Base-ten Blocks to Beginning Algebra: Unifying Computational Algorithms

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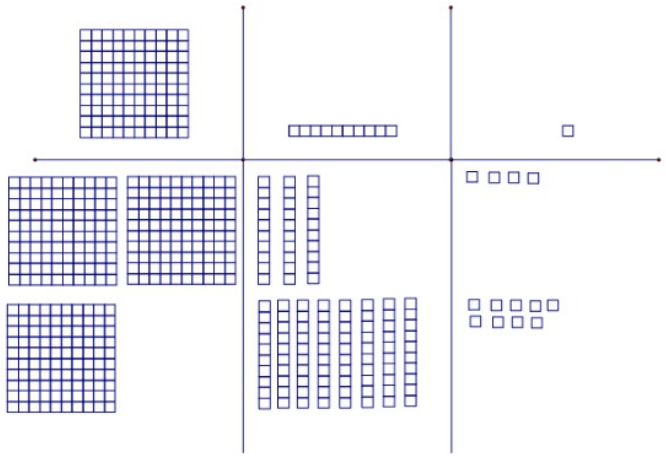
Whole Number Addition

Base-ten Blocks

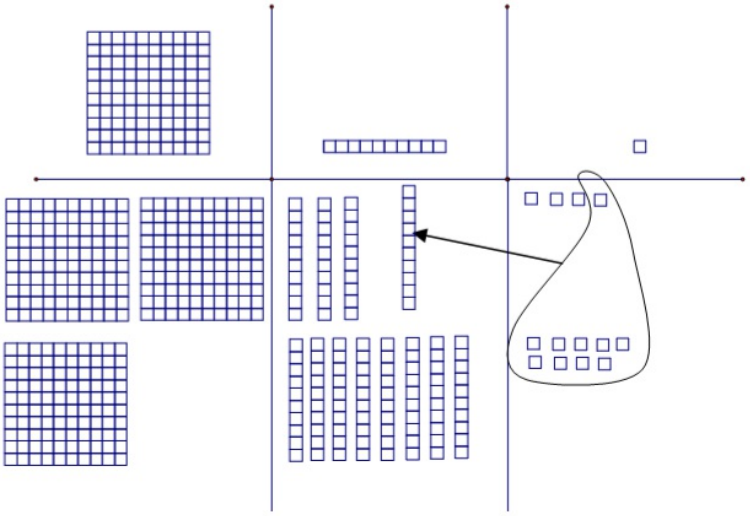
$$\begin{array}{r} 2\ 3\ 4 \\ +\ 1\ 8\ 9 \\ \hline \end{array}$$ means to combine 234 and 189.



Put the blocks out that represent the two numbers as shown to the left.

Whole Number Addition (cont.)

Base-ten Blocks (cont.)



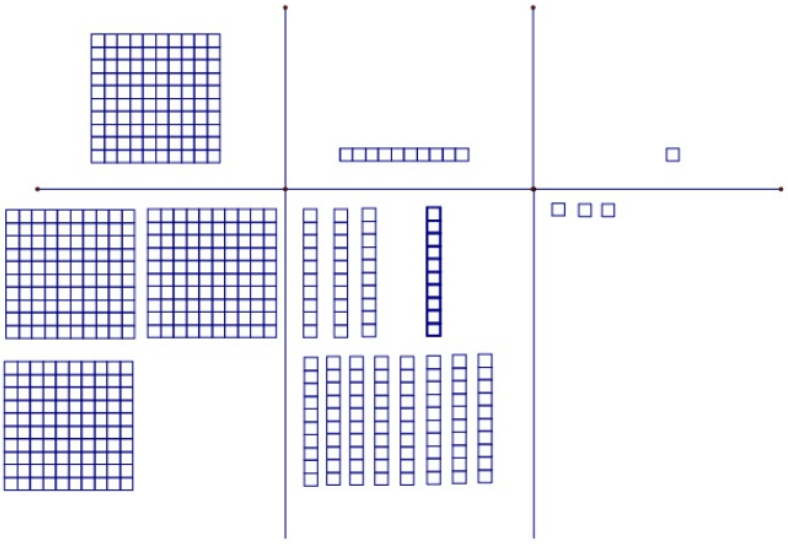
Combine the units to get 11 in total so we trade a long for 10 units.

Represents the long/ten traded for the 10 units.

$$\begin{array}{r} {\scriptstyle 1} \\ 234 \\ +\ 189 \\ \hline \end{array}$$

Whole Number Addition (cont.)

Base-ten Blocks (cont.)

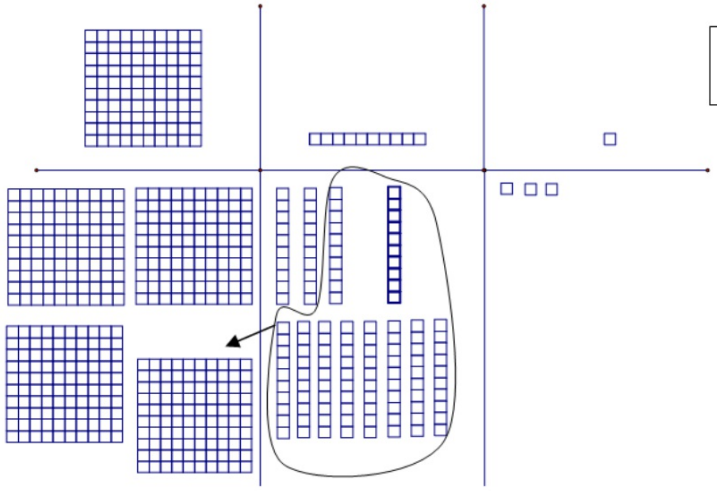


$$\begin{array}{r} {\scriptstyle 1} \\ 234 \\ +\ 189 \\ \hline 3 \end{array}$$

The number of units that remains.

Whole Number Addition (cont.)

Base-ten Blocks (cont.)



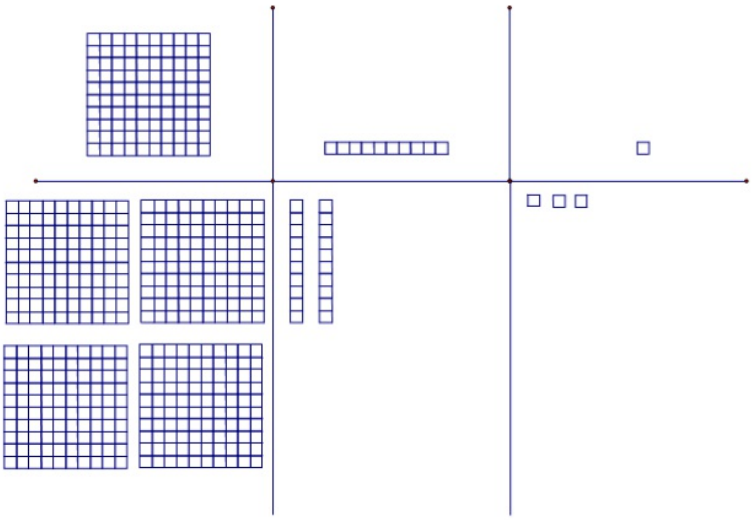
Combine the longs to get 12 longs and trade 10 for a flat.

