

*Your family is invited to attend...*  
**Phelps Luck Elementary School's**  
**2<sup>nd</sup> Annual**

*Creative Strategies for Math at Home Series*

<b>Why?</b>	<b>Who?</b>	<b>When?</b>
<p>Come check out how math instruction looks in the 21<sup>st</sup> Century classroom.</p> <p>We will look at how students are learning to solve computation both mentally and with creative strategies on paper.</p>	<p>All Phelps Luck Families Grades 1-5</p> <p>Separate sessions are planned for children and adults.</p>	<p style="text-align: center;"><b>Mental Math</b>  Thursday, October 9<sup>th</sup>  6:30 – 7:30</p> <p style="text-align: center;"><b>Basic Fact Night</b>  🕒 Families of Students Learning 3<sup>rd</sup> Grade Math Only🕒  Thursday, October 23<sup>rd</sup>  6:30 – 8:00</p> <p style="text-align: center;"><b>Addition and Subtraction</b>  Thursday, November 6<sup>th</sup>  6:30 – 8:00</p> <p style="text-align: center;"><b>Multiplication and Division</b>  Thursday, November 20<sup>th</sup>  6:30 – 8:00</p>

.....  
Student Name(s): \_\_\_\_\_

Grade(s): \_\_\_\_\_

\_\_\_\_\_ Yes! We will attend! Number of family members attending: \_\_\_\_\_

You may attend any or all of the sessions, each evening will be focused on the topics above.

Circle the night(s) you will be attending:

October 9<sup>th</sup>

October 23<sup>rd</sup>

November 6<sup>th</sup>

November 20<sup>th</sup>

\_\_\_\_\_ Check here for transportation (taxi) to be provided to and from school

\_\_\_\_\_ Interpreter (Language: \_\_\_\_\_)

\_\_\_\_\_ Childcare for NON School Age Child

Please return this form by Friday, September 26<sup>th</sup> to Mrs. Krownapple

RE: Math Night Series

# Sample Evaluation for Parent Feedback

Phelps Luck Elementary School  
Howard County Public School System  
FAMILY PROGRAM EVALUATION

Name of program: Grade 3 Just the Facts Night      Date: Thursday, October 23<sup>rd</sup>

Presenters: The 3<sup>rd</sup> Grade Math Team & The PLES Math Team

*Please circle your response to the following statements:*

**1. I understood the program topic.**

Strongly Agree      Agree      Disagree      Strongly Disagree

**2. I felt the information was presented clearly.**

Strongly Agree      Agree      Disagree      Strongly Disagree

**3. I was able to engage in the activities presented.**

Strongly Agree      Agree      Disagree      Strongly Disagree

**4. I will be able to use the information presented with my child at home.**

Strongly Agree      Agree      Disagree      Strongly Disagree

**\*5. I plan on using the take-home materials with my child.**

Strongly Agree      Agree      Disagree      Strongly Disagree      N/A

**\*6. Title I provides additional funds for family involvement. Do you feel this program was a good use of these funds?**

Strongly Agree      Agree      Disagree      Strongly Disagree

*Please rate the overall effectiveness of today's program:*

<b>Very Effective</b>		<b>Somewhat Effective</b>		<b>Not Effective</b>
<b>5</b>	<b>4</b>	<b>3</b>	<b>2</b>	<b>1</b>

ADDITIONAL COMMENTS, CONCERNS, CONSIDERATIONS.

# Mental Math Strategies

PLES Math Team

2014-2015

## What is Mental Math?

Calculations that are done in a student's head without the guidance of pencil and paper, calculators, or other aids. Mental math is often used as a way to calculate and estimate quickly, using math facts that a student has committed to memory.

## Mental Math Night

October 9, 2014, 6:30 - 7:30

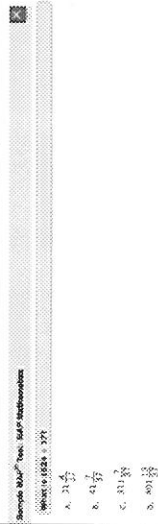
- I. Introduction/Rationale
  - a. Understanding Common Core
  - b. What is Mental Math?
  - c. Mental Math Strategies
- II. Mental Math and Testing
  - a. MAP example
  - b. PARCC example
- III. Mental Math Resources
  - a. Greg Tang Video Clip
  - b. Website
  - c. Printable Games
  - d. Literature
- IV. Did You Know? Video Clip
- V. Visit Computer Lab
- VI. Evaluation

## Understanding Common Core

The following clip will provide insight to the change of the State Curriculum to the Common Core Curriculum.

## How does Mental Math connect to Map?

Grades 2-5 MAP example



Would you like to see more? Visit: [http://www.nwea.org/warmup/sample\\_tests/map\\_short.html](http://www.nwea.org/warmup/sample_tests/map_short.html)

## Mental Math Strategies



## How does Mental Math connect to PARCC? Grade 3 PARCC Example

Name: \_\_\_\_\_ Score: \_\_\_\_\_

Grade 3 Mathematics (Fluency)

3.P.1.1.1.1

Check all the equations that are true.

$8 \times 9 = 81$   
  $54 \div 7 = 24 \times 5$   
  $7 \times 3 = 21$   
  $2 \times 3 = 6$   
  $96 \div 7 = 36 \times 4$

Would you like to see more? Visit: <http://practice.parcctestnav.com/>

## Mental Math Resource

Greg Tang is invested in creative mental math strategies. He shares his strategies via literature, websites, and games.

[Check this out!](#)

## Greg Tang Website

## Greg Tang Game

**Smart Sums™**  
Master Sums of 10

Roll a 20-sided die and write the number in the "top" of the "10".  
Use the number in the "top" of the die to find the missing number. You sometimes + 10

Roll a Die	$\square + \square = 10$	Roll a Die	$\square + \square = 10$
Roll a Die	$\square + \square = 10$	Roll a Die	$\square + \square = 10$

## Greg Tang Literature

## Final Thoughts...

[Did you know?](#)

Math Series! Mark your calendars

Grade 3 Basic Fact Night - Thursday, October 23rd @ 6:30

Addition and Subtraction Night - Thursday, November 6th @ 6:30

Multiplication and Division Night - Thursday, November 20th @ 6:30



# Mental Math Strategies

Removal or Counting Back  
Multiplication  
Partial Products  
Partial Quotients  
Making Tens  
Repeated Subtraction or Sharing/Dealing Out  
Adding Up  
Doubles/Near-Doubles  
Adding Up in Chunks  
Adjusting One Number to Create an Easier Problem  
Repeated Addition or Skip Counting  
Making Landmark or Friendly Numbers  
Subtraction  
Doubling and Halving  
Keeping a Constant Difference  
Breaking Each Number into its Place Value  
Division  
Breaking Factors into Smaller Factors  
Counting All/Counting On  
Compensation  
Place Value or Negative Numbers  
Proportional Reasoning  
Multiplying Up

**Just the Facts Night**  
**Grade 3**  
**Phelps Luck Elementary**  
**October 23, 2014**  
**6:30 – 8:00**

**Agenda:**

- **Presentation with Rationale**
  - Located in Cafeteria 6:30-6:45
  - Review importance of fact proficiency
  - Share goals for the evening
  
- **Rotation:**
  1. 6:45 -- 7:05
    - 1: Wendy Crockett, Karen Simcock (Wendy's Room)
      - SAW Games
  2. 7:05 -- 7:25
    - 2: Gretchen Gray, Casey Morseberger (Gretchen's Room)
      - Card and Dice Games
  3. 7:25 – 7:45
    - 3: Kelly Fleck (iMac Lab)
      - Intro to Smart Pages
      - Ipad and Iphone Apps
      - Math Fact Websites
      - [www.xtramath.com](http://www.xtramath.com)

**Juli and Kelly Krownapple - help with rotation transitions**

- **Conclusion**
  - Located in the Cafeteria 7:45-8:00
  - Parents fill out feed back forms.

# How can I help my child? : SMART PAGES

<http://smart.wikispaces.hcps.org>



- Wiki Home
- Recent Changes
- Pages and Files
- Members
- Settings
- Search

SMART Pages Welcome

- What Your Child Will Learn:**
- Kindergarten Common Core
  - Grade 1 Common Core
  - Grade 2 Common Core
  - Grade 3 Common Core
  - Grade 4 Common Core
  - Grade 5 Common Core
  - Grade 6 Common Core

**Mobile Apps**

- iTunes for grades K-2
- iTunes for grades 3-5
- Android

**For Parents:**

- Vision 2018: HCPSS Strategic

## SMART Pages

Welcome to the New Howard County Public Schools

### What are the SMART pages?

SMART standards for Student and Family Resources and Tools. They are designed to help students find information and resources for instruction in Howard County Public Schools. Click on the column to visit grade levels and resources.

