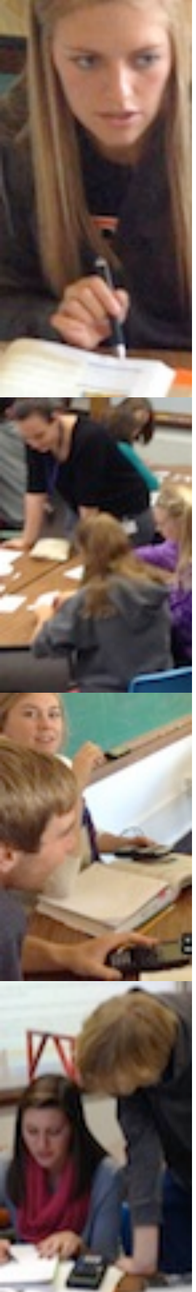

GeoGebra + Complex Number Arithmetic: Implementing CCSSM

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NCTM Conference
16 April 2015
BCEC 160A

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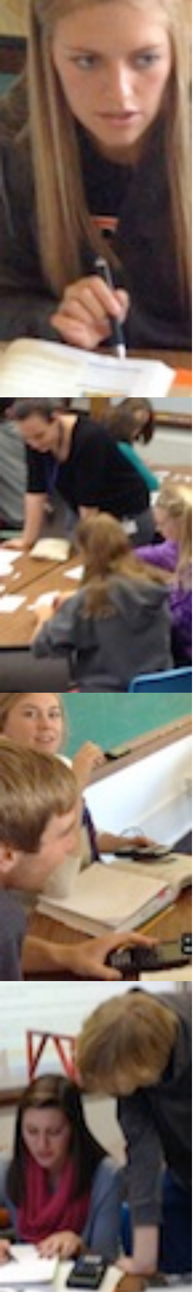

Objectives

- Develop a
  rich,
  meaningful, and
  connected framework
  for understanding complex numbers for teachers/students/learners.
- Use technology appropriately.

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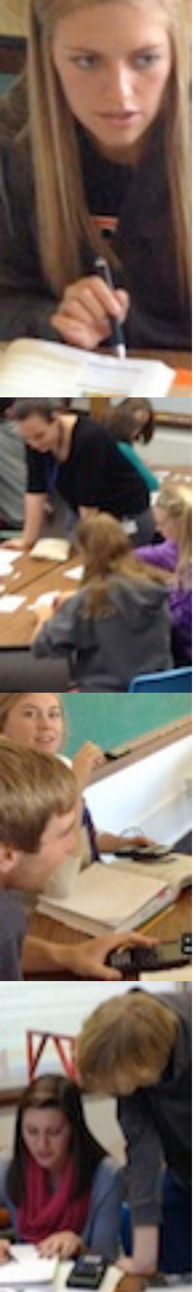

CCSS-Mathematics

High School Conceptual Category: Number and Quantity

Domain: The Complex Number System

Clusters:

- **Perform arithmetic operations with complex numbers.**
- **Represent complex numbers and their operations on the complex plane.**
- Use complex numbers in polynomial identities and equations.

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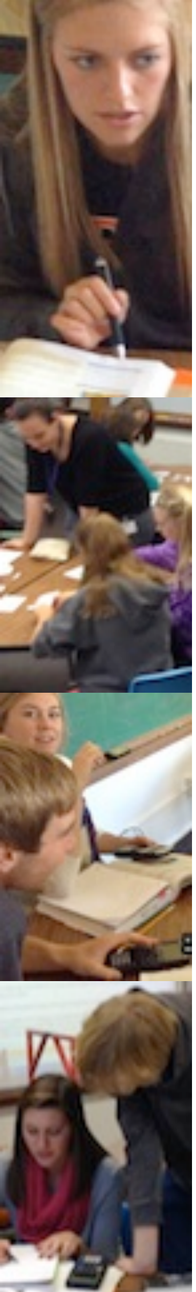

Table of Results In Complex Number Interpretation from Future Math Teachers in Junior Year Geometry Course (Similar results for Inservice Teachers)

Computation	Percent Success	Percent Graph
Add: (2+3i) + (4+2i)	100	50
Subtract: (5+4i) – (3+2i)	87	25
Multiply: (2+3i) * 2i	87	0
Divide: (4+6i)/2i	50	0

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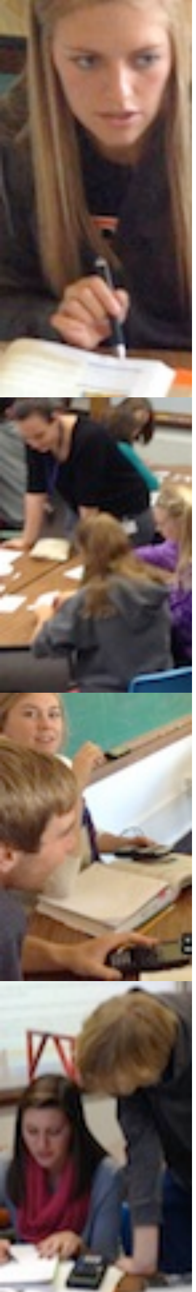