Represents the flat from the trade of the 10 longs.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 2\ 3\ 4 \\ +\ 1\ 8\ 9 \\ \hline 3 \end{array}$$

Whole Number Addition (cont.)

Base-ten Blocks (cont.)



$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 2\ 3\ 4 \\ +\ 1\ 8\ 9 \\ \hline 2\ 3 \end{array}$$

Number of longs that remain.

Combine the flats.

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 2\ 3\ 4 \\ +\ 1\ 8\ 9 \\ \hline 4\ 2\ 3 \end{array}$$

Number of flats that are combined.

Whole Number Addition (cont.)

Traditional Algorithm

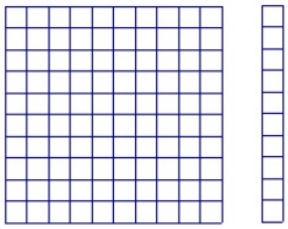
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 234 \\ +\ 189 \\ \hline 423 \end{array}$$

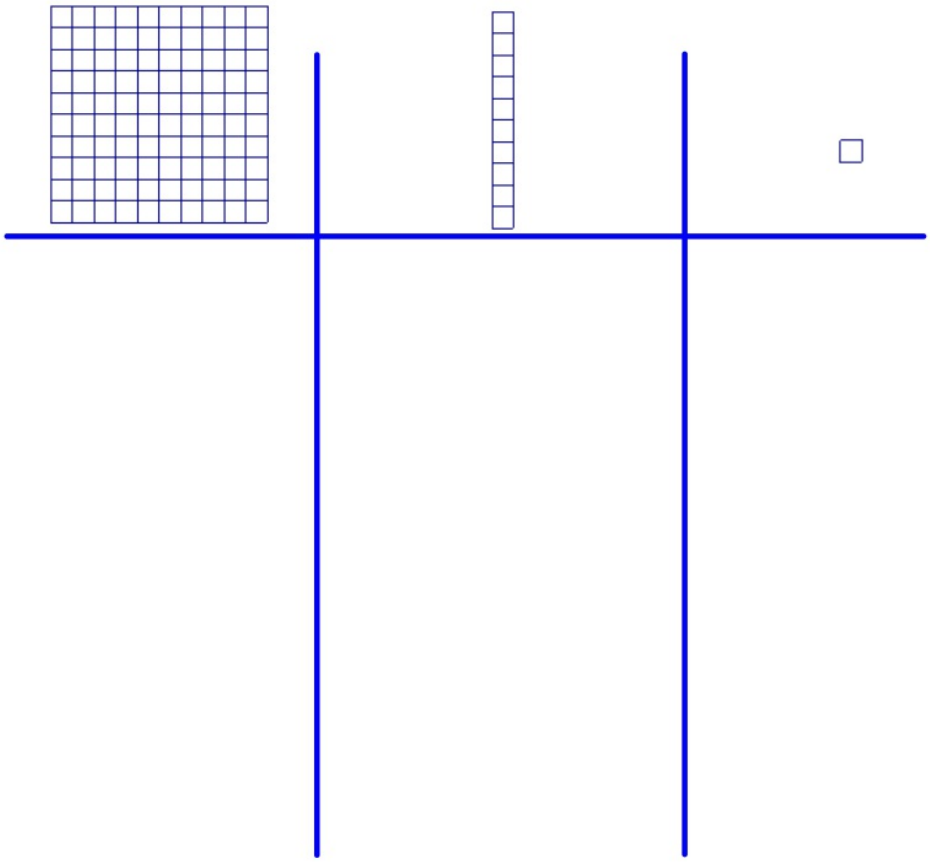
Alternative Algorithm

$$\begin{array}{r} 234 \\ +\ 189 \\ \hline 13 \\ 110 \\ 300 \\ \hline 423 \end{array}$$

Now let's try:

$$\begin{array}{r} 1\ 7\ 5 \\ +\ 1\ 8\ 9 \\ \hline \end{array}$$



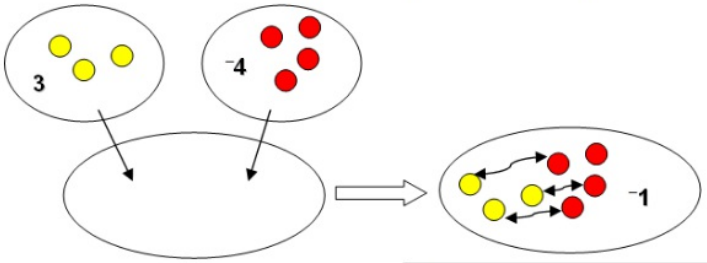


Integer Addition

Chip Model

Yellow chips represent positive integers and red chips represent negative integers. Each pair of yellow/red chips neutralize each other.

3 + (⁻4) means *Combine 3 yellow chips and 4 red chips.*

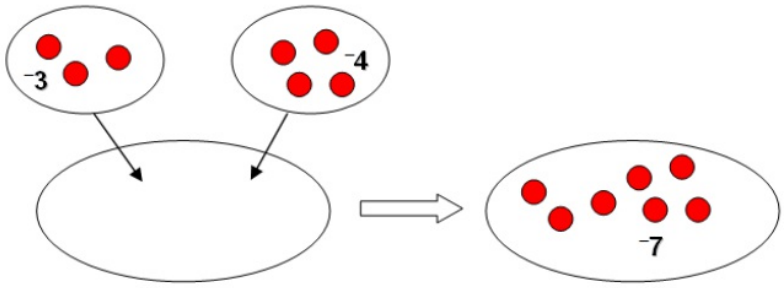


Since there is one more red chip than yellow chips, **the answer is ⁻1.**
So, 3 + (⁻4) = ⁻1.

Integer Addition (cont.)

Chip Model (cont.)