[Learn by grade level](#)

- Common Core standards
- Vocabulary
- Activities for home
- Online Links

[Mobile Apps that Support Mathematics](#)

[Information for Parents:](#)

- Common Core State Standards
- Assessment Information
- College and Career Advantage
- Online and Print Resources

## What Your Child Will Learn

- skills/concepts taught
  - vocabulary
- activities at home
- links for games

## Mobile Apps

## Resources for Parents

- Vision 2018
- Information about Common Core
  - Books

## Websites for Basic Fact Practice: +, -, X, ÷

[www.funbrain.com/math/](http://www.funbrain.com/math/)

<http://www.mathfactcafe.com/>

<http://www.multiplication.com/games/all-games>

<http://www.factmonster.com/>

<http://www.playkidsgames.com/games/mathfact/mathFact.htm>

<http://www.aplusmath.com/games/index.html>

<http://www.coolmath4kids.com/>

<http://www.fun4thebrain.com/>

<http://www.arcademics.com/>

[http://www.learninggamesforkids.com/math\\_multiplication\\_games.html](http://www.learninggamesforkids.com/math_multiplication_games.html)

<http://xtramath.org/>



# Creative Strategies for Math at

## Home

### Addition and Subtraction

Tonight's Agenda 11/6/14

6:30 – 6:55

Technology Session

7:00 – 7:25

Rotation 1

*Partial Sums & Differences*

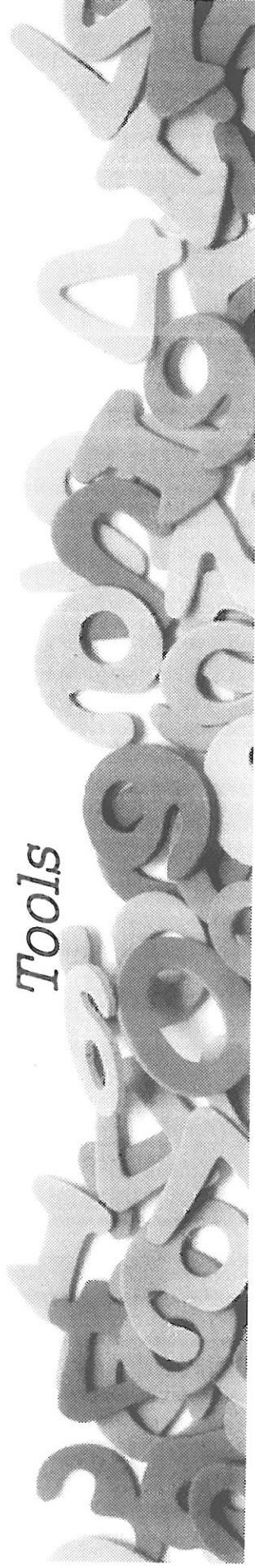
*Tools*

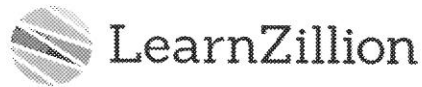
7:30 – 7:55

Rotation 2

*Partial Sums & Differences*

*Tools*





Go to [www.learnzillion.com](http://www.learnzillion.com).

LearnZillion is a website that provides teachers and parents with video lessons showing what your student needs to learn each year. The lessons include a 3-5 minute long lesson video, downloadable slides, and other resources for practice and assessment.

Your student can watch the videos alone or with you.

Some things to consider while watching the video:

- What is this lesson about?
- Review
- Mistakes to avoid
- The main thing to learn

You may also work on the “guided practice” video or other activities that come with the lesson.

Thank you for supporting your student’s learning at home.

## Adding and Subtracting with Partial

$$37 + 62$$

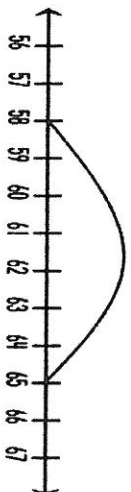
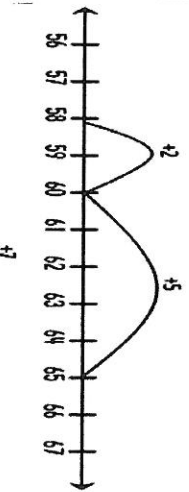
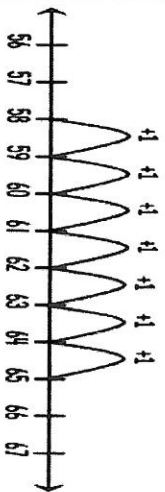
$$827 + 149$$

$$97 - 45$$

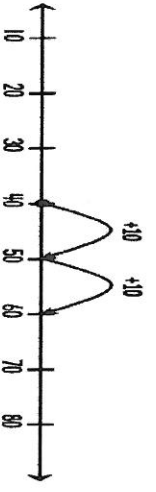
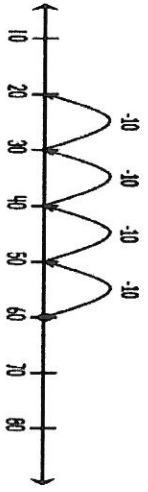
$$745 - 231$$



**Addition and Subtraction on Number Lines**  
 Number lines are useful for adding and subtracting. At first, students will jump in increments of 1. They will develop more efficient strategies and jumps as their number sense improves. This experience develops mental math skills and supports work with algorithms. These number lines show  $58 + 7$  with jumps of 1, by breaking 7 apart, and one single jump.



When subtracting, we can count back from a number or we can count up from one number to another to find the difference. The number lines show examples for  $60 - 40$ .



**Addition: Partial Sums**

Students may break numbers apart to add tens with tens and ones with ones. They find "parts" of the sum and can then combine to find the total sum.

$58 + 5 =$ $50 + (8 + 5) =$ $50 + 13 = 63$	
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**Subtraction: Place Value**

Students apply their knowledge of subtracting single digit numbers and place value to subtract multiples of 10.

$50 - 20 = 30$ $5 \text{ tens} - 2 \text{ tens} = 3 \text{ tens}$ $50 - 20 = 30$	$5 - 2 = 3$
--	-------------

**Addition: Properties**

Students can add numbers by using the properties of addition to rearrange or group the numbers.

$4 + 8 + 6 =$ $4 + 6 + 8 =$ $10 + 8 = 18$	<p><b>Commutative Property</b>                  rearranging the numbers  <math>8 + 6 = 6 + 8</math></p>	<p><b>Associative Property</b>                  grouping the numbers  <math>4 + 6 + 8</math></p>
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# Developing Computational Fluency

Grade 1

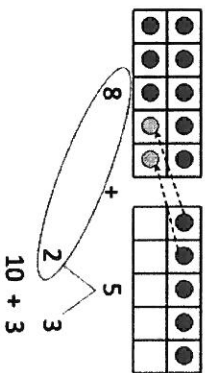


Elementary Mathematics Office  
 Howard County Public School System

This brochure highlights some of the methods for developing computational fluency. For more information about computation and elementary mathematics visit <http://smart.wikispaces.hcps.org>

### Addition: Making Tens

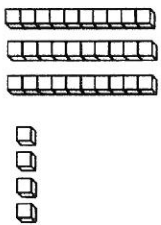
Making tens is an important strategy for fluency. Students work with ten-frames (below). They combine dots to fill a ten-frame. Below, we moved 2 dots from 5 to make a ten. The result is  $10 + 3$ .



We can apply the combinations of tens to add other numbers. In  $58 + 5$ , we might break apart 5 into  $2 + 3$  and then add the 2 to 58 making the next ten which is 60.

$$\begin{array}{r} 58 + 5 \\ 2 \quad 3 \\ \hline 60 + 3 \end{array}$$

### Working with Base Ten Blocks



34

Base ten blocks are a math tool that help us build numbers. The rod is equal to 10 and the single cube is equal to 1.

### Hundreds Chart

The hundreds chart is a useful math tool. Students can add and subtract on the hundreds chart.

$$14 + 30 = 44$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

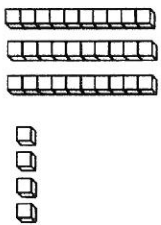
A student started at 14 and jumped down 3 rows of 10 (30) to equal 44.

$$90 - 40 = 50$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A student started at 90 and jumped up 4 rows of 10 (40) to equal 50.

### Working with Base Ten Blocks



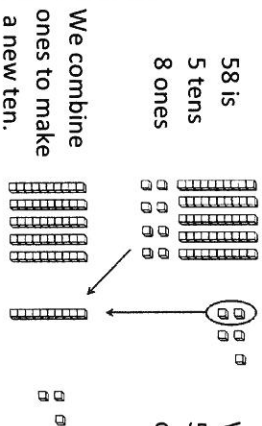
34

Base ten blocks are a math tool that help us build numbers. The rod is equal to 10 and the single cube is equal to 1.

### Addition with Base Ten Blocks

$$58 + 5$$

58 is 5 tens 8 ones. We add 5 more ones.



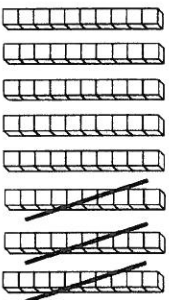
We combine ones to make a new ten.

