

NCTM-Boston 2015

## Geometric Habits of Mind

How do we encourage their development?

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## *Fostering Geometric Thinking:*

*A guide for Teachers, Grades 5-10*

Mark Driscoll with Rachel Wing DiMatteo, Johannah Nikula, and Michael Egan (2007) of Education Development Center.

## *5 Practices*

*for Orchestrating Productive Mathematics Discussions*

Margaret S. Smith and Mary Kay Stein (2011)  
National Council of Teachers of Mathematics

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### Task 1:

In two minutes or less, can you draw a quadrilateral that has two right angles but no pair of parallel sides?



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## Geometric Habits of Mind

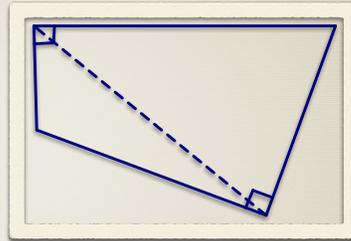
- \*Reasoning with relationships
- \*Generalizing geometric ideas
- \*Investigating invariants
- \*Balancing exploration and reflection

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GHOM:

**Reasoning with Relationships-**

reasoning of parallel line relationships and relationships the angles have with each other

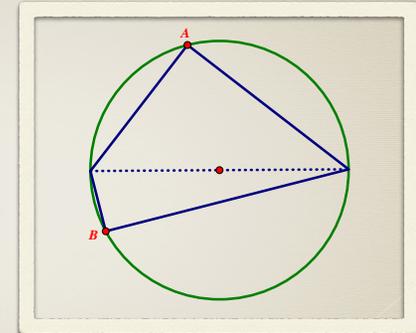


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GHOM:

**Generalizing Geometric Ideas-**

using your knowledge of circles (semicircles) to create an infinite class of examples (right angles inscribed in a semicircle)



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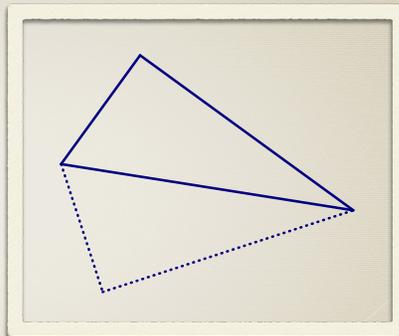
GHOM:

**Investigating**

**Invariance**-properties and relationships with motions-reflecting a right triangle across the hypotenuse (what stays invariant under reflection)

Or

**Reasoning with Symmetry Relationships**

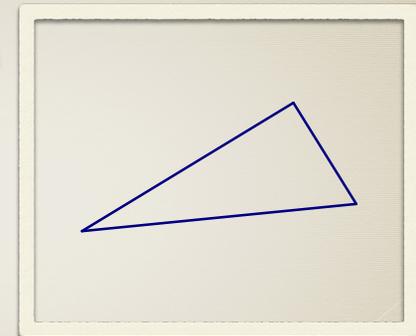


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GHOM:

**Balancing Exploration and Reflection-**

trying various ways to solve a problem and stepping back to reassess what you found and adjust your thinking



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# Rationale for Questioning

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# Reasoning with Relationships

**Rationale:** Looking for relationships within and between geometric figures

- \* How are the two shapes alike?
- \* In what ways are they alike?
- \* How are these shapes different? In what ways are they different?
- \* What other shapes fit this description?

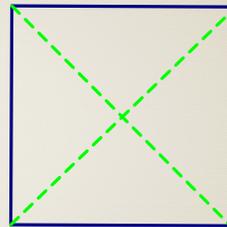


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# Generalizing Geometric Ideas

**Rationale:** Attempts to understand and describe the “always” and “every” related to geometric concepts

- \* Does this happen every time?
- \* Why would this happen every time?
- \* Have I found all of the possibilities that fit this description?
- \* Is there a time when this is not true?



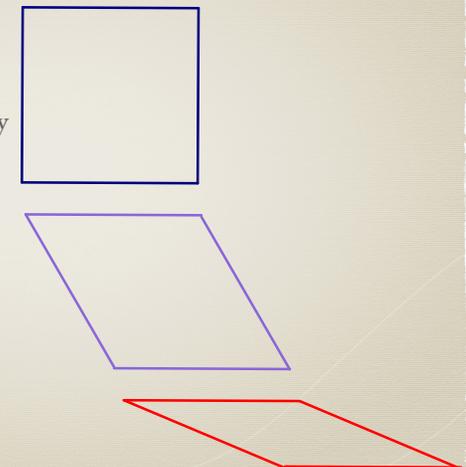
Are the diagonals of a square perpendicular bisectors?

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# Investigating Invariants

**Rationale:** Analyzes which attributes of a figure stay the same and which change as the figure is transformed in some way

- \* How did that figure get from here to there?
- \* Is it possible to transform this figure so that it becomes that one?
- \* What changes? Why?
- \* What stays the same? Why?