$^{-}3 + (^{-}4)$ means *Combine 3 red chips and 4 red chips.*



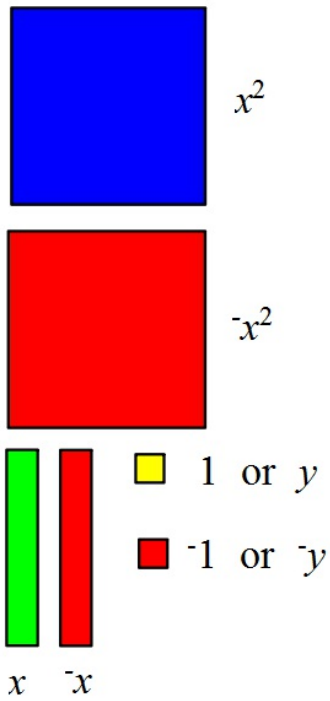
Since there 7 red chips all, **the answer is $^{-}7$. So, $^{-}3 + (^{-}4)$**

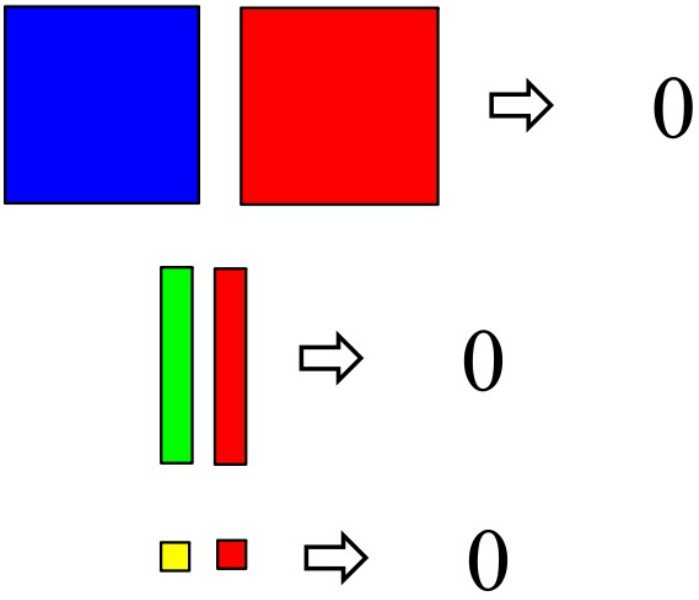
Integer Addition (cont.)

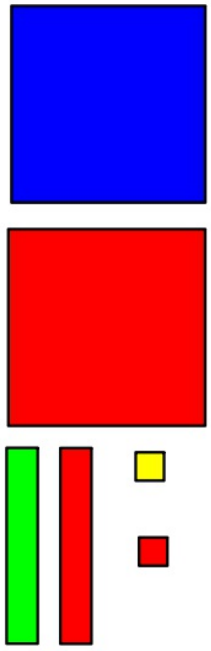
When adding two integers with the same signs:
Ignore the signs and add the two numbers; then, append the sign of the two numbers.

When adding two integers with different signs:
Ignore the signs and find the difference; then attach the sign of the number with the greatest absolute value.

Algebra Tiles







Algebra Tiles: Addition

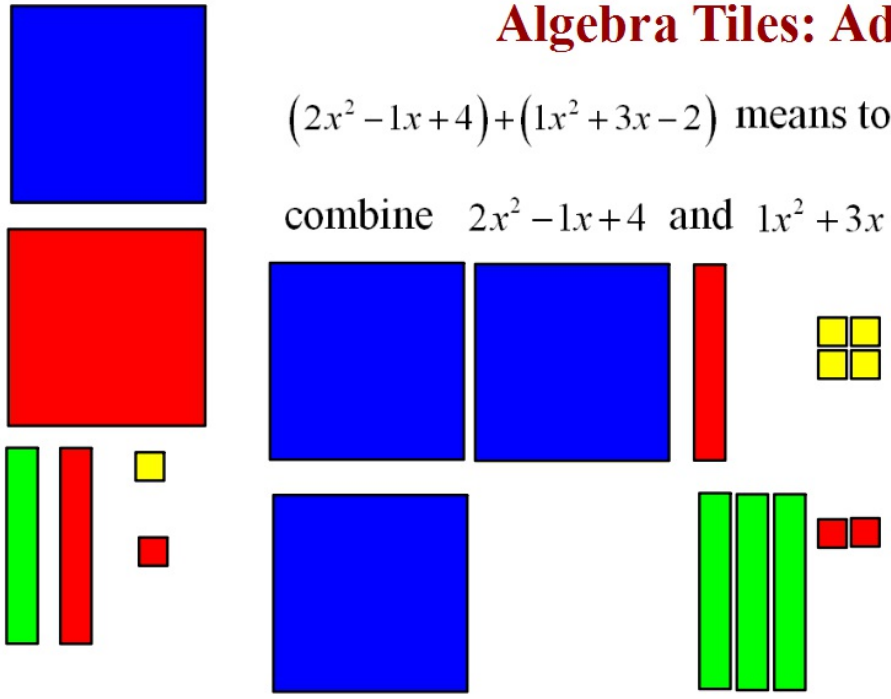
$(2x^2 - 1x + 4) + (1x^2 + 3x - 2)$ means to

combine $2x^2 - 1x + 4$ and $1x^2 + 3x - 2$.

Algebra Tiles: Addition (cont.)

$(2x^2 - 1x + 4) + (1x^2 + 3x - 2)$ means to

combine $2x^2 - 1x + 4$ and $1x^2 + 3x - 2$.

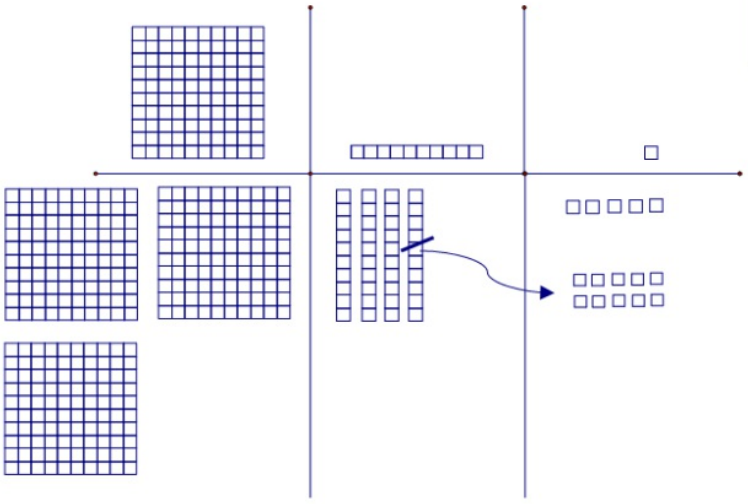


$$\begin{array}{r} 2x^2 - 1x + 4 \\ +\ 1x^2 + 3x - 2 \\ \hline 3x^2 + 2x + 2 \end{array}$$

Whole Number Subtraction

Base-ten Blocks

$$\begin{array}{r} 3\ 4\ 5 \\ -\ 1\ 6\ 8 \\ \hline \end{array}$$ means to take 168 away from 345.