In  $58 + 5$ , we make a new ten from the ones (8 + 2). This leaves us with 6 tens and 3 ones leftover. So,  $58 + 5 = 63$

### Subtraction with Base Ten Blocks

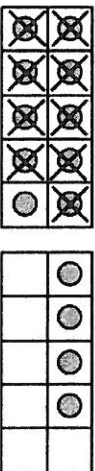
$$80 - 50$$

In first grade, we work with taking tens from tens. Below, there are 8 tens (80) and we take away 3 tens (30) leaving 5 tens (50). So,  $80 - 30 = 50$



### Subtraction Using Ten Frames

We can also use tens for subtraction. The ten-frame below shows 14. To subtract 14-9, we can break 9 into 4 and 5. We can subtract the 4 from 14 giving us 10. Then, 5 less than 10 is 5. Eventually, this will become automatic for students.



$$14 - 9$$

$$4 \quad 5$$

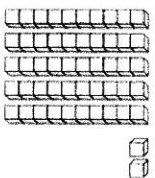
$$14 - 4 = 10, \text{ then } 10 - 5 = 5$$

$$\text{So, } 14 - 9 = 5$$

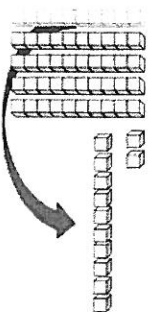
### Subtraction: Regrouping

Sometimes a problem requires a group of ten to be regrouped so that ones can be taken away. Consider  $52 - 28$

This is 52. There aren't enough ones to take away.



This is 52 after regrouping 1 ten to make 4 tens and 12 ones.



Now, we can take away 8 ones from 12 ones.

### Subtraction: Adjusting

We can use "friendlier numbers" to solve problems.  $500 - 239$  can be challenging to regroup. But the difference between these numbers is the same as the difference between  $499 - 238$ . Now, we don't need to regroup.

$$\begin{array}{r} \text{(Original problem)} \\ 500 \\ - 239 \\ \hline \end{array} \qquad \begin{array}{r} - \\ 239 \\ - 1 \\ \hline \end{array} = 261$$
  
$$\begin{array}{r} \text{(Compensation)} \\ 499 \\ - 238 \\ \hline \end{array} = 261$$

### Subtraction: Unknown Addend or Think Addition

Many people think of subtraction as unknown addition problems. Instead of finding the difference, they think about what the missing addend is. Consider the problem below.

(Original problem)

$$347 - 249 = ?$$

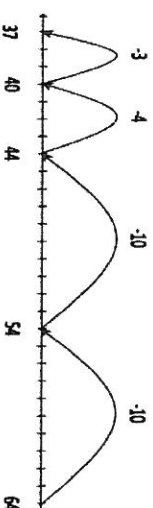
(Think Addition) "249 plus what number equals 347?"

$$249 + ? = 347$$

### Subtraction: Number Line

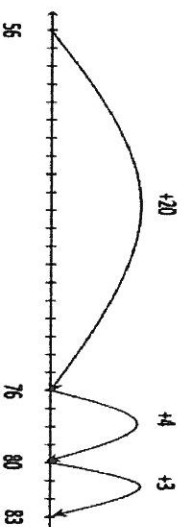
#### Counting Back on the Number Line:

Consider  $64 - 27$ . We can start at 64 and count back using friendly numbers. After counting back 27, we land on 37. So,  $64 - 27 = 37$ .



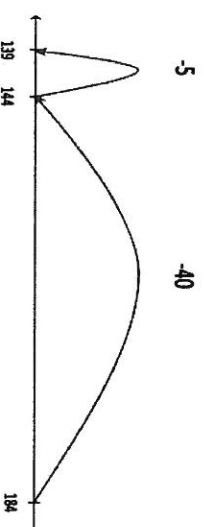
#### Counting Up on the Number Line:

Consider  $83 - 56$ . Another strategy is to find the difference by counting up. To do this, start with 56 and count up to 83. We can add 20, add 4, and add 3 (27). So,  $83 - 56 = 27$ .



#### Using an Open Number Line:

Students begin to work with open or empty number lines as they become more comfortable with numbers and number lines. These number lines do not have individual tick marks. Consider  $184 - 139 = 45$ .



# Developing Computational Fluency

Grade 2

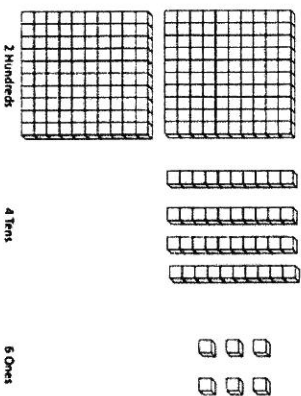


Elementary Mathematics Office  
Howard County Public School System

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### Working with Base Ten

Computation in 2<sup>nd</sup> grade is grounded in place value. Students need to understand what the digits in a number represent.

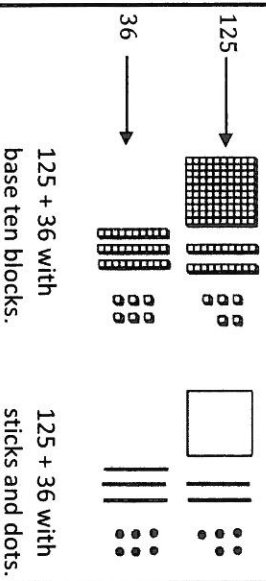


The image above shows 246 as 2 hundreds, 4 tens, and 6 ones. We must understand that numbers can be taken apart in different ways to use them efficiently. Some other ways to think about 246 include:

- 2 hundreds, 4 tens, and 6 ones ( $200 + 40 + 6$ )
- 23 tens and 16 ones ( $230 + 16$ )
- 22 tens and 26 ones ( $220 + 26$ )
- 1 hundreds, 14 tens, and 6 ones ( $100 + 140 + 6$ )

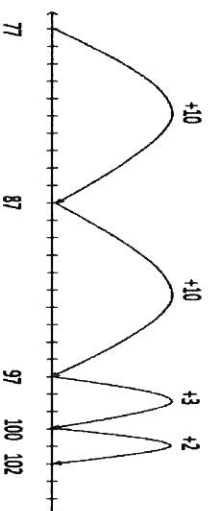
### Addition with Base Ten Blocks

We use base ten blocks when beginning to learn addition. Students may progress to drawing pictures of these blocks with sticks and dots. Later, students work with numbers only. Each picture below shows  $125 + 36$ .



### Addition with Number Lines

Number lines are useful for adding numbers. Students use them as they transition away from blocks. Numbers can be broken apart and added in different ways. The number line below shows one way to add  $77 + 25$ .



### Addition with Partial Sums

We can break apart addends by place value and then add the parts. This gives us partial sums that we can add back together at the end.

Consider  $248 + 345$

$$\begin{array}{r} 248 = 200 + 40 + 8 \\ + 345 = 300 + 40 + 5 \\ \hline 500 + 80 + 13 \\ 500 + 93 \\ 593 \end{array}$$

Partial sums is a strategy many people use to add numbers mentally.

### Addition: Adjusting

Sometimes, it makes sense to give a value from one addend to the other for a "more friendly" problem.

Consider  $28 + 37$

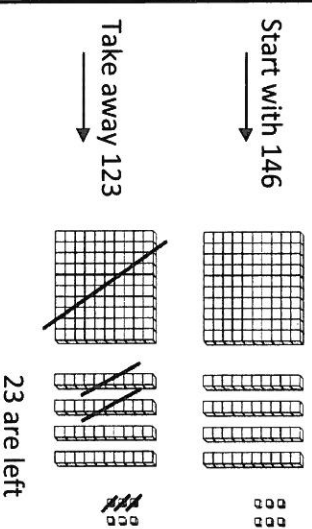
$$\begin{array}{r} 28 + 37 = 65 \\ +2 \quad -2 \\ 30 + 35 = 65 \end{array}$$

Consider  $49 + 27$

$$\begin{array}{r} 49 + 27 = 76 \\ +1 \quad -1 \\ 50 + 26 = 76 \end{array}$$

### Subtraction with Base Ten Blocks

We begin with the total amount when subtracting with base ten blocks. Consider  $146 - 123$ . We begin with 146. We take away or cross off 123. We are left with 23. So,  $146 - 123 = 23$ .



Start with 146

Take away 123

23 are left

### Subtraction: Partial Differences

Sometimes, it is more efficient to break apart a number and subtract the parts.

Consider  $436 - 128$

$$\begin{array}{r} 436 - 100 = 336 \\ 336 - 20 = 316 \\ 316 - 8 = 308. \end{array} \quad \text{So, } 436 - 128 = 308$$

We might break 128 into  $(100 + 20 + 8)$ .