Standards

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

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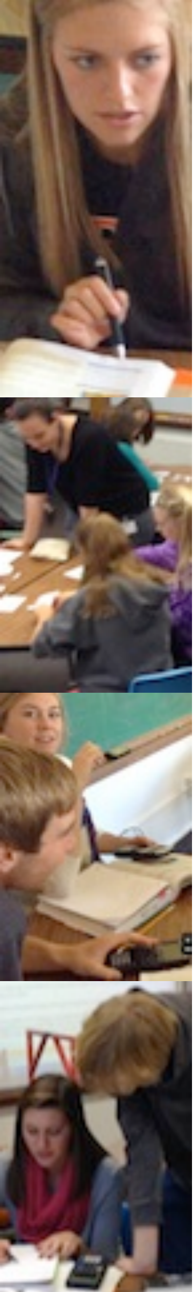

In a prior life:

- Before the *1980 Agenda for Action* or the *1989 NCTM Standards*, a common method was to tell students the following:

  $i = \sqrt{-1}$

  $i^2 = -1$

  $i^3 = -i$

  $i^4 = 1$

  And every complex number has the form $a + bi$ where a and b are real.

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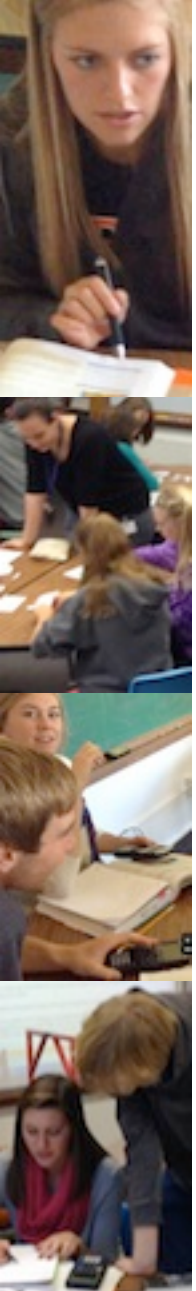

Before-

- To add complex numbers, add the real parts, the *a*’s together and the *b*’s together.

$$(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2) i$$

- To multiply complex numbers, FOIL

$$(a_1 + b_1 i)(a_2 + b_2 i) = (a_1 a_2) + (a_1 b_2) i + (b_1 a_2) i + (b_1 b_2) i^2$$

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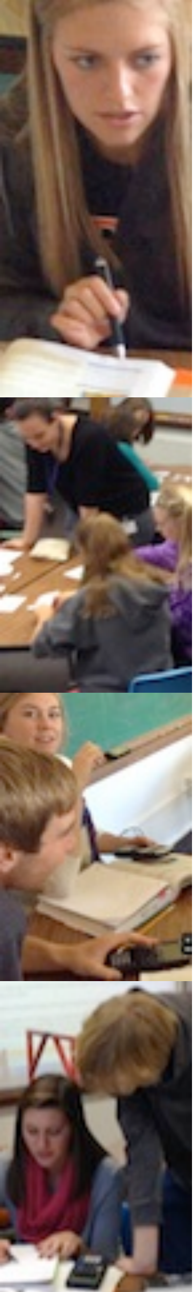

Standards

5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

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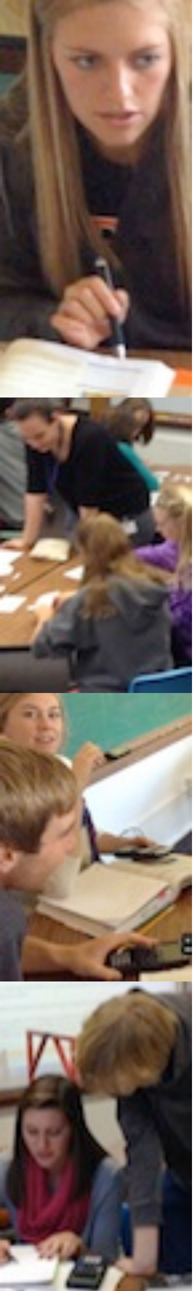
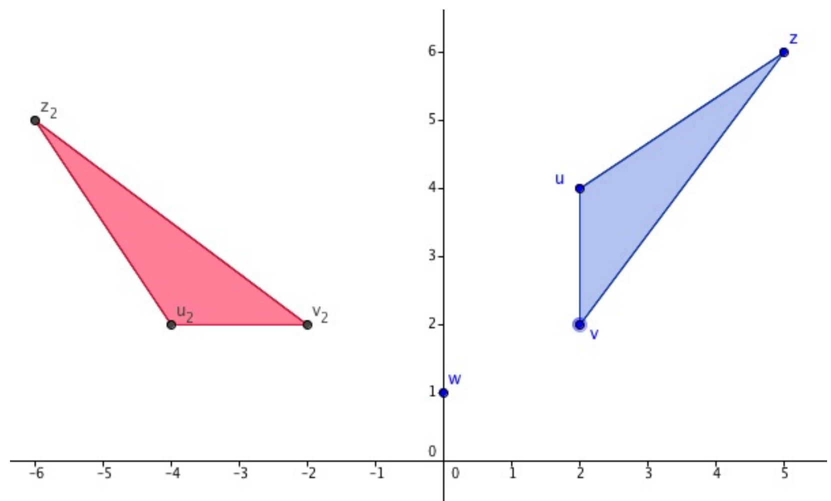