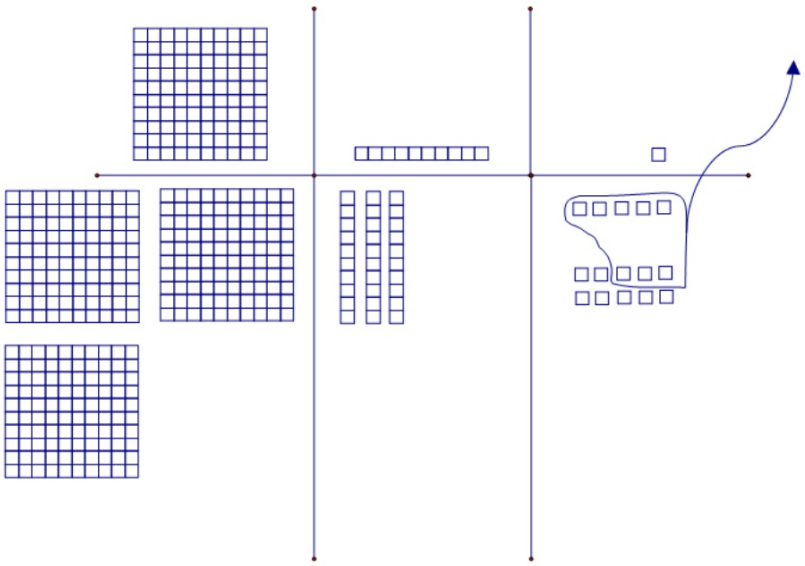


Since we cannot take 8 units from the 5 units, we need to trade a long for 10 units.

$$\begin{array}{r} 3\ 4\ 5 \\ -\ 1\ 6\ 8 \\ \hline \end{array}$$

Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)



Represents the long that are left.

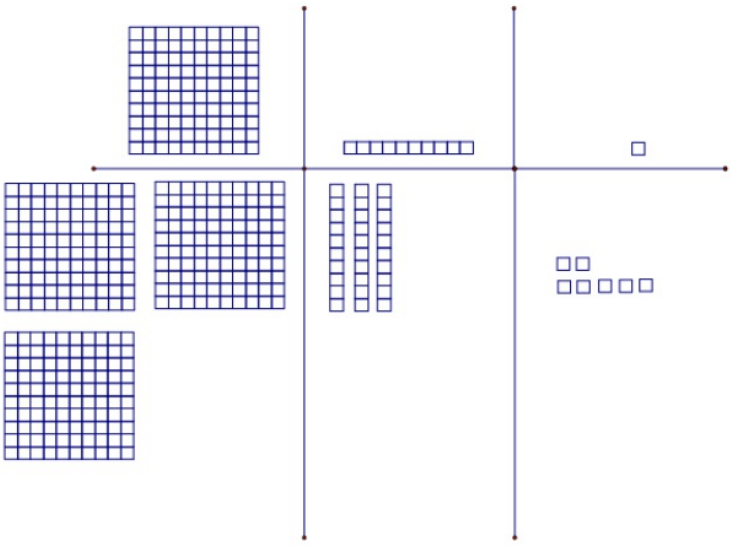
Number of units after trade.

$$\begin{array}{r} 3\ 15 \\ 3\ \cancel{4}\ \cancel{5} \\ -\ 1\ 6\ 8 \\ \hline \end{array}$$

Now take away the 8 units.

Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)

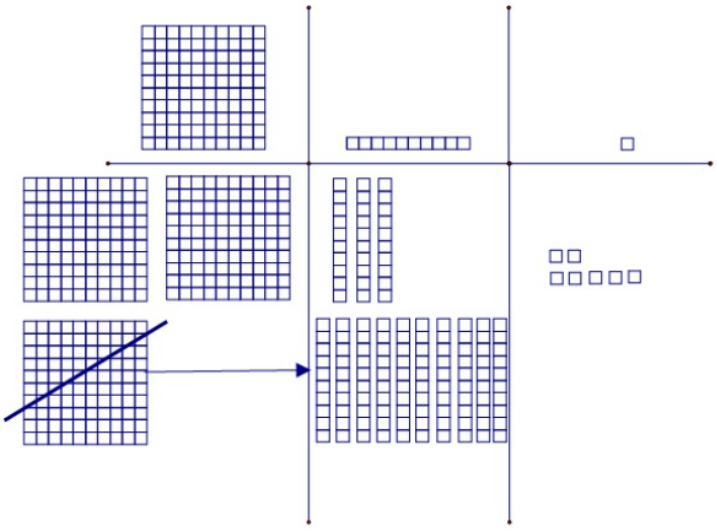


$$\begin{array}{r} \ 3\ \ 15 \\ 3\ \not{4}\ \not{5} \\ -\ 1\ \ 6\ \ 8 \\ \hline \ \ \ \ 7 \end{array}$$

Number of units that remain.

Whole Number Subtraction (cont.)

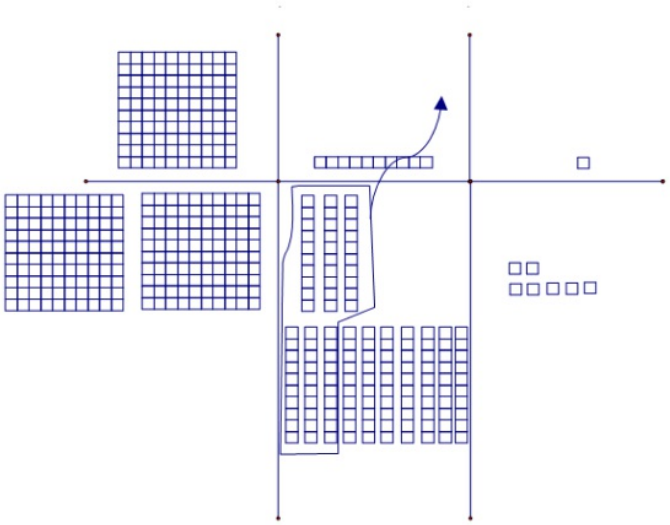
Base-ten Blocks (cont.)



Since we cannot take 6 longs from the 3 longs we have we need to trade a flat for 10 longs.

Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)



Represents the flats that are left.

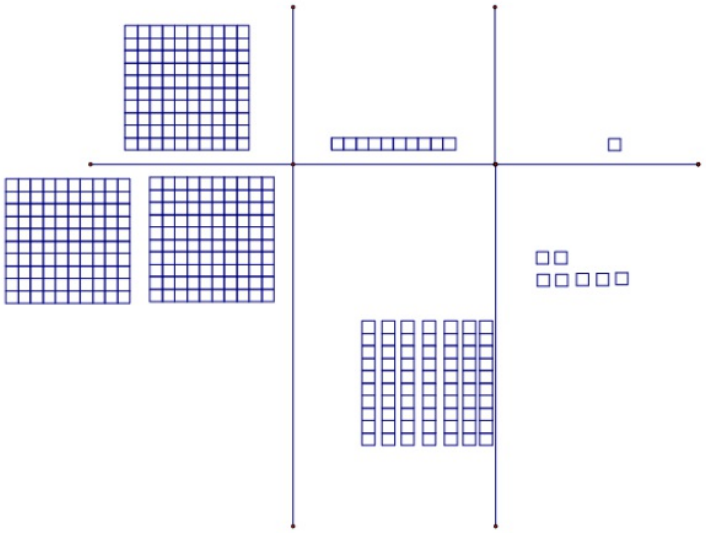
Number of longs after trade.

$$\begin{array}{r} \ 13 \\ \ 2\ \ \cancel{3}\ \ 15 \\ \ \cancel{3}\ \ \cancel{4}\ \ \cancel{5} \\ -\ 1\ \ 6\ \ 8 \\ \hline \ \ \ \ \ 7 \end{array}$$

Now take away the six longs.

Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)

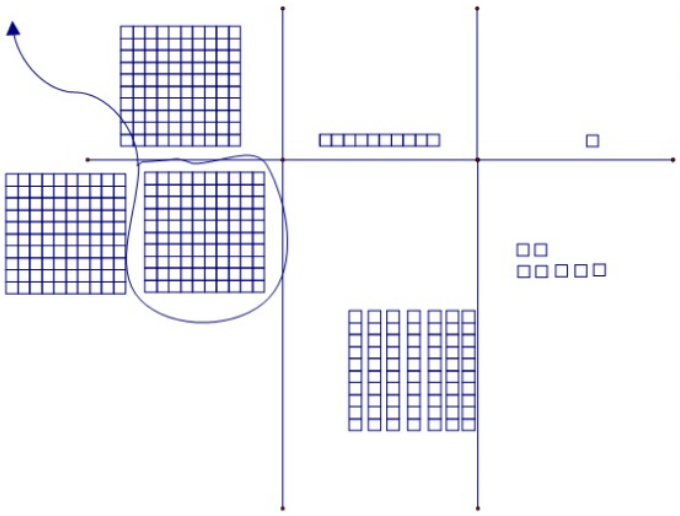


$$\begin{array}{r} \,13 \\ \,2\;\not{3}\;15 \\ -\;1\;6\;8 \\ \hline \,7\;7 \end{array}$$

Number of longs that remain.

Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)

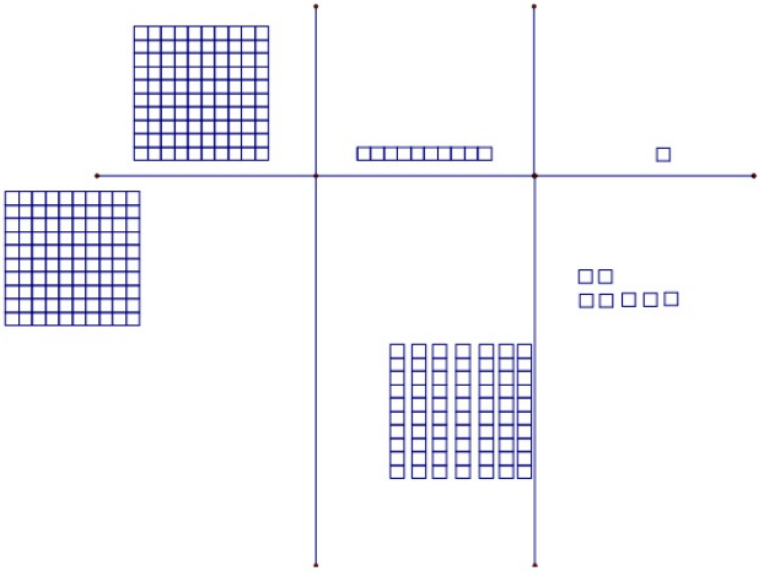


Now take away one flat from the two that are left.

$$\begin{array}{r} \;1\;3 \\ 2\;\not{3}\;15 \\ \not{3}\;\not{4}\;\not{5} \\ -\;1\;6\;8 \\ \hline \;7\;7 \end{array}$$

Whole Number Subtraction (cont.)

Base-ten Blocks (cont.)



$$\begin{array}{r} \;\;13 \\ \;2\;\;\not{3}\;\;15 \\ \;\not{3}\;\;\not{4}\;\;\not{5} \\ -\;\;1\;\;6\;\;8 \\ \hline \;\;1\;\;7\;\;7 \end{array}$$

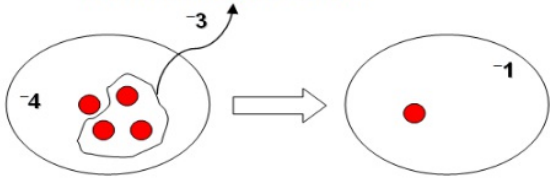
Number of flats that remain.

Integer Subtraction

Chip Model (take–away model)

- Create one group. The group should have the color and quantity of chips corresponding to the minuend.
- Take away or move out the quantity and color of chips corresponding to the subtrahend.
- The resultant set is the difference.

a. $^-4 - (^-3)$ means *Start with a set equivalent to 4 red chips. Then take away 3 red chips.*

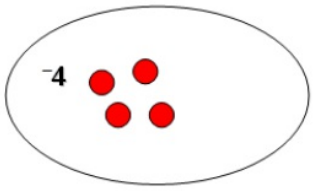


Since one red chip is left, **the answer is $^-1$. So, $^-4 - (^-3)$ =**

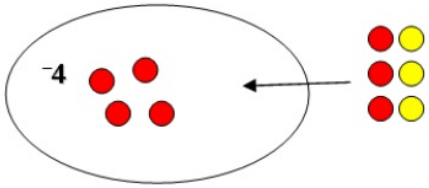
Integer Subtraction (cont.)

Chip Model (cont.)

b. $^{-}4 - 3$ **means** *Start with 4 red chips. Then take away three yellow chips.*



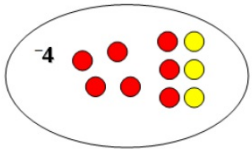
However, there are not three yellow chips to be removed!



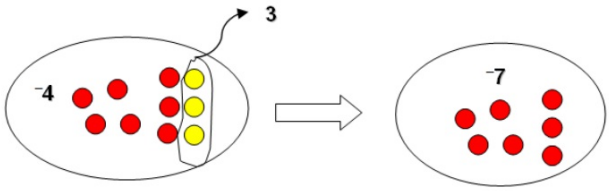
Three yellow / red pairs are put in the group. This is the same as adding zero to the group.

Integer Subtraction (cont.)

Chip Model (cont.)



Now, three yellow chips can be removed.



Since seven red chips are left, **the answer is ⁻7. So, ⁻4 – 3 = ⁻7.**

Integer Subtraction (cont.)

Subtraction Using the Additive Inverse

Use your two color counters to find the solutions to each of the following problems.

a. $2-8$	a. $2+(^{-}8)$
b. $2-(^{-}8)$	b. $2+8$
c. $^{-}12-(^{-}5)$	c. $^{-}12+5$
d. $^{-}12-5$	d. $^{-}12+(^{-}5)$

What do you notice about the two columns of problems? That is, how are they similar and how do they differ? What does this tells about the subtractions of integers?