Consider  $847 - 637$

We don't have to use place value. We might break 637 into  $(600 + 37)$ .

$$\begin{array}{r} 847 - 600 = 247 \\ 247 - 37 = 210 \end{array} \quad \text{So, } 847 - 637 = 210$$

Creative Strategies for Math at Home  
Multiplication and Division Alternative Algorithms  
Thursday, November 20, 2014  
Phelps Luck Elementary School

6:30 - 6:40 Welcome and Travel to Rooms  
PPT showing with kids doing math!

6:40 - 7:15 Rotation 1  
Partial Products (Kelly K. and Karen)  
Wendy Crockett's Room

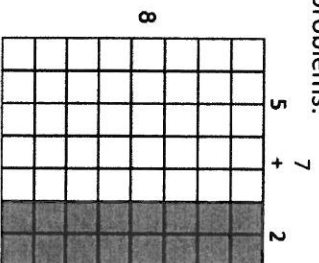
7:15 - 7:50 Rotation 2  
Partial Quotients (Gretchen and Kelly F.)  
Gretchen Gray's Room

### Multiplication: Area/Array Model

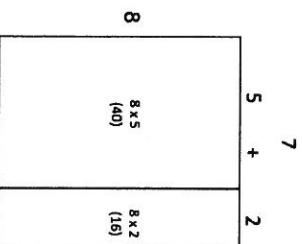
The area/array model for multiplication and the distributive property are used to solve multiplication problems.

Model for  $8 \times 7$ :

$$\begin{aligned} 8 \times 7 &= \\ (8 \times 5) + (8 \times 2) &= \\ 40 + 16 &= \\ 56 & \end{aligned}$$



This is the same model without inner squares. It is considered an "open model."



Students move from area/array models to working with partial products and the distributive property.

$$\begin{aligned} 8 \times 7 \\ (8 \times 5) + (8 \times 2) \\ 40 + 16 \\ 56 \end{aligned}$$

### Multiplication: Multiples of 10

$$\begin{aligned} 3 \times 1 &= 3 \\ 3 \times 1 \text{ ten} &= 3 \text{ tens} \\ 3 \times 10 &= 30 \end{aligned}$$

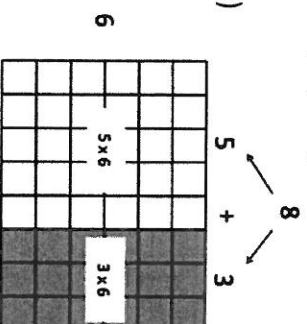
$$\begin{aligned} 3 \times 4 &= 12 \\ 3 \times 4 \text{ tens} &= 12 \text{ tens} \\ 3 \times 40 &= 120 \end{aligned}$$

### The Distributive Property

This property allows us to break apart factors. It can make computation more efficient. It will be used later in algebra.

In  $8 \times 6$ , we can break the 8 into  $(5 + 3)$ .  $8 \times 6$  becomes  $(5 \times 6) + (3 \times 6)$ .

$$\begin{aligned} 8 \times 6 \\ (5 \times 6) + (3 \times 6) \\ 30 + 18 \\ 48 \end{aligned}$$



### Division: Think Multiplication

Multiplication and division are related.

When working with division, it sometimes makes sense to "think multiplication."  $12 \div 4$  could be thought of as "4 times what equals 12."

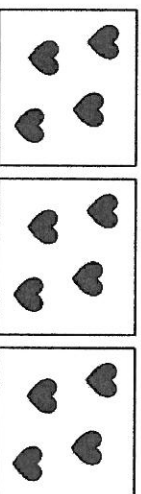
How many groups of 4 are in 12 hearts?

What is  $12 \div 4$ ?



What times 4 equals 12?

$3 \times 4 = 12$  so there are 3 groups of 4 hearts.



# Developing Computational Fluency

Grade 3



Elementary Mathematics Office  
Howard County Public School System

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### Addition: Partial Sums

Many times it is easier to break apart addends. Often it makes sense to break them apart by their place value. Consider  $248 + 345$

$$248 = 200 + 40 + 8$$

$$345 = 300 + 40 + 5$$

$$500 + 80 + 13 = 593$$

Sometimes we might use partial sums in different ways to make an easier problem.

Consider  $484 + 276$

$$484 = 400 + 84$$

$$276 = 260 + 16$$

$$660 + 100 = 760$$

### Addition: Adjusting

We can adjust addends to make them easier to work with. We can adjust by giving a value from one addend to another.

Consider  $326 + 274$ . We can take 1 from 326 and give it to 274.

$$326 + 274$$

$$\begin{array}{r} -1 \\ +1 \end{array}$$

$$\rightarrow 325 + 275 = 600$$

Consider  $173 + 389$ . We can take 27 from 389 and give it to 173 to make 200.

$$173 + 389$$

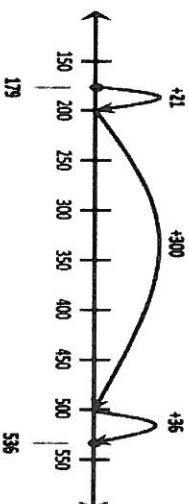
$$\begin{array}{r} +27 \\ -27 \end{array}$$

$$\rightarrow 200 + 362 = 562$$

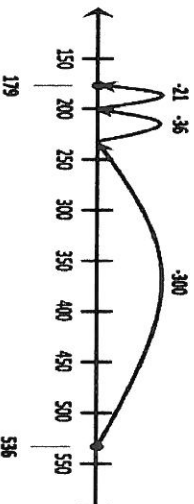
### Subtraction: Count Up or Count Back

When subtracting, we can count back to find the difference of 2 numbers. In many situations, it is easier to count up.

Consider  $536 - 179$



We can count up from one number to the other. The difference is  $300 + 21 + 36$  or 357. (above)



We can count back from one number to the other. The difference is  $-300$  (land at 236),  $-36$  (land at 200)  $- 21$  (end at 179).

### Subtraction: Adjusting

We can use "friendlier numbers" to solve problems.  $4,000 - 563$  can be challenging to regroup. But the difference between these numbers is the same as the difference between  $3,999 - 562$ . Now, we don't need to regroup.

$$\begin{array}{r} \text{(Original problem)} \\ \text{(Compensation)} \end{array} \begin{array}{r} 4,000 \\ -1 \end{array} \quad \begin{array}{r} - \\ -1 \end{array} \quad \begin{array}{r} 563 \\ -1 \end{array} =$$

$$3,999 \quad - \quad 562 = 3,437$$

### What Is Multiplication?

Multiplication has different representations based on the context. Regardless of the representation, the product of any 2 factors remains the same. Representations for 3<sup>rd</sup> grade include:

#### Repeated Addition:

$$6 + 6 + 6 + 6$$

$$4 + 4 + 4 + 4 + 4 + 4$$

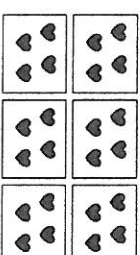
These examples are for  $6 \times 4$ .

#### Equal Groups / Sets:

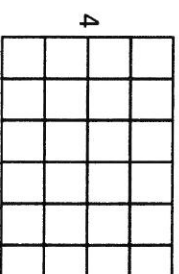
4 groups of 6 hearts



6 groups of 4 hearts



#### Area/Array Model:



$6 \times 4 = 24$  square units -or-  
 $4 \times 6 = 24$  square units

### The Commutative Property

This property allows us to reverse the order of factors. It is useful in many situations.

Examples above show that  $6 \times 4$  is equal to  $4 \times 6$  regardless of the representation.



# Algorithm: Partial Products



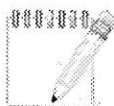
## When do I use it?

- Students use this algorithm to find products of multiplication problems.



## Description:

- Each factor is thought of as a sum of ones, tens, hundreds, and so on. Each part of one factor is multiplied by each part of the other factor.



## Example:

### Example 1

Multiply  $6 \times 43$

Think of 43 as 4 tens and 3 ones.

Multiply each part of 43 by 6

$$6 \text{ ones} \times 4 \text{ tens:} \quad 6 \times 40 = 240$$

$$6 \text{ ones} \times 3 \text{ ones:} \quad 6 \times 3 = 18$$

Add these two parts:  $240 + 18 = 258$

$$6 \times 43 = 258$$

### Example 2

Multiply  $25 \times 37$

Think of 25 as 2 tens and 5 ones

Think of 37 as 3 tens and 7 ones

Multiply each part of 25 by each part of 37

$$2 \text{ tens} \times 3 \text{ tens:} \quad 20 \times 30 = 600$$

$$2 \text{ tens} \times 7 \text{ ones:} \quad 20 \times 7 = 140$$

$$5 \text{ ones} \times 3 \text{ tens:} \quad 5 \times 30 = 150$$

$$5 \text{ ones} \times 7 \text{ ones:} \quad 5 \times 7 = 35$$

Add these four parts:  $600 + 140 + 150 + 35 = 925$

$$25 \times 37 = 925$$



## Milestones:

- If students can give a ballpark estimate of the product and understands place value and the distributive property, they are ready to move on to the traditional algorithm.

