QUALITY QUESTIONING

FOR

EFFECTIVE MATHEMATICS TEACHING

A Presentation by

DesLey V. Plaisance, Ph. D.

Department of Mathematics

desley.plaisance@nicholls.edu

NICHOLLS STATE UNIVERSITY

NCTM 2015 Annual Meeting and Exposition

Nicholls State University
Thibodaux, LA

Nicholls State University

OVERVIEW

- ***WHAT IS QUALITY QUESTIONING?***
- ***PURPOSE OF QUALITY QUESTIONING***
- ***STANDARDS FOR MATHEMATICAL PRACTICE***
- ***QUALITY QUESTIONING within MATHEMATICAL STRANDS***
- ***DISCUSSION/QUESTIONS***

GOAL OF PRESENTATION

For each audience member to not only *think about the questions that are posed as mathematical problems*,

but also to *think about the questions asked during the problem-solving process*

and *to think about the questions asked after students believe they have solved the problem*.

WHAT IS QUALITY QUESTIONING?

QUALITY QUESTIONING?

- …NOT a simple tool for extracting memorized information Walsh and Sattes, 2011
- …is a dynamic process through which a teacher intentionally engages students in both cognitive and metacognitive operations Walsh and Sattes, 2011
- …invites students to think and to understand as well as sharing their "mathematical journey" with other students and teachers Schuster and Anderson, 2004
- …sets the stage for meaningful classroom discussion and learning Schuster and Anderson, 2004

QUALITY QUESTIONING?

Careful, intentional, and mindful questioning is one of the most powerful tools a skillful teacher possesses.

Costa and Kallick, 2000

THE "LOOK" OF QUALITY QUESTIONS

- Assist students to make sense of the math
- Are open-ended
- Empower students to unravel misconceptions
- Require application of facts and procedures AND encourage students to make connections and generalizations
- Are accessible to all students
- Lead students to **WONDER** more about the math

Schuster and Anderson, 2004

STANDARDS FOR MATHEMATICAL PRACTICE

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STANDARDS FOR MATHEMATICAL PRACTICE

- **MP1** Make sense of problems and persevere in solving them.
- **MP2** Reason abstractly and quantitatively.
- **MP3** Construct viable arguments and critique the reasoning of others.
- **MP4** Model with mathematics.
- **MP5** Use appropriate tools strategically.
- **MP6** Attend to precision.
- **MP7** Look for and make use of structure.
- **MP8** Look for and express regularity in repeated reasoning.