Integer Subtraction (cont.)

Subtraction Using the Additive Inverse

For all integers a and b

$$a - b = a + {}^{-}b.$$

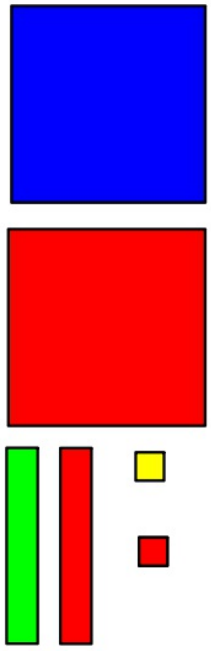
Using the fact that ***a-b=a+ (¯b)***, solve the following problems.

a. $5-9$

b. $7-\left({}^{-}8\right)$

c. ${}^{-}10-\left({}^{-}3\right)$

d. ${}^{-}9-5$

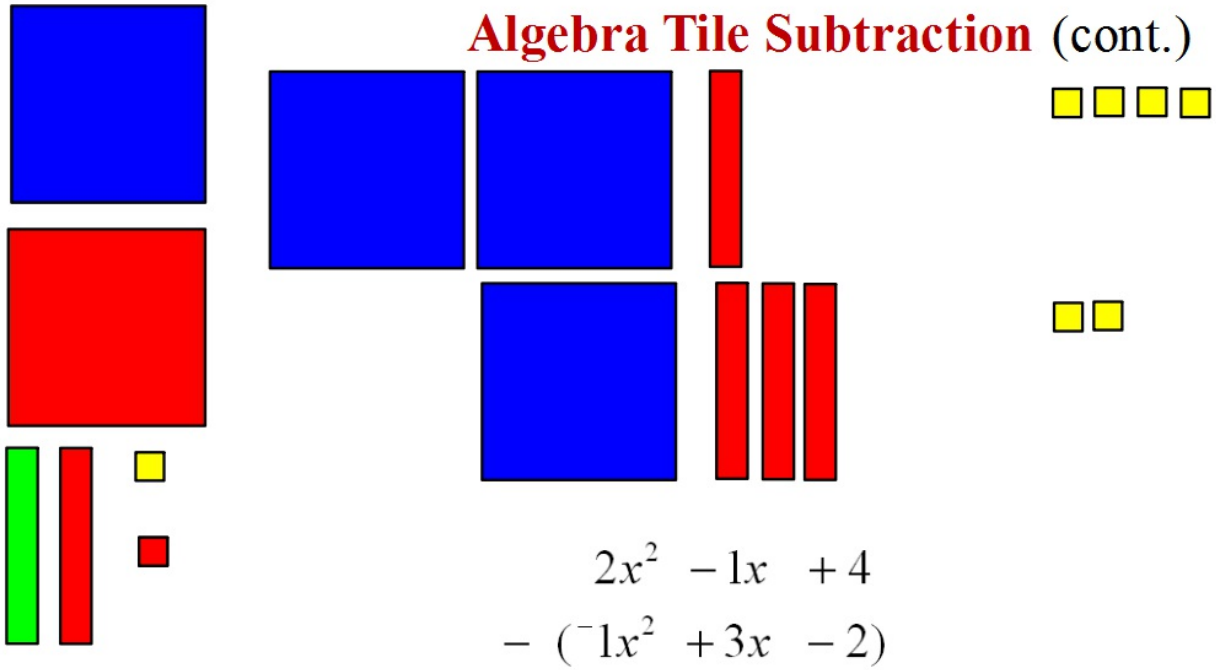


Algebra Tile Subtraction

$(2x^2 - 1x + 4) - (^{-}1x^2 + 3x - 2)$ means to take away

$^{-}1x^2 + 3x - 2$ from $2x^2 - 1x + 4$.

Algebra Tile Subtraction (cont.)



$$\begin{array}{r} 2x^2 - 1x + 4 \\ -\ (^{-}1x^2 + 3x - 2) \\ \hline 3x^2 - 4x + 6 \end{array}$$

Algebra Tile Subtraction (cont.)

Subtraction Using the Additive Inverse

Remember:

For all integers a and b

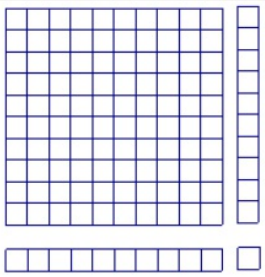
$a - b = a + {}^{-}b$.

$$\begin{array}{r} 2x^2 \ -1x \ +4 \\ -\ (^{-}1x^2 \ +3x \ -2) \end{array} \Longrightarrow \begin{array}{r} 2x^2 \ -1x \ +4 \\ +\ (1x^2 \ +^{-}3x \ +2) \end{array}$$

$$\begin{array}{r} \\ 3x^2 \ -4x \ +6 \end{array} \qquad \begin{array}{r} \\ 3x^2 \ -4x \ +6 \end{array}$$

OR

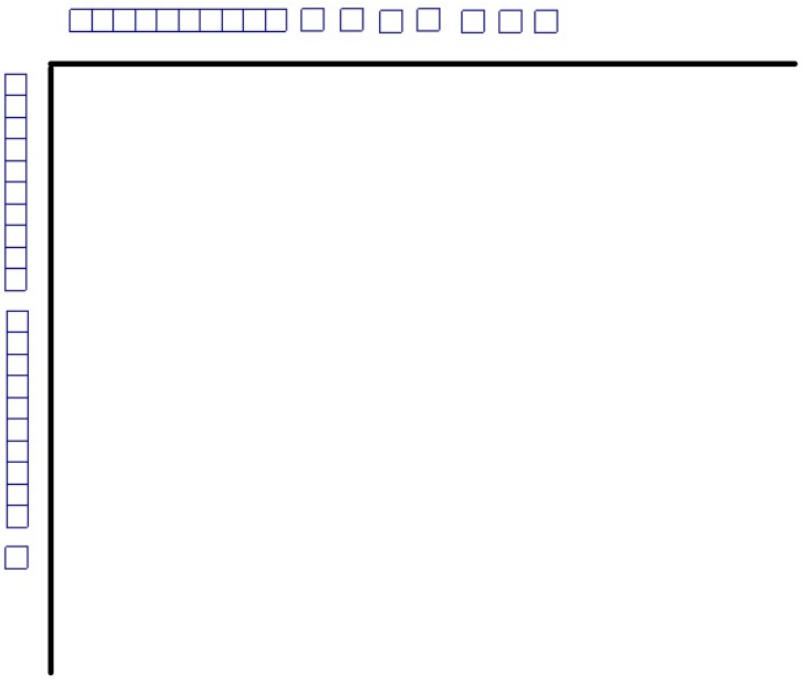
$$\begin{aligned} &\left(2x^2 - 1x + 4\right) - \left(^{-}1x^2 + 3x - 2\right) \\ &= \left(2x^2 + {}^{-}1x + 4\right) - \left(^{-}1x^2 + 3x + {}^{-}2\right) \\ &= \left(2x^2 + {}^{-}1x + 4\right) + {}^{-}\left(^{-}1x^2 + 3x + {}^{-}2\right) \\ &= \left(2x^2 + {}^{-}1x + 4\right) + \left(1x^2 + {}^{-}3x + 2\right) \\ &= \left(2x^2 + 1x^2\right) + \left(^{-}1x + {}^{-}3x\right) + (4 + 2) \\ &= 3x^2 + {}^{-}4x + 6 \quad \text{OR} \quad 3x^2 - 4x + 6 \end{aligned}$$