2x1

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

--	--

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

--	--

$$\underline{\quad} \times \underline{\quad} = \underline{\quad}$$

--	--

\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

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\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

--	--	--

\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

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\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

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\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

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\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

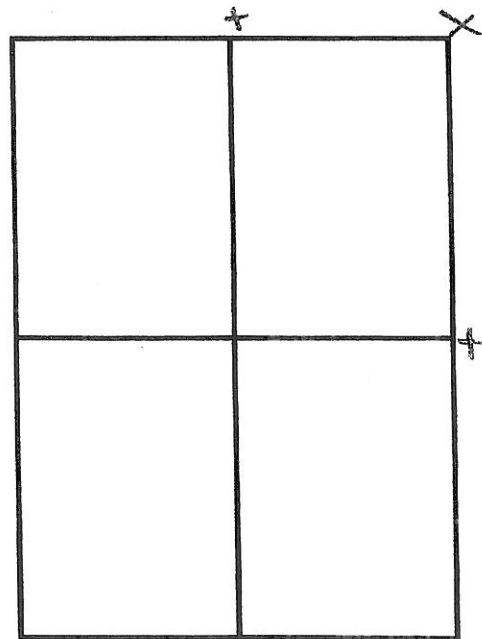
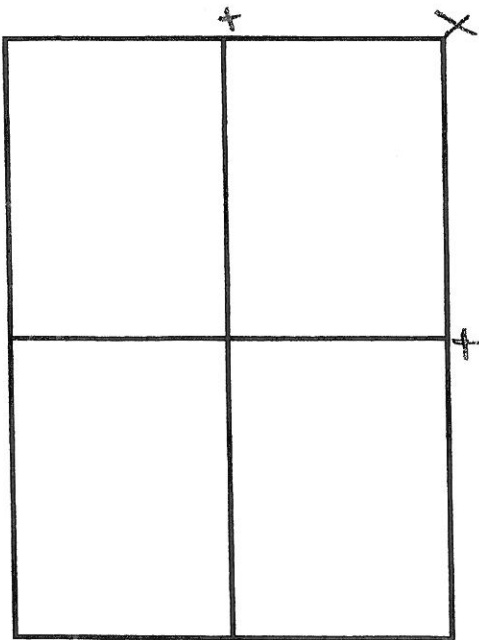
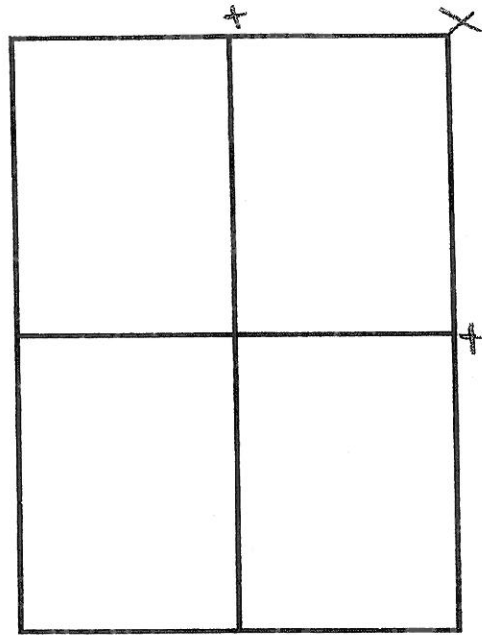
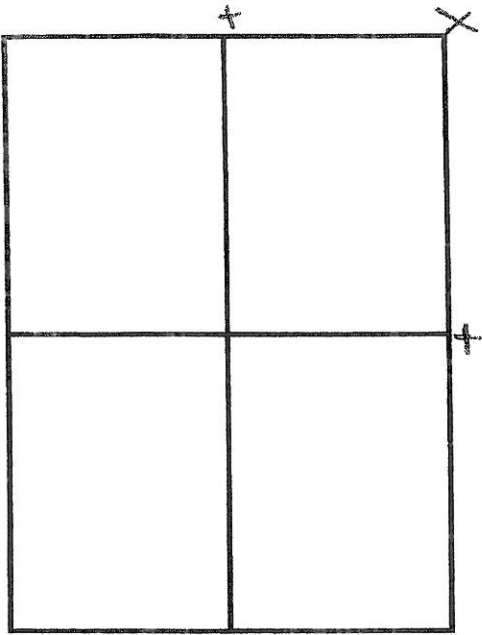
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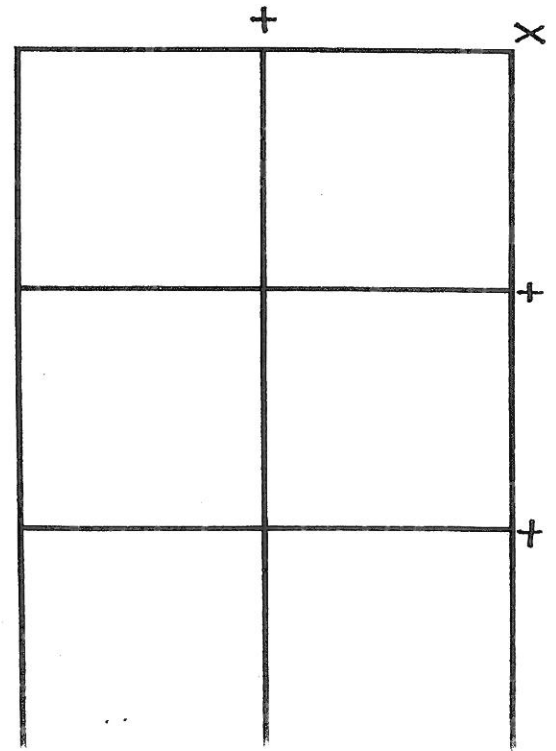
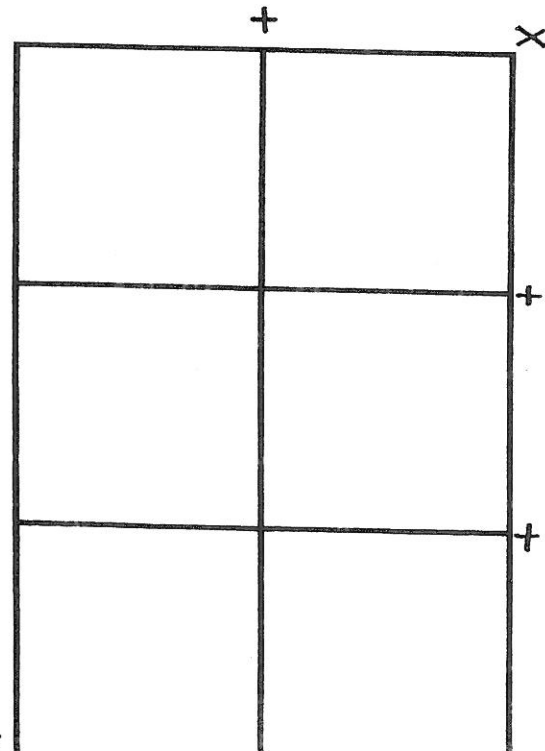
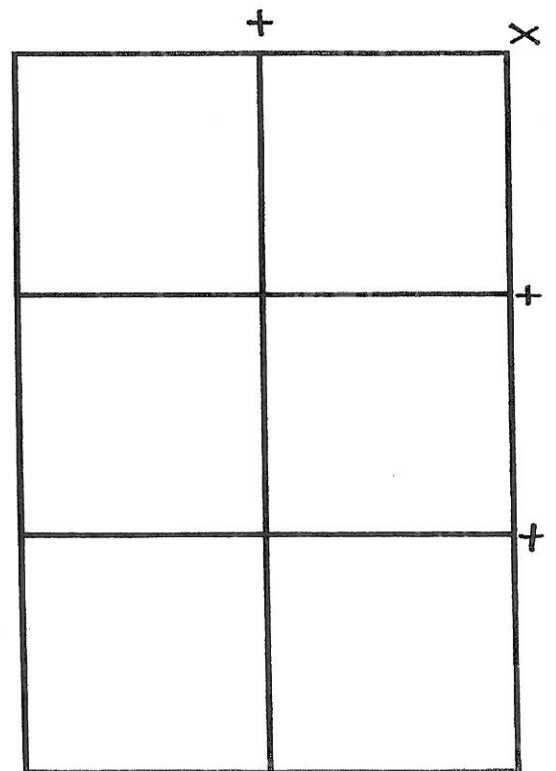
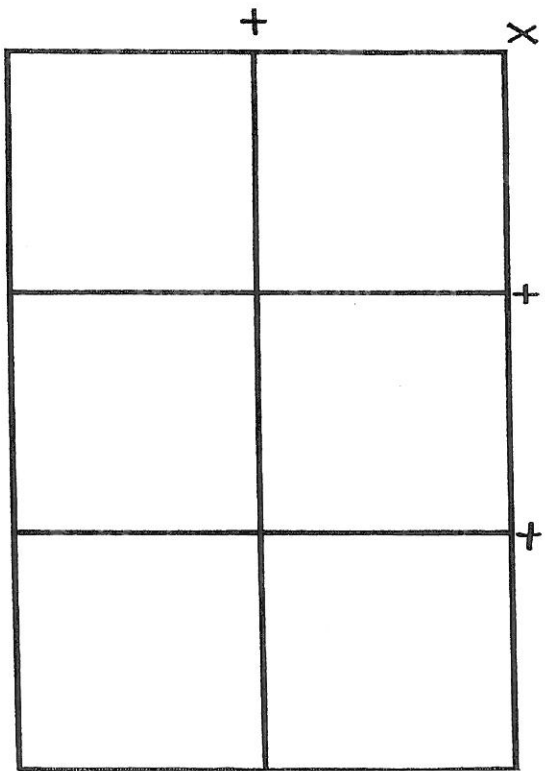
\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