PURPOSE OF QUALITY QUESTIONING

PURPOSE OF QUALITY QUESTIONS

Assess Formatively

Monitor Progress

Engage Students

Differentiate Instruction

Encourage Student Metacognition

PURPOSE OF QUALITY QUESTIONS

Check for understanding to guide instruction

Assess Formatively

Assess students' academic performance to evaluate instructional effectiveness

Monitor Progress

Increase student skills such as critical thinking and communicating

Engage Students

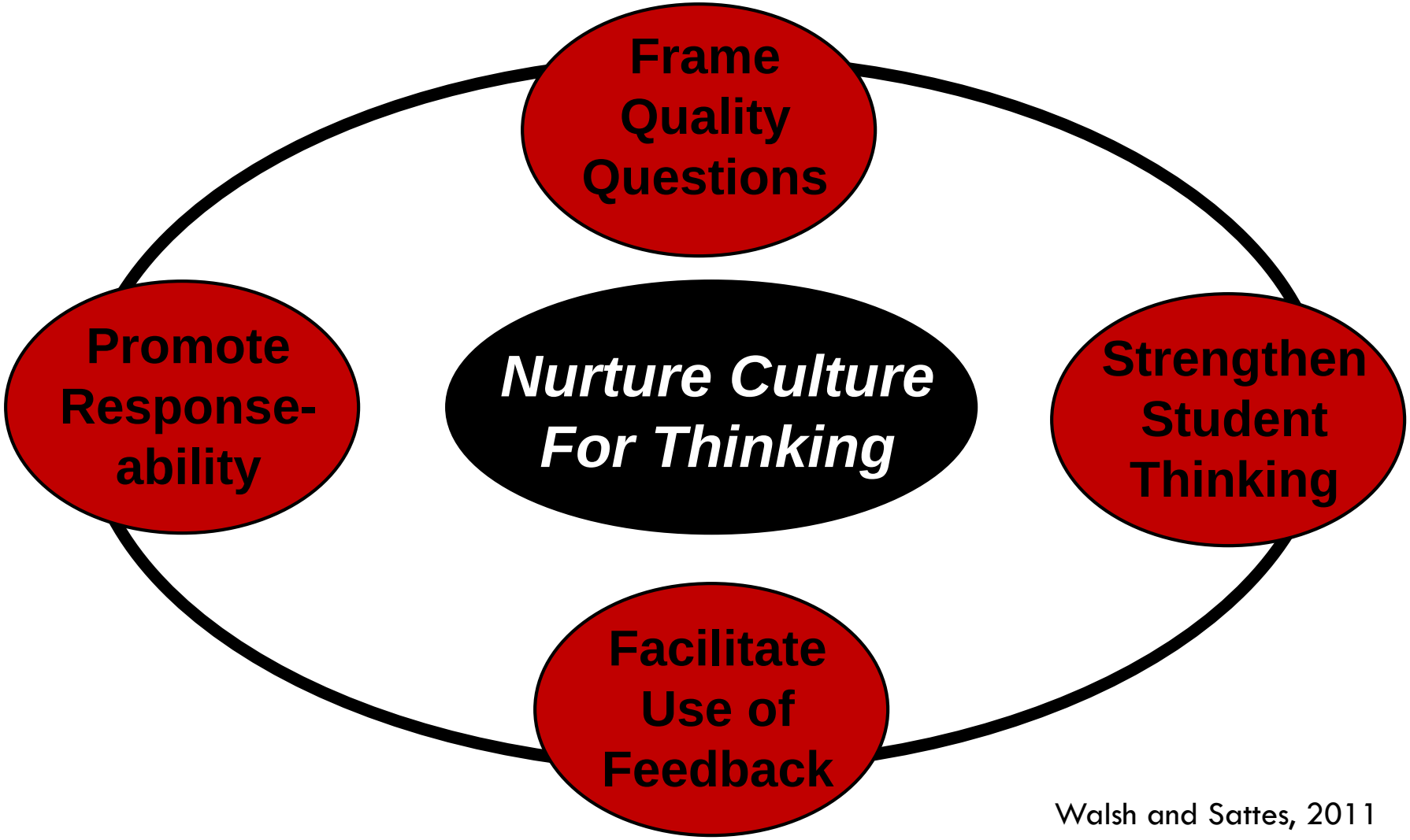
Customize instruction but have all students working on the same objective

Differentiate Instruction

Assist students in recognizing what is valuable about different strategies

Encourage Student Metacognition

PURPOSE OF QUALITY QUESTIONS



Walsh and Sattes, 2011

QUALITY QUESTIONING WITHIN DIFFERENT MATHEMATICAL STRANDS

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CONTENT STRANDS

- NUMBER & OPERATIONS
- ALGEBRA
- GEOMETRY
- MEASUREMENT
- DATA ANALYSIS & PROBABILITY

QUALITY QUESTIONS within NUMBER AND OPERATIONS

Some Big Ideas

- There are many ways to represent numbers.
- Numbers indicate how many or how much.
- Number benchmarks are helpful in relating numbers and estimating amounts.

Small, 2009

QUALITY QUESTIONS within NUMBER AND OPERATIONS

OPEN QUESTIONS

Different Ways to Represent Numbers

1) The answer is 56. What is the question?
2) You add two numbers and the sum is 1½. What are the two numbers?
3) Describe 10 thousand in as many different ways as you can.
4) Create a sentence that includes the following words: 20, percent, some, 100

QUALITY QUESTIONS within NUMBER AND OPERATIONS

OPEN QUESTIONS

Numbers Tell How Many or How Much

1) What makes 1 a special number?
2) A number describes the number of students in an elementary classroom. What might the number be?
3) A number can be written .33333333…. What do you know about the size of the number?
4) The square root of a number can be easily found. What might the number be?

QUALITY QUESTIONS within NUMBER AND OPERATIONS

OPEN QUESTIONS

Number Benchmarks are Helpful in Relating Numbers and Estimating Amounts

1) Choose two numbers to compare. Tell which is the smaller number and how you know it is the smaller number.
2) You divide two numbers and the quotient is almost 3. What could the two numbers be? Explain your answer.
3) Choose a decimal and a percent. Tell which is smaller. Explain your answer.

Teaching Tip

"One of the simplest strategies for differentiating instruction is allowing students to choose the numbers with which they will work."

Small, 2009 (p. 26)

Teaching Tip

"If a mathematically strong student always seems to select values that make a question too easy, the teacher should allow the student make the initial choice, but then challenge him or her to try other values as well."

Small, 2009 (p. 31)

QUALITY QUESTIONS within NUMBER AND OPERATIONS

OPEN QUESTIONS

There are many different ways to perform each operation.