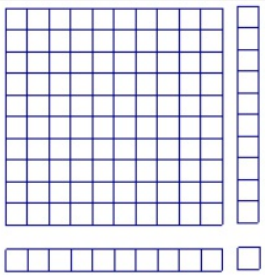


Whole Number Multiplication

$$\begin{array}{r} 2\ 1 \\ \times\ 1\ 7 \\ \hline \end{array}$$

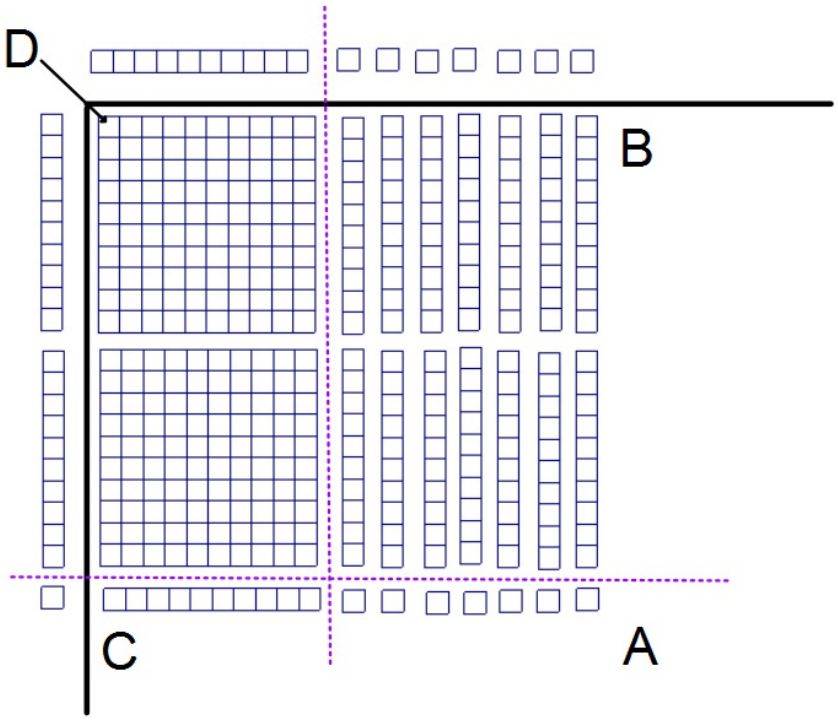
means 21 rows of 17.





Whole Number Multiplication (cont.)

$$\begin{array}{r} 2\ 1 \\ \times\ 1\ 7 \\ \hline \end{array}$$ **means 21 rows of 17.**



$$\begin{array}{rrrl} & 2 & 1 & \\ \times & 1 & 7 & \\ \hline & & 7 & \textbf{Region A} \\ 1 & 4 & 0 & \textbf{Region B} \\ & 1 & 0 & \textbf{Region C} \\ 2 & 0 & 0 & \textbf{Region D} \\ \hline 3 & 5 & 7 & \end{array}$$

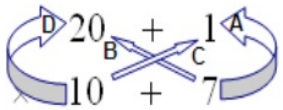
Whole Number Multiplication (cont.)

$$\begin{array}{r} 21 \\ \times\ 17 \\ \hline \end{array}$$

7	**Region A**
140	**Region B**
10	**Region C**
200	**Region D**
357	

$$\begin{array}{r} 21 \\ \times\ 17 \\ \hline \end{array} \Rightarrow \begin{array}{r} 20 + 1 \\ \times\ 10 + 7 \\ \hline \end{array}$$

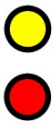
$$\Rightarrow \begin{array}{r} 20 + 1 \\ \times\ 10 + 7 \\ \hline \end{array}$$



7	A
140	B
10	C
200	D
357	

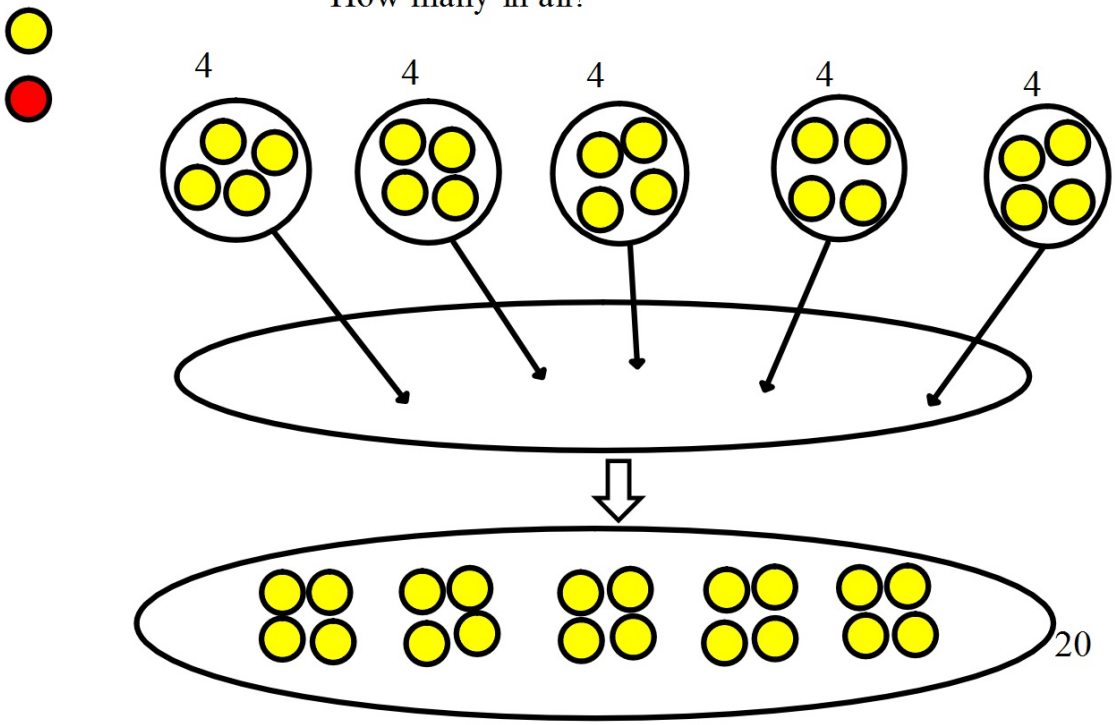
Integer Multiplication

Chip Model: 5×4 means to combine 5 groups of 4 yellow chips.
How many in all?