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\_\_\_\_\_ X \_\_\_\_\_ = \_\_\_\_\_

--	--	--	--





Still Partial Product but organized like the standard algorithm

$$\begin{array}{r}
 \begin{array}{|c|c|} \hline 2 & 5 \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline 2 & 3 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|} \hline 1 & 5 \\ \hline \end{array} \quad (3 \times 5) \\
 \begin{array}{|c|c|c|} \hline & 6 & 0 \\ \hline \end{array} \quad (3 \times 20) \\
 \begin{array}{|c|c|c|} \hline 1 & 0 & 0 \\ \hline \end{array} \quad (20 \times 5) \\
 + \begin{array}{|c|c|c|c|} \hline & 4 & 0 & 0 \\ \hline \end{array} \quad (20 \times 20) \\
 \hline
 \begin{array}{|c|c|c|} \hline & 5 & 7 & 5 \\ \hline \end{array}
 \end{array}$$

Standard Algorithm

$$\begin{array}{r}
 \begin{array}{|c|c|} \hline 2 & 5 \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline 2 & 3 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|} \hline & 7 & 5 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline & 5 & 0 & \otimes \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|} \hline & 5 & 7 & 5 \\ \hline \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \\
 \hline
 \end{array}$$



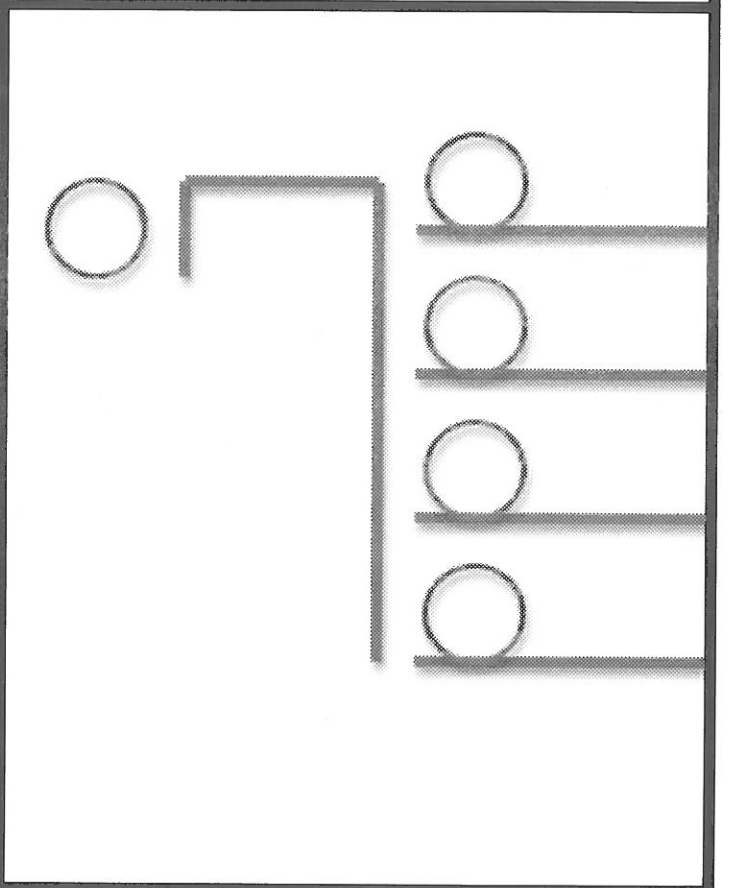
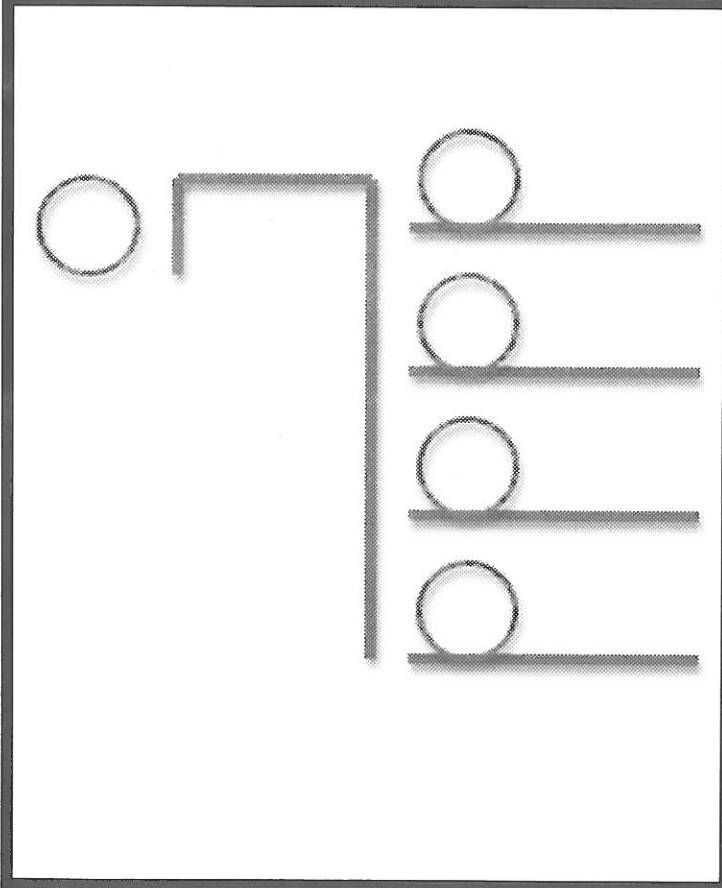
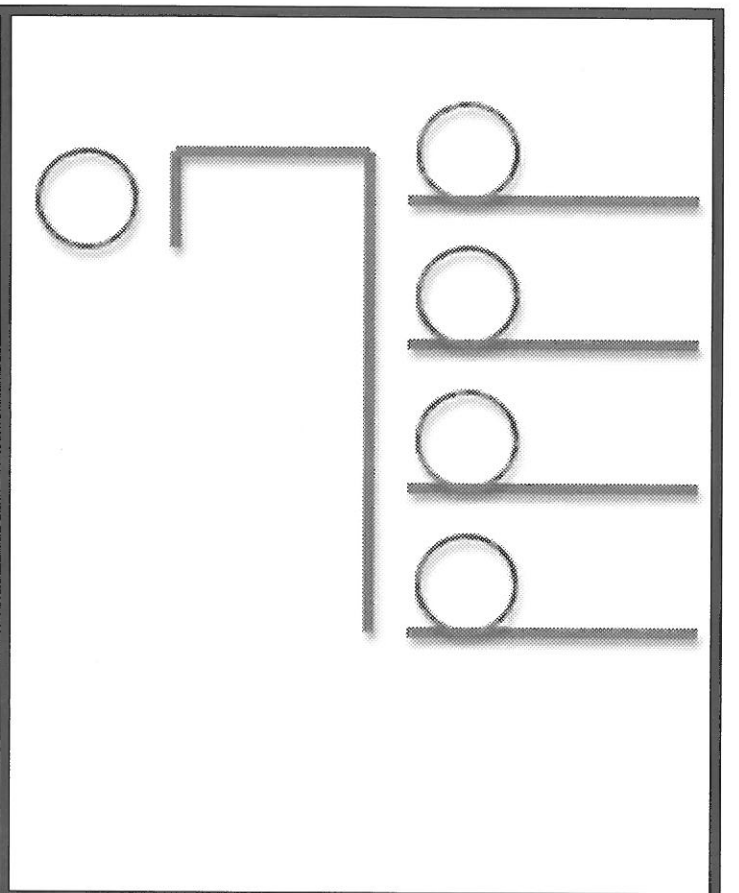
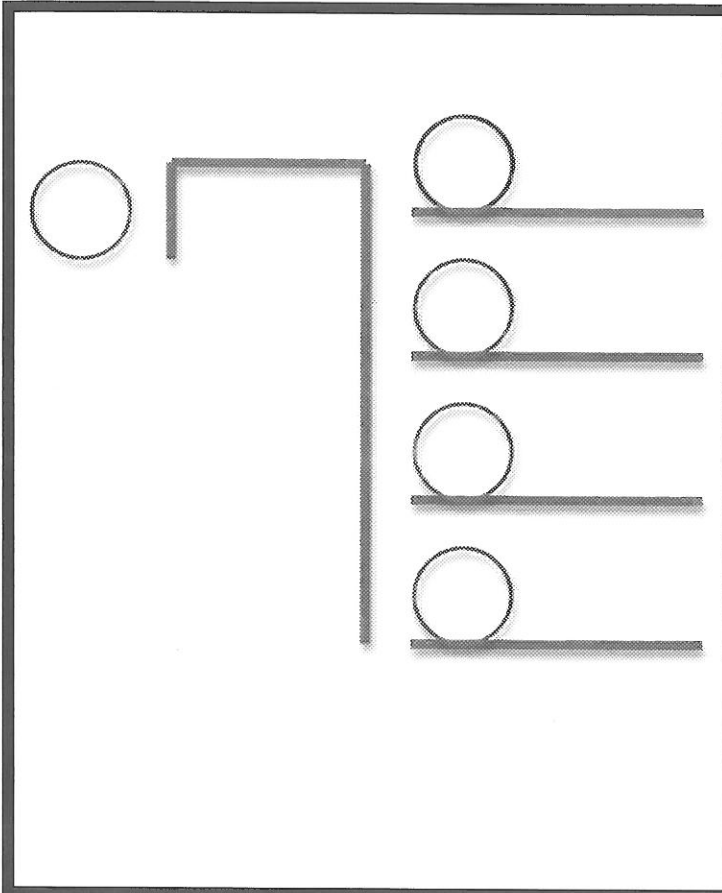
$$\begin{array}{r}
 \begin{array}{|c|c|c|} \hline 2 & 4 & 3 \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline 2 & 4 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|} \hline 1 & 1 & 2 \\ \hline \end{array} \quad (4 \times 3) \\
 \begin{array}{|c|c|c|} \hline 1 & 1 & 6 & 0 \\ \hline \end{array} \quad (4 \times 40) \\
 \begin{array}{|c|c|c|c|} \hline & 8 & 0 & 0 \\ \hline \end{array} \quad (4 \times 200) \\
 \begin{array}{|c|c|c|} \hline & 6 & 0 \\ \hline \end{array} \quad (20 \times 3) \\
 \begin{array}{|c|c|c|c|} \hline & 8 & 0 & 0 \\ \hline \end{array} \quad (20 \times 40) \\
 + \begin{array}{|c|c|c|c|c|} \hline & 4 & 0 & 0 & 0 \\ \hline \end{array} \quad (20 \times 200) \\
 \hline
 \begin{array}{|c|c|c|c|} \hline 5 & 8 & 3 & 2 \\ \hline \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|} \hline 2 & 4 & 3 \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline 2 & 4 \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|} \hline 1 & 1 & 2 \\ \hline \end{array} \\
 \begin{array}{|c|c|c|} \hline 1 & 9 & 7 & 2 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|c|} \hline & 4 & 8 & 6 & \textcircled{x} \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|} \hline 5 & 8 & 3 & 2 \\ \hline \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \\
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline \end{array} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \\
 \times \begin{array}{|c|c|} \hline & \\ \hline \end{array} \\
 \hline
 \begin{array}{|c|c|c|c|} \hline & & & \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline \end{array} \\
 \hline
 \end{array}$$