GeoGebra

free application software from geogebra.org

How could we transform the blue triangle into the red triangle?



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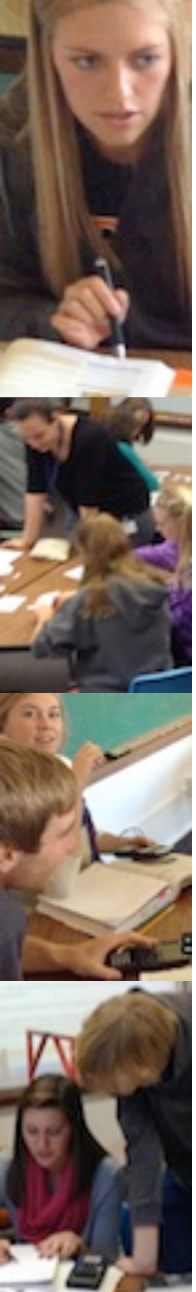
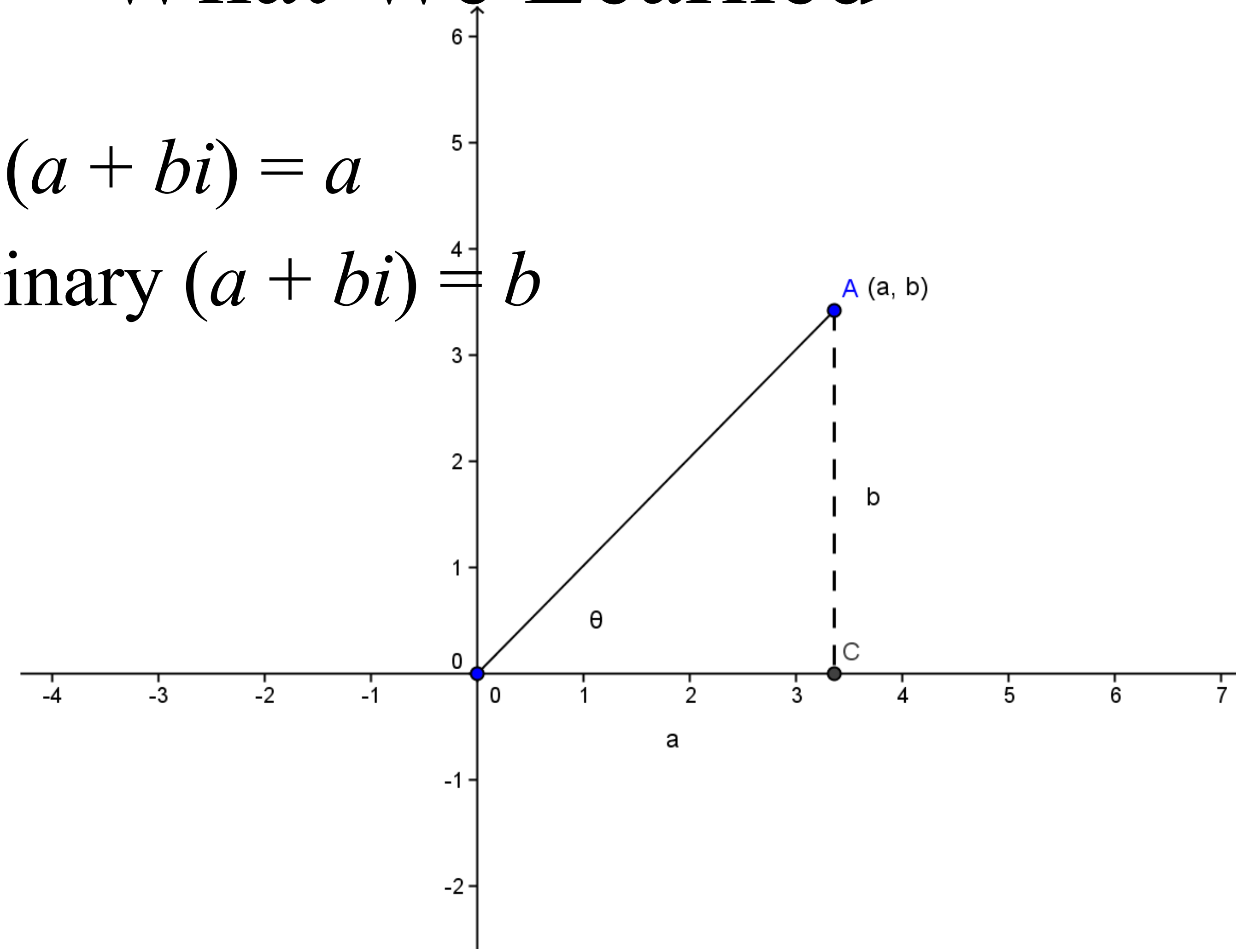