Integer Multiplication (cont.)

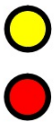
Chip Model: 5×4 means to combine 5 groups of 4 yellow chips.
How many in all?



So, $5\times4=20$

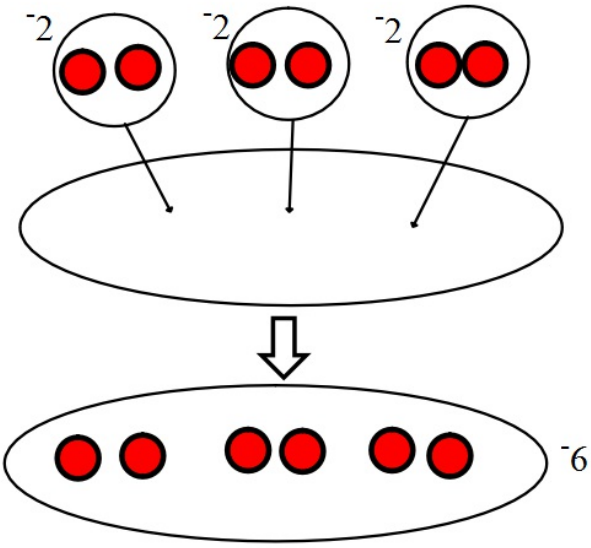
Integer Multiplication (cont.)

$3 \times {}^{-}2$ means to combine 3 groups of 2 red chips.
How many in all?



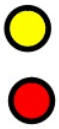
Integer Multiplication (cont.)

$3 \times {}^{-}2$ means to combine 3 groups of 2 red chips.
How many in all?



So, $3 \times {}^{-}2 = {}^{-}6$.

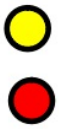
Integer Multiplication (cont.)



-3×2 means take away 3 groups of 2 yellow chips.
What is the remaining value?

Integer Multiplication (cont.)

$^{-}3 \times {}^{-}2$ means take away 3 groups of 2 red chips.
What is the remaining value?



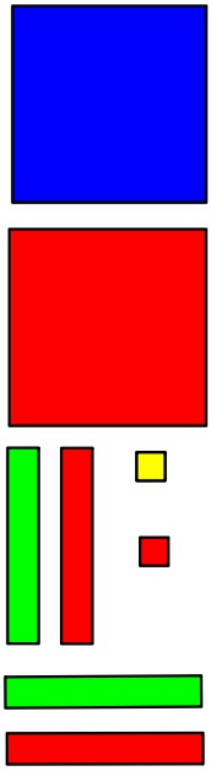
Integer Multiplication (cont.)

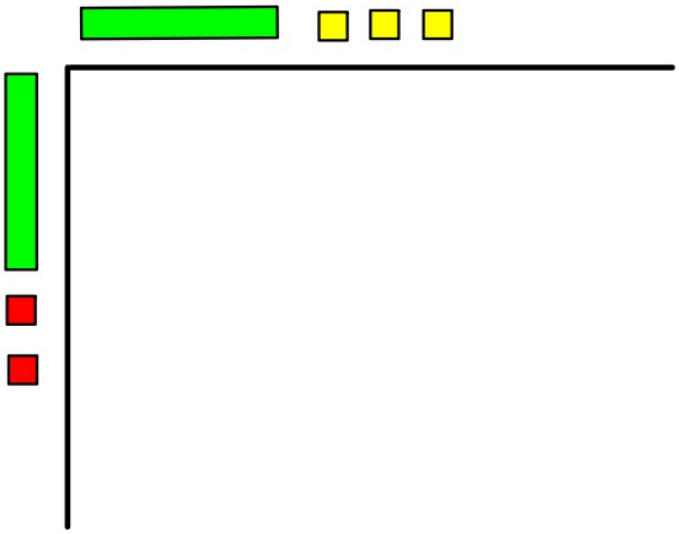
When multiplying two integers with the same signs (both positive or both negative): The product will be positive.

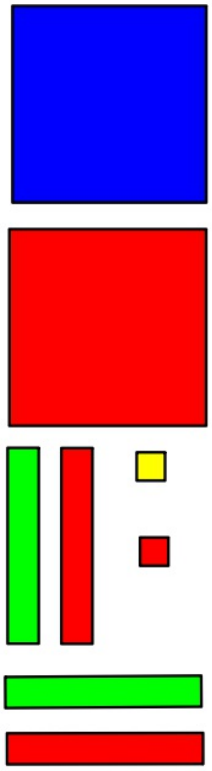
When multiplying two integers with different signs (one positive and one negative):The product will be negative.

Algebra Tile Multiplication (cont.)

$(x-2)(x+3)$ means $(x-2)$ rows of $(x+3)$

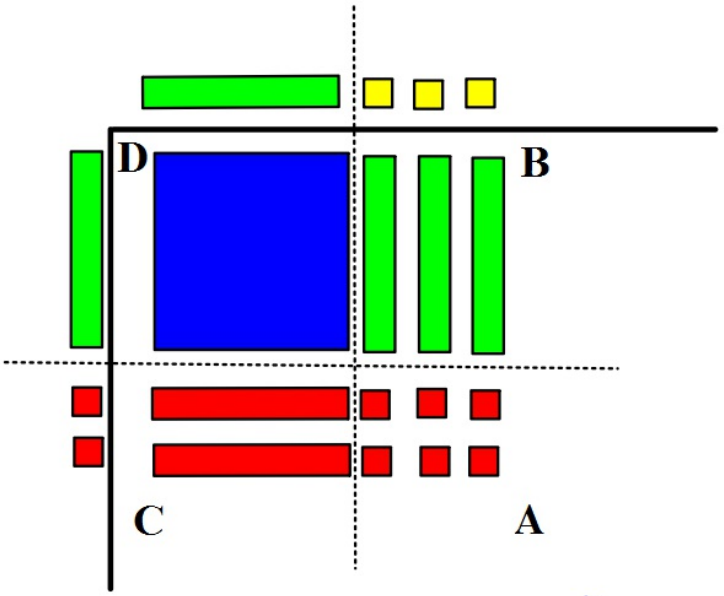




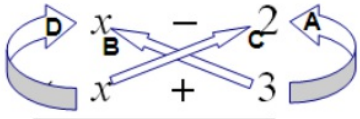


Algebra Tile Multiplication (cont.)

$(x\text{-}2)(x\text{+}3)$ means $(x\text{-}2)$ rows of $(x\text{+}3)$



$$(x - 2)(x + 3) = x^2 + 3x - 2x - 6$$
$$= x^2 + 1x - 6$$



		-6	A
	$3x$		B
	$-2x$		C
x^2			D
x^2	$+x$	-6	