1) Make up an addition problem that includes the digits 2, 3, and 4.

2) You multiply two fractions and the denominator of the quotient is 24. What could the fractions be?

Teaching Tip

"Teachers need to provide a significant amount of computational practice so students will gain procedural fluency. Rather than assigning large numbers of similar questions, posing one open question can efficiently create opportunities to practice in the context of lively class discussion."

Small, 2009 (p. 30)

QUALITY QUESTIONS within ALGEBRAIC THINKING

Some Big Ideas Relating to Algebra

- Patterns are all around us in the everyday world.
- Arranging information in charts and tables can make patterns easier to see
- Variables can be used to describe relationships.

Small, 2009

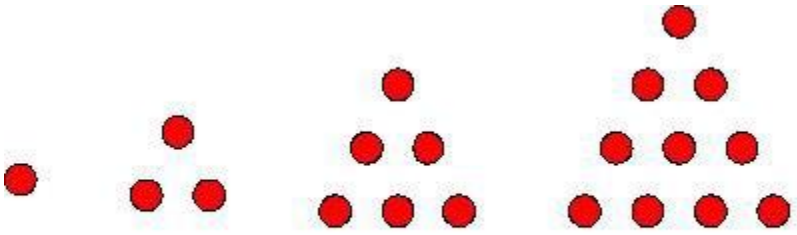
QUALITY QUESTIONS within ALGEBRAIC THINKING

Task 1

The Math Club has a way of greeting each other at the club meetings. They have a secret handshake. If each member shakes hands with each other member exactly once, how many handshakes will there be if there are 2 people in the club? 4 people? 10 people?

Task 2

The triangular numbers can be viewed by arranging dots in triangular patterns. How many dots will there be in the eleventh sketch?



QUALITY QUESTIONS within ALGEBRAIC THINKING

Task 1

Make up two different "equations" that use "variables" and are true ALL of the time.

Examples:

♦ + 0 = ♦

3 ♦ = ♦ + ♦ + ♦

Task 2

Make up two different "equations" that use "variables" and are true only SOME of the time.

Examples:

□ + 2 = 5

2□ = 0

Teaching Tip

Parallel tasks are a good way to ask open questions with same idea but "are designed to suit the needs of students at different developmental levels. The tasks are similar enough in context that all students can participate fully in a single follow-up discussion." Note that when two tasks are given, alternating the difficulty of the tasks is recommended. Students will not know which is the "simpler" task and will consider both possibilities.

Small, 2009 (p. 136)

QUALITY QUESTIONS within GEOMETRY Focus on REASONING and PROOF

- Not an isolated topic to be taught at a specific time of the year – but a "way of thinking" that should be integrated into all parts of the mathematics curriculum
- Essential part of a rich mathematical task in which students share and justify their reasoning
- Active questioning by teachers supporting students and moving them forward without "doing the thinking" for them
- Classroom interactions in which students reason mathematically, justify ideas, and evaluate their thinking

Annenberg Learner (www.learner.org)

REASONING AND PROOF?

Mathematics as reasoning is a focus on providing the why, the explanation, the clarity to what we're doing in mathematics. Not trying to learn things to be done just in isolation without any other way to validate than using the student's answer key or the teacher's authority. In this way, we're trying to have student say, "I can show you why it's true."

Hank Kepner (Past NCTM President)

Retrieved from Annenberg Learner (www.learner.org)

REASONING AND PROOF?

Proof is the essence of mathematics – it is just what makes mathematics valid. The making of conjectures about the use of mathematics and the justification of conjectures as solutions. Anything less can not present the true nature of mathematics.

Classrooms must empower students to make conjectures, create solutions, and justify the methods for attaining the results; instead of focusing on justifying their use from teacher taught procedures.

Robert Sweetland

Retrieved from www.homeofbob.com

QUALITY QUESTIONS within REASONING AND PROOF

Secret Sort

Student or teacher creates a collection of about five shapes that fit a certain rule.

Students have to determine rule – but have them explain why and explain their reasoning.

Van de Walle, Karp, & Bay-Williams, 2013

Questioning Following Problem Solving

Think about the type of questions to ask students in order to support development of reasoning skills.

- How do you know the shapes fit your rule?
- Is there another rule that would fit the group of shapes? What is that rule? How do you know the shapes fit this rule?
- Does anyone agree or disagree with the rule? Tell why.

QUALITY QUESTIONING

Creating rich mathematical tasks and asking questions <u>related to justification</u> encourage students to examine what constitutes a valid justification for a general statement. Such reasoning bridges the mathematical activity that occurs at the elementary level and the reasoning that is necessary at the high school level.