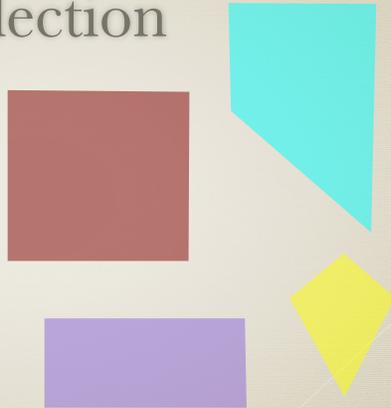


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# Balancing Exploration and Reflection

**Rationale:** Stepping back to take stock of where you are in the problem-solving process

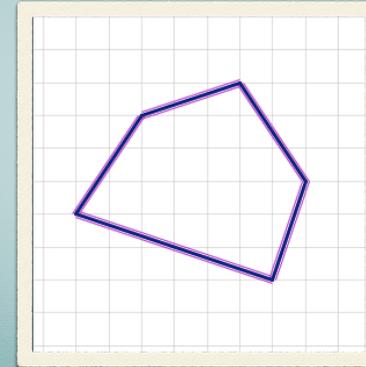
- \* What if . . . ?
- \* What happens if I draw a picture? take apart the figure?
- \* What happens if I work backward from the end?
- \* What have I learned?
- \* What did that tell me?



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## Area—High-Level Task

Problem: Find several ways to calculate the area of the irregular figure below.

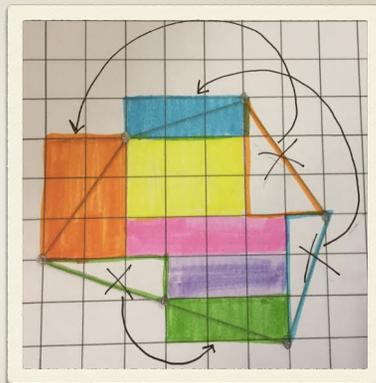


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### Solution 1:

...found the area of each rectangle and added all 6 together...moving around congruent triangles to make rectangles.

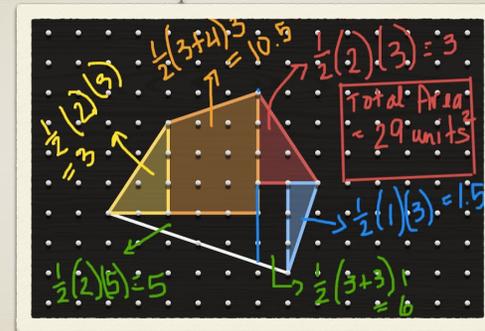
**Reasoning with Relationships**  
(of congruent figures)



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### Solution 2:

...first ways I saw to split the pentagon up where there were obvious parallel lines and right angles to know for sure that the inner shapes were what I wanted them to be.



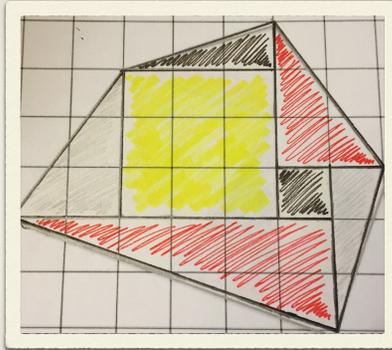
### Balancing Explorations and Reflections

(expand on my thinking for each way)

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### Solution 2:

...create the largest square and then draw triangles around the outside of that square...into shapes that were easy to find the area of.



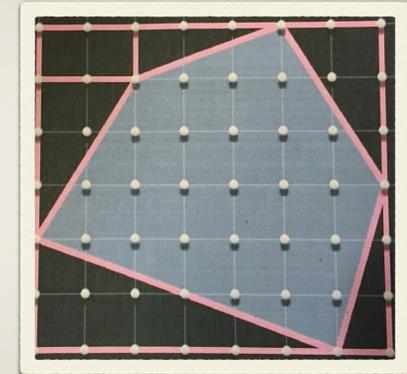
### Balancing Exploration and Reflection

(kept repeating process until he could find figures he could calculate area for)

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### Solution 3:

...found area of the large rectangle and then subtracted the areas that were not part of the shaded region.



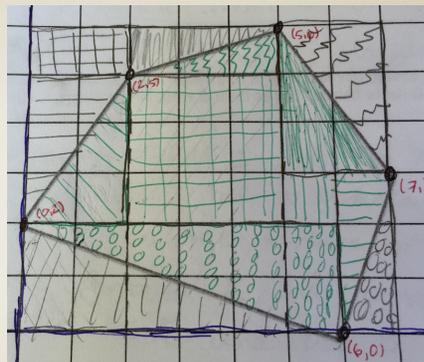
### Balancing Explorations and Reflections

(stepped back and tried combination of my first two processes)

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### Solution 4:

...create a coordinate plane then use area of a polygon formula.



### Reasoning with Relationships

(visualize the space inside and outside of the polygon and how I can manipulate it)

$$\frac{[(x_1y_2 - y_1x_2) + (x_2y_3 - y_2x_3) + \dots + (x_ny_1 - y_nx_1)]}{2}$$

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## Works Cited

- \* Driscoll, M. J., DiMatteo, R. W., Nikula, J., & Egan, M. (2007). *Fostering Geometric Thinking: A Guide for Teachers, Grades 5-10*. Portsmouth, NH: Heinemann.
- \* Smith, M. S. & Stein, M. K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.
- \* Van de Walle, J. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Boston: Pearson Education, Inc.

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