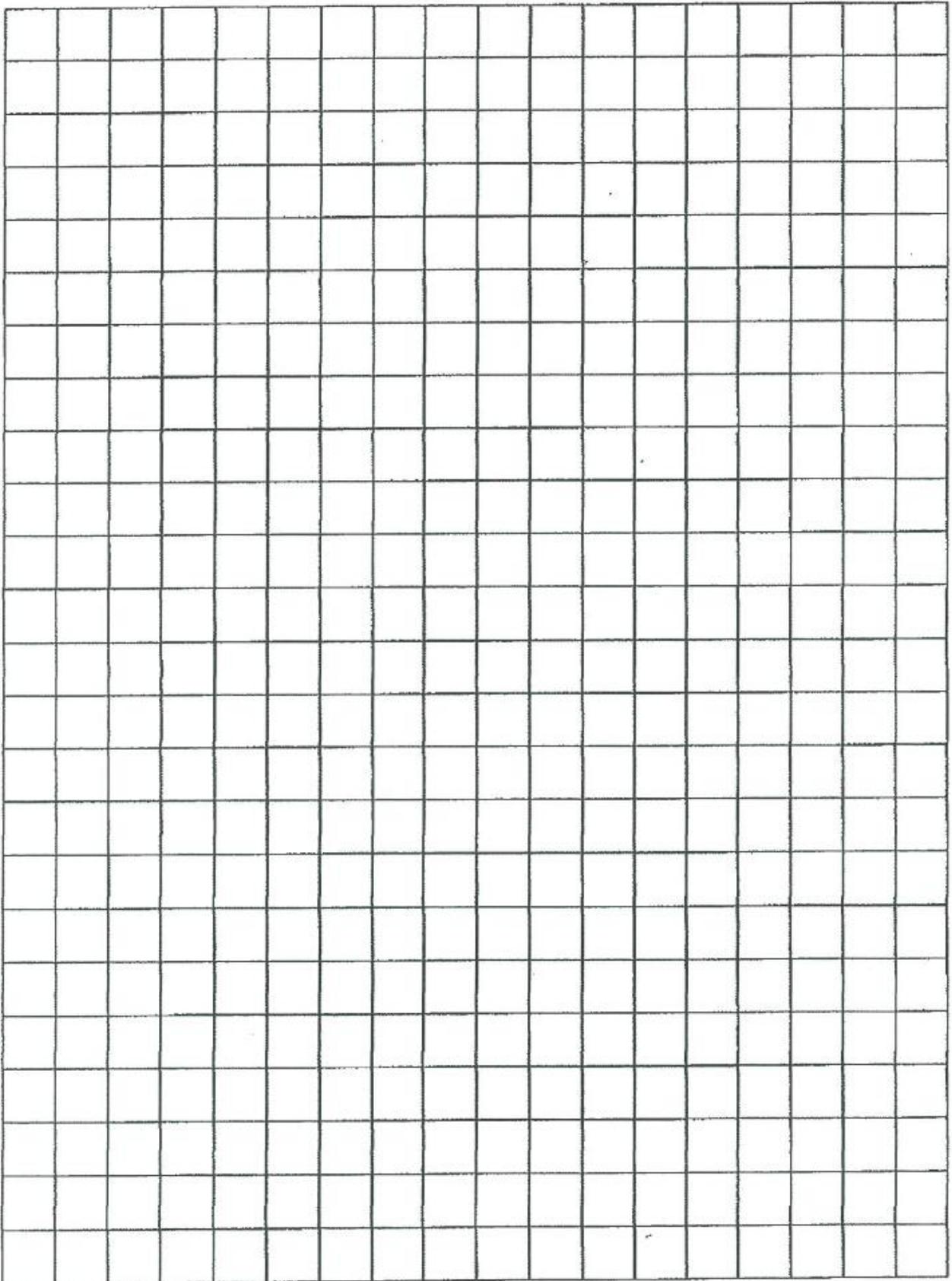
Area Model of Multiplication using Base-Ten Blocks

The diagram illustrates the area model for the multiplication 13×14 using base-ten blocks. A large green rectangle is divided into a grid of 182 small squares, representing the total product. This grid is composed of three horizontal sections: a top section of 10 rows by 14 columns (140 blocks), a middle section of 3 rows by 14 columns (42 blocks), and a bottom section of 10 rows by 4 columns (40 blocks). To the right of the grid, the blocks are shown in their individual forms: three vertical rods of 10 blocks each (representing the 10 in 13), three horizontal rods of 4 blocks each (representing the 4 in 14), and 12 small cubes (representing the 3 in 13 multiplied by the 4 in 14).