Lannin, Barker, and Townsend, 2006, p. 443

QUALITY QUESTIONS within REASONING AND PROOF

Pick one of the following statements and prove that it is true or false:

Statement 1: Two odd numbers always have a sum that is even.

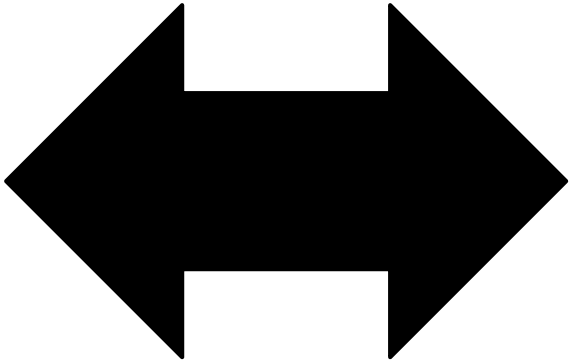
$$2a + 1 + 2b + 1$$
$$= 2a + 2b + 1 + 1$$
$$= 2a + 2b + 2$$
$$= 2(a + b + 1)$$

Statement 2: Two odd numbers always have a sum that is odd.

$$3 + 5 = 8$$

QUALITY QUESTIONS within REASONING AND PROOF

Prove that this shape has symmetry. You can use a transparent mirror or a ruler. Small, 2009



Research has indicated that using a ruler to test for symmetry is a more developmentally demanding task than using a transparent mirror. Small, 2007

QUALITY QUESTIONS within MEASUREMENT

Some Big Ideas

- The same object can be described using different measurements.
- Measurement formulas allow us to rely on measurements that are simpler to access to calculate measurements that are more complicated to access.
- Knowledge of the size of benchmarks assists in measuring.

Small, 2009

QUALITY QUESTIONS within MEASUREMENT

OPEN QUESTION

The same object can be described using different measurements.

A shape has an area of 400 square inches. What could be the length and width of the shape?

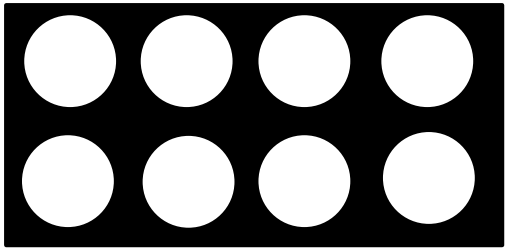
QUALITY QUESTIONS within MEASUREMENT

Parallel Tasks

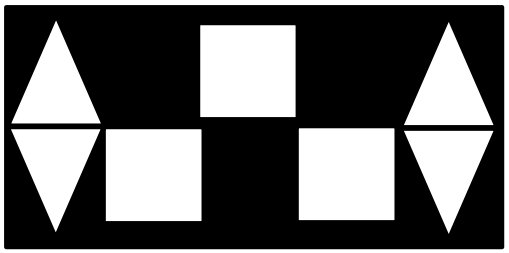
Measurement formulas allow us to rely on measurements that are simpler to access to calculate measurements that are more complicated to access.

What fraction of the rectangle is white?

Option 1:



Option 2:



QUALITY QUESTIONS within MEASUREMENT

OPEN QUESTION

Knowledge of the size of benchmarks assists in measuring.

A rectangular prism has a volume of about 200 cubic centimeters. Describe the dimensions of a cylinder with about the same volume.

QUALITY QUESTIONS within DATA ANALYSIS AND PROBABILITY

Some Big Ideas

- Graphs are powerful data displays because they quickly reveal a great deal of information.
- An experimental probability is based on past events, but a theoretical probability is based on analyzing what could happen. An experimental probability approaches a theoretical one when enough random samples are used.

Small, 2009

QUALITY QUESTIONS within DATA ANALYSIS AND PROBABILITY

Parallel Tasks

Graphs are powerful data displays because they quickly reveal a great deal of information.

Consider the set of data below which includes the ages of actors in a play production:

12, 3, 14, 8, 22, 2, 7, 19, 12, 22, 23, 12, 12, 3

Option 1.

Draw a line plot representing the data.

Option 2:

Create a histogram representing the data.

QUALITY QUESTIONS within DATA ANALYSIS AND PROBABILITY

OPEN QUESTION

An experimental probability is based on past events, but a theoretical probability is based on analyzing what could happen. An experimental probability approaches a theoretical one when enough random samples are used.

You roll two dice. You want an event that is only a bit less likely than rolling a difference of 2. What could it be?

