Enriching Recursion Lessons with Activities

Cheryl Gann

gann@ncssm.edu

North Carolina School of Science and Mathematics

Goals of the Activities

- To provide students with concrete examples that will help them better understand what can happen in recursive systems.
- To build student intuition for predicting long-term behavior in recursive systems.

Simple Recursive System

New value = old value + change

$$A_0 = \text{starting value}$$

$$A_n = A_{n-1} + \Delta A \text{ for } n = 1, 2, 3, \ldots$$

We want to consider what will happen when we complete "many" iterations…

Activity # 1: Frogs in a Pond

- Start with 16 “frogs” in your pond.
- Each year half the frogs, rounded down, leave and then 2 more frogs join the pond.
- Repeat this process, keeping track of the number of frogs in the pond each year (I suggest a table). Take note of what eventually happens in the pond.
- Please look at handout for other variations.

Activity # 1: Frogs in a Pond

As you are working, consider the following questions…

- What is the purpose of the different variations of the problem? What questions do you hope will occur to the students as they work?
- What other variations could you try?
- What might give the students some trouble? How could you help them get through this difficulty?
- What would be a good way to structure the follow up to this activity?

Activity # 1: Frogs in a Pond

- This activity helps give the students a concrete demonstration that equilibrium is reached when the amount going in is equal to the amount going out (or, equivalently, when the net change is zero)

- This provides an interesting example because we are dealing with <u>discrete</u> values (no pieces of frogs…). Because of the rounding, either 4 or 5 frogs could be the equilibrium value.

Activity # 1: Frogs in a Pond

- Implementation Ideas:
  - Put students into pairs and assign each pair a different starting number of frogs.
  - Have different groups with different "rules" for what portion of frogs leave and how many frogs join each year.
  - Compare results by having students share with the class their initial number, "rules" for change, and equilibrium.

Activity #2: Credit Card Debt

- In a spreadsheet or on a calculator, work out the following scenario:
  - You make purchases for a total of $10,000 on a credit card and then do not make any more charges.
  - Each month, the credit card company charges you a monthly interest rate of 2%.
  - Assume after the interest is added in, you make a monthly payment. If you pay $100 per month, what happens? Why?
- Please look at handout for other variations.

Activity #2: Credit Card Debt

As you are working, consider the following questions…

- What is the purpose of the different variations of the problem? What questions do you hope will occur to the students as they work?
- What other variations could you try?
- What might give the students some trouble? How could you help them get through this difficulty?
- What would be a good way to structure the follow up to this activity?

Activity #2: Credit Card Debt

- In this scenario, the equilibrium is only achieved if the monthly payment is equal to the amount of interest on the initial balance. If that is the case, you will never make any progress paying off your credit card.
- If the monthly payments are lower than the interest for the initial balance, your balance will increase over time.
- If the monthly payments are higher than the interest for the initial balance, your balance will decrease to zero. Your balance will decrease more quickly as the balance gets lower.

Activity #2: Credit Card Debt

- Implementation Ideas:
  - If doing this activity as an introduction to recursion, a spreadsheet works well because the students do not have to know how to write recursive equations.
  - This activity can also be a nice example to use after you have introduced recursive equations, in which case you could spend some time having the students write down the recursive equations and explore on their calculators.
  - You can have the students come up with purchases they would like to make and research current interest rates.

Take-Home Activity: Water Reservoirs

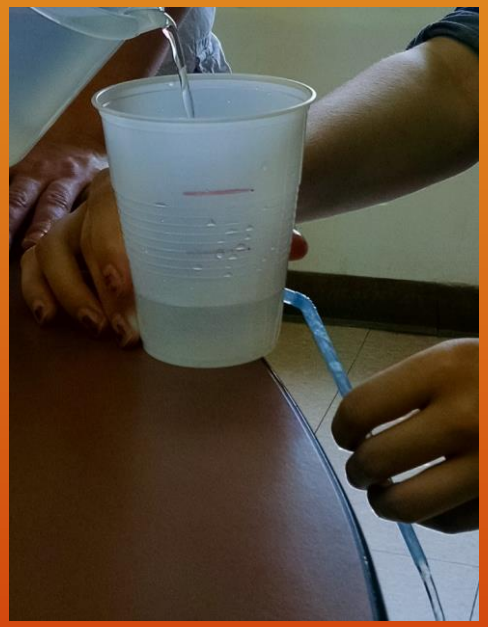
- Students work in groups– one person will be in charge of draining; one person filling; and others monitoring/directing/timing.

To affect the speed at which the water drains, you will use your fingers to compress the straw.

- On the handout, I have listed 5 different scenarios that students can try out in the "reservoirs". Students should take pauses between

the scenarios rather than speeding through the list.

Take-Home Activity: Water Reservoirs



Take-Home Activity: Water Reservoirs

- This activity helps students understand how different equilibrium levels can happen in a system.
- Having the students vary input/output rates will help students think about how they can manipulate recursive systems to get desired results.
- This provides a nice contrast to the Frog Pond scenario because we are now dealing with a <u>continuous</u> quantity.

Take-Home Activity: Water Reservoirs

- Implementation Ideas:
  - This activity was initially used in an investigation about water levels in aquifers. Putting the problem in that context would provide a great opportunity for discussion and research on the impact of increased water flow to aquifers due to storm water runoff and glacier melting.
  - As an additional step to the process, you could have students record the levels at various times to give data that can later be modeled by recursive equations.

Additional Activity: Pass the Candy

- Students sit in groups of 3 in a circle. Begin with one of the following distributions of candy (or other objects):
  - 16, 2, 12
  - 1, 12, 18
  - 1, 11, 17
- When teacher say 'Pass the Candy' each pass half of his/her candy (rounded down) to the person to his/her left. Everyone in the group should pass at the same time.
- After each iteration, everyone makes note of how much candy each person has.

Additional Activity: Pass the Candy

Initial Starting Values	**Equilibrium Value(s) and Type of Equilibrium**	**# Iterations to Reach Equilibrium**
16, 2, 12	*Values*: 10, 10, 10 *Type*: all equal values	4
1, 12, 18	*Values*: 11, 10, 10 *Type*: stable & unequal values	3
1, 11, 17	*Values*: 10, 10, 9 *Type*: unstable & unequal values	4

KEY IDEAS

- There are multiple, equivalent ways to think about what it means for a recursive system to be at equilibrium. Equilibrium is achieved when:
  - **Amount going out = Amount coming in**
  - **Total change = 0**
  - **Current value = previous value**
- Sometimes equilibrium can only be reached if the system starts at the equilibrium value.
- Sometimes different long-term behaviors (or even different equilibriums) are possible depending on the initial set up.

NCSSM Recursion Materials

- http://www.dlt.ncssm.edu/stem/content/lesson-1-introduction-recursion
  - This lesson includes a video and lesson notes for 'Pass the Candy'. In this lesson you will also find tips for how to do recursive equations on a TI-83/84
- http://www.dlt.ncssm.edu/stem/lesson-2recursion

Opportunities at NCSSM

- Teaching Contemporary Mathematics Conference
  - North Carolina School of Science and Math in Durham, NC
  - http://www.ncssm.edu/courses/math/tcm/
- Full-Time Permanent Math Instructor Position
  - Position until May 1, 2015
  - Go to http://www.oshr.nc.gov/jobs/. Search Job Positions then search Departments – NCSSM.

Please contact me with any questions!

Cheryl Gann
gann@ncssm.edu

North Carolina School of Science and Mathematics