What We Learned

- $\text{Real}\,(a + bi) = a$
- $\text{Imaginary}\,(a + bi) = b$



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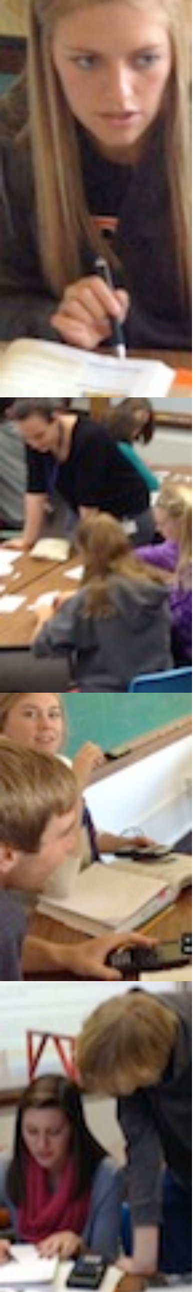

What We Learned

Let $m = |a + bi|$ and $n = |c + di|$. Also, let $\theta = \arg(a + bi)$ and $\varphi = \arg(c + di)$.

$$(a + bi) \times (c + di) = m \cdot n \operatorname{cis}(\theta + \varphi)$$

$$(a + bi) \div (c + di) = m/n \operatorname{cis}(\theta - \varphi)$$

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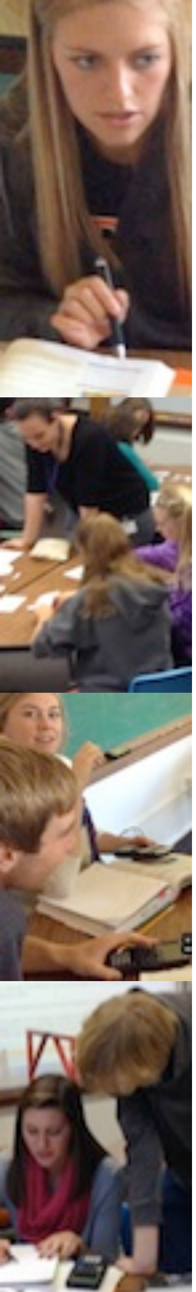

Geometric effects of complex multiplication

- Multiplying z by w scales the modulus of z by a factor of $|w|$ and increases the argument of z by $\arg(w)$ radians (counterclockwise).
- Definition: A *dilation* of a polygon is the uniform scaling of the sides of a polygon.
- Definition: A *rotation* is a uniform radial reorientation of the vertices of a polygon.

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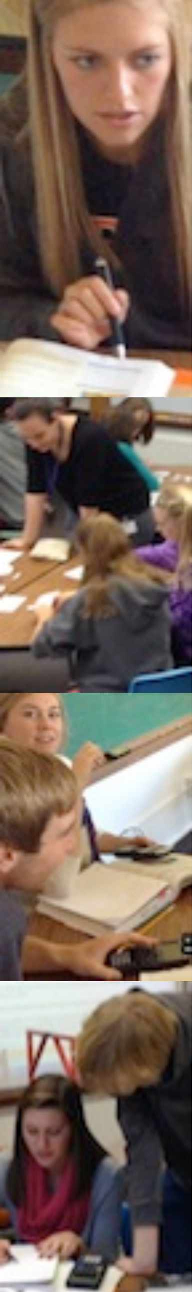

Geometric effects of complex arithmetic

- Definition: Two geometric shapes are *similar* provided that one can be obtained from the other by dilation, possibly with additional translation, rotation and reflection.
- Definition: Two geometric shapes are *congruent* provided that one can be transformed into the other by an isometry, i.e., a combination of translations, rotations and reflections, i.e. no dilation allowed.

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Questions