Classroom-Tested Tip

Give students an opportunity to record their thinking and share with the rest of the class. Use a document camera to have students project their work from their notebooks for classmates to see. You can also use transparencies with overhead projectors. Use what is available to you to have students share their thinking and reasoning. Provide students with opportunities to "rethink" their work and revise if necessary. Allow them to incorporate new knowledge and thinking into their written explanations.

The Math Process Standard Series

Introduction to Reasoning and Proof (2008)

A Few More Teaching Tips

1) Don't just ask a student for an answer
   – ask how and why!

2) "Listen" to what a student says AND
   what he doesn't say.

3) Take the opportunity to "extend" the concept by
   asking related questions.

4) Seize every teachable moment and make the most
   of it. That moment may never come around again!

DISCUSSION

QUESTIONS

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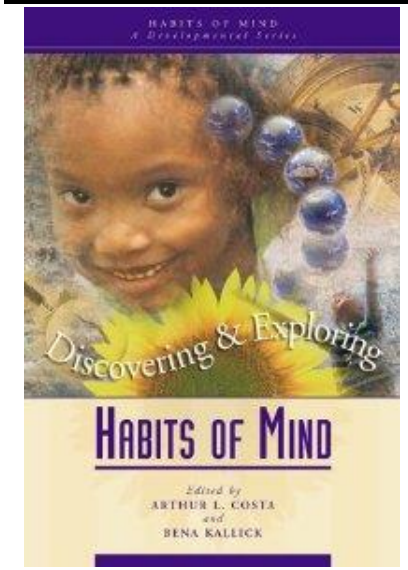

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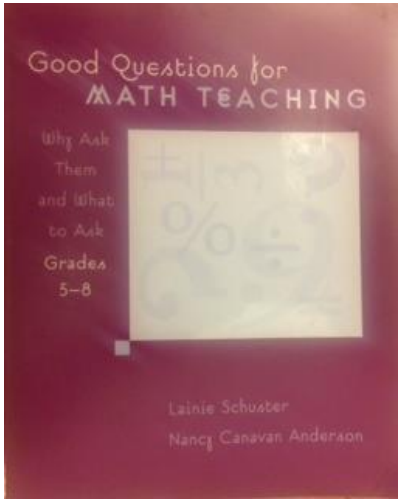

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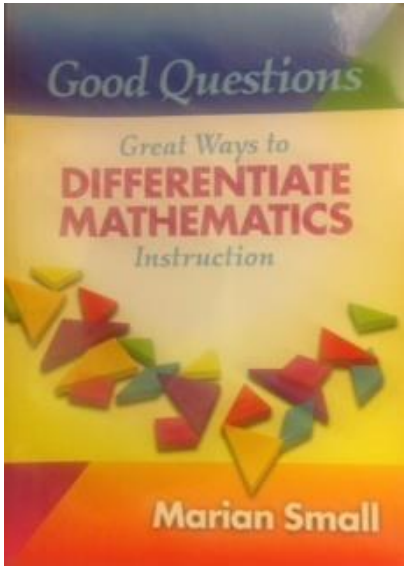

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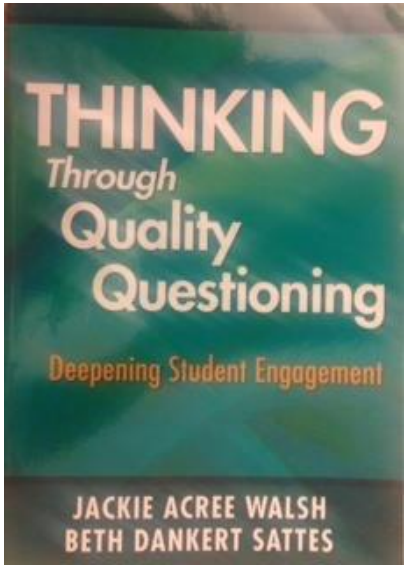

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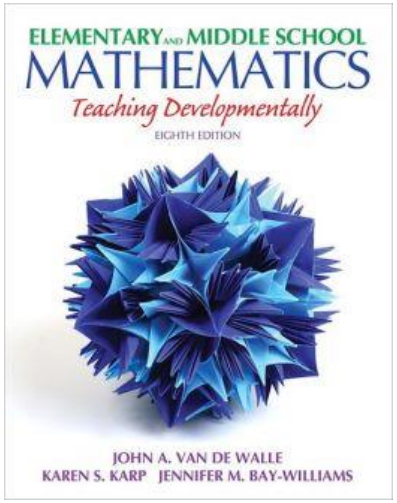

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CONTACT INFO

DesLey V. Plaisance, Ph.D.
Nicholls State University
Thibodaux, LA
desley.plaisance@nicholls.edu