Dividing – Partial Quotients



### Multiplication: Partial Products

Students move from area/array models (other side) to working with numbers.

Consider  $26 \times 45$ , we can break apart each factor by its place value.

$26 = (20 + 6)$  We can then multiply each 45 =  $(40 + 5)$  of the "parts" and add them back together.

$$(20 \times 40) + (20 \times 5) + (40 \times 6) + (6 \times 5)$$

$$800 + 100 + 240 + 30$$

$$900 + 240 + 30$$

$$1,140 + 30$$

$$\text{So, } 26 \times 45 = 1,170$$

It might seem like a lot of numbers above. But, when we think about it, the multiplication is quite simple. This understanding develops mental math, the traditional algorithm, and algebraic concepts including factoring polynomials.

Sometimes, it makes sense to work with different parts. Consider  $51 \times 21$ . We might think of 21 as  $10 + 10 + 1$ :

$$(51 \times 10) + (51 \times 10) + (51 \times 1)$$

$$510 + 510 + 51$$

$$1,020 + 51$$

$$\text{So, } 51 \times 21 = 1,071$$

Another example, consider  $4 \times 327$ . We can break 327 into  $(300 + 20 + 7)$  then multiply.

$$4 \times 300 = 1,200$$

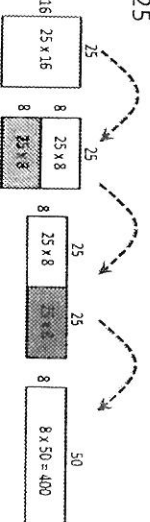
$$4 \times 20 = 80$$

$$+ \quad 4 \times 7 = 28$$

$$\text{So, } 4 \times 327 = 1,308$$

### Doubling and Halving

There are many strategies we can take advantage of so that computation is efficient. Doubling and halving is an example. When multiplying, we can double one factor and halve the other. The product is unchanged. This makes some numbers easier to work with. Consider  $16 \times 25$



The image shows that we can halve  $16 (8 + 8)$  and then double 25. So,  $16 \times 25$  is the same as  $8 \times 50$ .

### Division

4<sup>th</sup> grade students are beginning to develop an understanding of division with larger numbers. One approach is to take groups of numbers, usually "friendly numbers" out.

Consider this:

We have 252 buttons to put in 4 boxes. How many buttons can we put in each box? ( $252 \div 4$ )

We can put 50 in each box ( $4 \times 50$ ) = 200

We can put 10 in each box ( $4 \times 10$ ) = 40

We can put 3 in each box ( $4 \times 3$ ) = 12

$$\underline{252}$$

So, we can put 63 buttons in each box.  
 $252 \div 4 = 63$

Another approach is to break apart the dividend into "friendly numbers." Consider  $252 \div 4$ . We could break 252 into  $(240 + 12)$  and divide each by 4.

$$240 \div 4 = 60 \quad 60 + 3 = 63$$

$$12 \div 4 = 3 \quad \text{So, } 252 \div 4 = 63$$

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Grade 4



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More Friendly Problem

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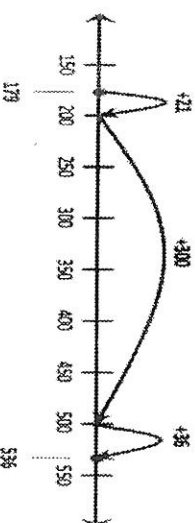
$$\begin{array}{r} 173 + 389 \\ +27 \quad -27 \\ \hline 200 + 362 = 562 \end{array}$$

More Friendly Problem

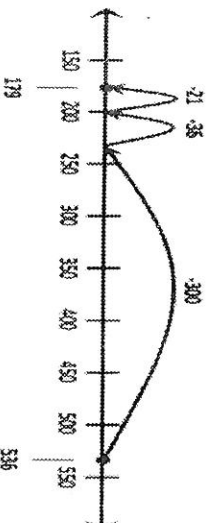
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We can count up from one number to the other. The difference is  $300 + 21 + 36$  or 357. (above)



We can count back from one number to the other. The difference is  $-300$  (land at 236),  $-36$  (land at 200),  $-21$  (end at 179).

### Subtraction: Adjusting

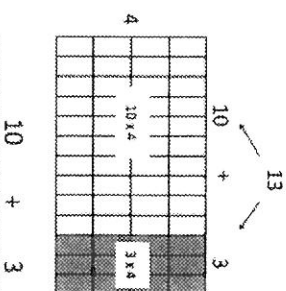
We can use "friendlier numbers" to solve problems.  $4,000 - 563$  can be challenging to regroup. But the difference between these numbers is the same as the difference between  $3,999 - 562$ . Now, we don't need to regroup.

$$\begin{array}{r} \text{(Original problem)} \quad 4,000 \\ \text{(Compensation)} \quad -1 \\ \hline 3,999 \end{array} \quad \begin{array}{r} - \\ - \\ \hline 563 \\ -1 \\ \hline 562 \end{array} = 3,437$$

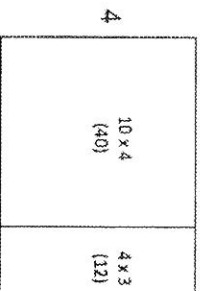
### Multiplication: Area/Array

The area/array model for multiplication and the distributive property are used to solve multiplication problems

$$\begin{aligned} 13 \times 4 &= \\ (10 \times 4) + (3 \times 4) &= \\ 40 + 12 &= \\ 52 \end{aligned}$$



This is the same model without grid lines. It is considered an "open model."