13×14
Also,
13
 $\times 14$

3 x 4 = 12
3 x 10 = 30
10 x 4 = 40
10 x 10 = 100

182



Directions for Creating a Base-Ten Baggie of Bundled Craft Sticks

When you get back home, I hope you choose to use the grouped craft sticks idea with your students. I usually buy a box of 1,000 craft sticks at Walmart for a little less than \$5/box. I buy size-16 rubber-bands at Office Depot, because they are small enough to only need a few twists around the bundles of 10 sticks. I also buy quart-sized plastic baggies.

When I taught 5th grade, I had my students create the baggies at the start of the school year. This kinesthetic experience of grouping 10 craft sticks to make a bundle and then grouping 10 bundles to make a baggie helped learners better understand the 1:10 ratio of our Base-Ten place value system. At the end of every year, I had students take the sets apart to prepare for the next year's group of learners.

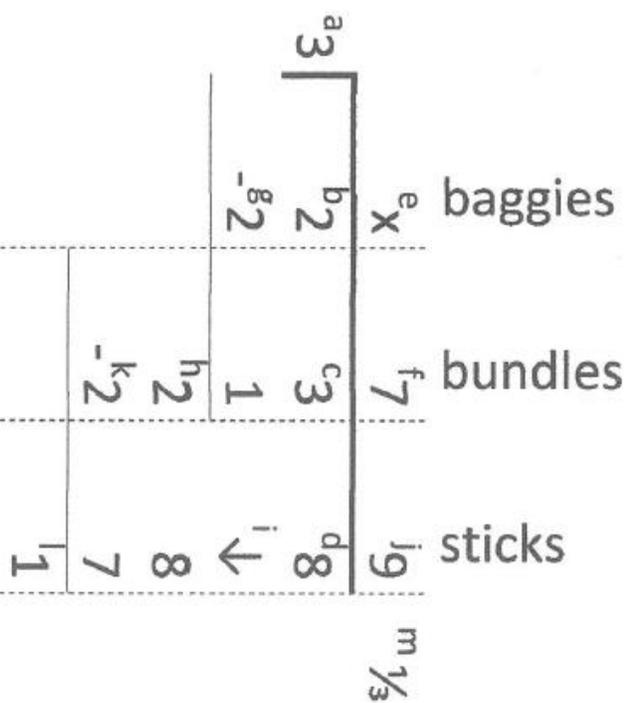
Your goal is to group 10 individual craft sticks and secure them with a rubber-band. Do this 10 times (i.e., create 10 bundles) and place all bundles (of 10 sticks per bundle) inside your baggie.

Base-Ten Baggie of Grouped Craft Sticks

1 stick; 10 sticks = 1 bundle; 10 bundles (100 sticks) = 1 baggie



Division with Craft Sticks Worked Example



- a) Number of people you are sharing the sticks with
- b) "2 baggies" equivalent to 200
- c) "3 bundles" equivalent to 30
- d) "8 sticks" equivalent to 8
- e) (Always begin with your largest place value.) Question 1: "I have 2 baggies. Is this enough to share fairly with 3 people?" No. Place an "x" in the quotient above the ^b2 to prevent accidentally placing another number there later on.
- f) Comment and Question 2: "Since I can't share 2 baggies fairly, I need to exchange them for 20 bundles. I already have 3 bundles, so I will combine the 20 and 3 bundles and share 23 bundles (notice the 23 in the dividend) among 3 people. Is that enough? Yes. Each of the 3 people get 7 bundles."
- g) "21" is the total number of bundles fairly shared among 3 people.
- h) "2" is the number of bundles left over. I can't share them fairly with 3 people, so I need to exchange them for 20 sticks.
- i) Comment and Question 3: "After exchanging the leftover 2 bundles for 20 sticks and combining them with my original 8 sticks, I have 28 sticks to share fairly with 3 people. Do I have enough? Yes."
- j) "9" is the number of sticks each of the 3 people get
- k) "27" is the total number of sticks passed out
- l) "1" is the number of sticks left over. It's not enough to share fairly with 3 people, so it becomes a remainder.
- m) " $\frac{1}{3}$ " is the fraction of the leftover stick each person gets