$$40 + 12 = 52$$

The open model also works well with 2 or 3-digit factors. This supports development of algorithms later, as well as mental mathematics. Consider  $29 \times 14$

$$\begin{array}{r} 20 \\ + \\ 9 \end{array}$$

10	200 <small>(20 x 10)</small>	90 <small>(9 x 10)</small>
4	80 <small>(20 x 4)</small>	36 <small>(9 x 4)</small>

$$200 + 90 + 80 + 36 = 406 \quad \text{So, } 29 \times 14 = 406$$

### Multiplication: Multiples of 10

Understanding why we "add zeros."

$$\begin{aligned} 3 \times 6 &= 18 & 20 \times 40 &= \\ 3 \times 6 \text{ tens} &= 18 \text{ tens} & (2 \times 10) \times (4 \times 10) &= \\ 3 \times 60 &= 180 & 2 \times 4 \times 10 \times 10 &= \\ & & 8 \times 100 &= 800 \end{aligned}$$

**Multiplication: Partial Products Algorithm**  
 In this algorithm, we break apart the numbers by place value to find parts of the product. We add them back together to get the final product. This algorithm begins in the ones place.

$$\begin{array}{r}
 48 \\
 \times 32 \\
 \hline
 16 \quad \longleftarrow (2 \times 8) \\
 80 \quad \longleftarrow (2 \times 40) \\
 240 \quad \longleftarrow (30 \times 8) \\
 + 1200 \quad \longleftarrow (30 \times 40) \\
 \hline
 1,536
 \end{array}$$

**Multiplication: Partial Products Algorithm**  
 In this algorithm, we break apart the numbers by place value to find parts of the product. We add them back together to get the final product. This algorithm begins in the tens place.

$$\begin{array}{r}
 48 \\
 \times 32 \\
 \hline
 1200 \quad \longleftarrow (40 \times 30) \\
 240 \quad \longleftarrow (30 \times 8) \\
 80 \quad \longleftarrow (40 \times 2) \\
 + 16 \quad \longleftarrow (8 \times 2) \\
 \hline
 1,536
 \end{array}$$

**Multiplication: Traditional Algorithm**  
 This is a digit-based algorithm. It is useful for multiplying large numbers. We begin in the ones place and proceed to multiply each digit. We combine products of each place value.

$$\begin{array}{r}
 48 \\
 \times 32 \\
 \hline
 96 \quad \longleftarrow (2 \times 8) + (2 \times 40) \\
 + 1440 \quad \longleftarrow (30 \times 8) + (30 \times 40) \\
 \hline
 1,536
 \end{array}$$

### Division\*

5<sup>th</sup> grade students continue to develop an understanding of division with larger numbers. One approach is to take groups of numbers, usually “friendly numbers” out.

Consider this:  
 We have 252 buttons to put in 4 boxes. How many buttons can we put in each box?  $(252 \div 4)$

$$\begin{array}{l}
 \text{We can put 50 in each box } (4 \times 50) = 200 \\
 \text{We can put 10 in each box } (4 \times 10) = 40 \\
 \text{We can put } \underline{3} \text{ in each box } (4 \times 3) = \underline{12} \\
 \hline
 63
 \end{array}$$

So, we can put 63 buttons in each box.  
 $252 \div 4 = 63$

Another approach is to break apart the dividend into “friendly numbers.” Consider  $252 \div 4$ . We could break 252 into  $(240 + 12)$  and divide each by 4.

$$\begin{array}{l}
 240 \div 4 = 60 \qquad 60 + 3 = 63 \\
 12 \div 4 = 3 \qquad \text{So, } 252 \div 4 = 63
 \end{array}$$

We may also consider Think Multiplication to work with division. Consider  $932 \div 45$ .

We can think of “What times 45 equals 932?”

We might think  $45 \times 10 = 450$ , so...  
 $45 \times 20 = 900$

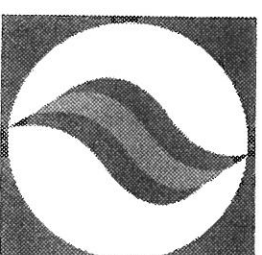
20 groups of 45 is 900. We have 32 leftover but that is not enough for another group.

$932 \div 45 = 20$  with 32 leftover.

\* The long division algorithm is introduced in grade 6 after students develop deep understanding of grouping and division.

# Developing Computational Fluency

Grade 5



Elementary Mathematics Office  
 Howard County Public School System

This brochure highlights some of the methods for developing computational fluency. For more information about computation and elementary mathematics visit <http://smart.wikispaces.hcps.org>

### Addition: Partial Sums

Many times it is easier to break apart addends. Often it makes sense to break them apart by their place value. Consider  $248 + 345$

$$\begin{aligned} 248 &= 200 + 40 + 8 \\ 345 &= 300 + 40 + 5 \end{aligned}$$

$$500 + 80 + 13 = 593$$

Sometimes we might use partial sums in different ways to make an easier problem. Consider  $484 + 276$

$$\begin{aligned} 484 &= 400 + 84 \\ 276 &= 260 + 16 \end{aligned}$$

$$660 + 100 = 760$$

### Addition: Adjusting

We can adjust addends to make them easier to work with. We can adjust by giving a value from one addend to another. Consider  $326 + 274$ . We can take 1 from 326 and give it to 274.

$$\begin{array}{r} 326 + 274 \\ -1 \quad +1 \\ \hline \end{array}$$

$$\begin{array}{r} \text{More Friendly} \\ \text{Problem} \longrightarrow \\ \hline 325 + 275 = 600 \end{array}$$

Consider  $173 + 389$ . We can take 27 from 389 and give it to 173 to make 200.

$$\begin{array}{r} 173 + 389 \\ +27 \quad -27 \\ \hline \end{array}$$

$$\begin{array}{r} \text{More Friendly} \\ \text{Problem} \longrightarrow \\ \hline 200 + 362 = 562 \end{array}$$

### Addition: Traditional Algorithm

This algorithm is useful for adding large numbers. We add place values and regroup when needed.

$$\begin{array}{r} \phantom{0}^1 \phantom{0}^1 \\ 13,089 \\ + 4,684 \\ \hline 17,773 \end{array}$$

### Subtraction: Count Up or Count Back

When subtracting, we can count back to find the difference of 2 numbers. In many situations, it is easier to count up. Consider  $536 - 179$ .

#### Counting Up

$$\begin{aligned} 179 + 21 &= 200 \\ 200 + 300 &= 500 \\ 500 + 36 &= 536 \end{aligned}$$

The total of our

counting up is 357.

$$\text{So, } 536 - 179 = 357$$

#### Counting Back

$$\begin{aligned} 536 - 36 &= 500 \\ 500 - 300 &= 200 \\ 200 - 21 &= 179 \\ (-) 357 \end{aligned}$$

The total of our

counting back is 357.

$$\text{So, } 536 - 179 = 357$$

### Subtraction: Adjusting

We can use "friendlier numbers" to solve problems.  $4,000 - 563$  can be challenging to regroup. But the difference between these numbers is the same as the difference between  $3,999 - 562$ . Now, we don't need to regroup.

$$\begin{array}{r} \text{(Original problem)} \\ \text{(Compensation)} \\ \hline 4,000 \\ -1 \end{array}$$

$$3,999$$

$$\begin{array}{r} - \\ 563 = \\ -1 \end{array}$$

$$562 = 3,437$$

### Subtraction: Traditional Algorithm

This algorithm is useful for subtracting large numbers. We regroup when necessary.

$$\begin{array}{r} \phantom{0}^8 \phantom{0}^1 \\ 14,290 \\ - 3,236 \\ \hline 11,054 \end{array}$$

### Multiplication: Partial Products

Students move from area/array models to working with numbers.

Consider  $26 \times 45$ , we can break apart each factor by its place value.

$$\begin{aligned} 26 &= (20 + 6) \\ 45 &= (40 + 5) \end{aligned}$$

We can then multiply each of the "parts" and add them back together.

$$(20 \times 40) + (20 \times 5) + (40 \times 6) + (6 \times 5)$$

$$800 + 100 + 240 + 30$$

$$900 + 240 + 30$$

$$1,140 + 30$$

$$1,170$$

$$\text{So, } 26 \times 45 = 1,170$$

It might seem like a lot of numbers above. But, when we think about it, the multiplication is quite simple. This understanding develops mental math, the traditional algorithm, and algebraic concepts including factoring polynomials.

Sometimes, it makes sense to work with different parts. Consider  $51 \times 21$ . We might think of 21 as  $10 + 10 + 1$ :

$$(51 \times 10) + (51 \times 10) + (51 \times 1)$$

$$510 + 510 + 51$$

$$1,020 + 51$$

$$1,071$$

$$\text{So, } 51 \times 21 = 1,071$$

Another example, consider  $4 \times 327$ . We can break 327 into  $(300 + 20 + 7)$  then multiply.

$$4 \times 300 = 1,200$$

$$4 \times 20 = 80$$

$$+ \quad 4 \times 7 = 28$$

$$1,308$$

$$\text{So, } 4 \times 327 = 1,308$$