

Exploring Wireless Mathematics for Algebra and Geometry

In this hands-on workshop, we look at a variety of problems in both Algebra and Geometry, and solve them using the HP Prime graphing calculator and the HP Wireless Classroom.

Problem 1: Does the equation $y^2 - 5x = 27$ have any integer solutions?

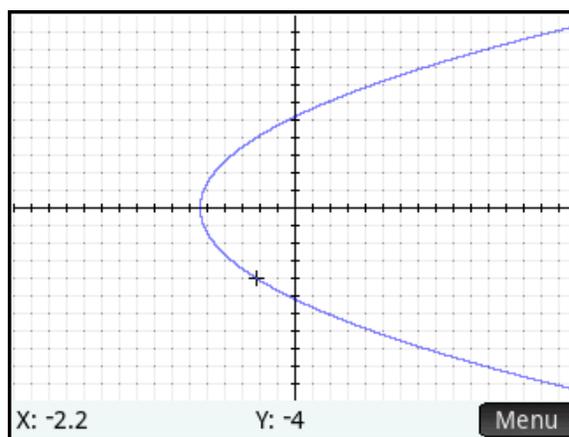
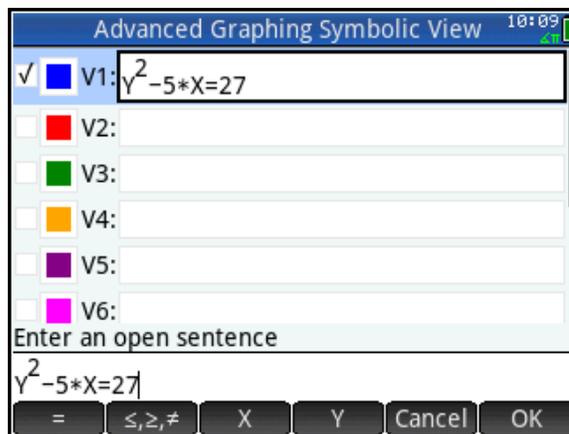
This problem comes to us from *Challenging Problems in Algebra* by Alfred S. Posamentier and Charles T. Salking (Dover 1996).

1. Press **Apps** to open the App Library and select the **Advanced Graphing** app.
2. The app opens in Symbolic view, where you can enter up to 10 equations or inequalities. In V1, enter our equation. Tap the menu keys at the bottom to enter =, X, and Y. Tap **OK** or press **Enter** when you are done.
3. Press **Plot** to see the graph

Inspection does not reveal any examples of integer solutions to our equation. Let's use Numeric view to get a closer and more exact sampling of points on this curve.

4. Press **Shift** **Num** to open Numeric Setup view. By default, the X-Step and Y-step values are set to 0.1. Change them both to 1.
5. Press **Num** to open Numeric view. Tap **Trace** and select **Edge**.

The table shows the range of integer y-values visible in Plot view, and the corresponding x-values. You can zoom out in Plot view (press **Plot** **+** or pinch) and then return to Numeric view to see more values. Why would there be no integer solutions to this equation?



Y	X	X
-6	1.8	
-5	-0.4	
-4	-2.2	
-3	-3.6	
-2	-4.6	
-1	-5.2	
0	-5.4	
1	-5.2	
2	-4.6	
3	-3.6	
4	-2.2	
5	-0.4	
6	1.8	
7	4.4	

We now re-write our equation as $x^2 = 5y + 27$. The left side of this equation is a square. We now investigate whether or not the right side can also be a square.

6. Press  to open the App Library and select the Spreadsheet app.
7. Tap the column header A to select the entire column. To fill column A with squares, type $=\text{Row}^2$. Press   to enter the equal sign. Press , then tap  and select Spreadsheet, Numeric, and Row to enter the variable Row. Press  to see column A fill with the squares of the positive integers.

You will notice that the last digits of these squares are from the set $\{0, 1, 4, 5, 6, 9\}$.

8. Tap the column header B and enter $=5*\text{Row}+27$.

You will notice that the last digits of the values in column B are from the set $\{2, 7\}$.

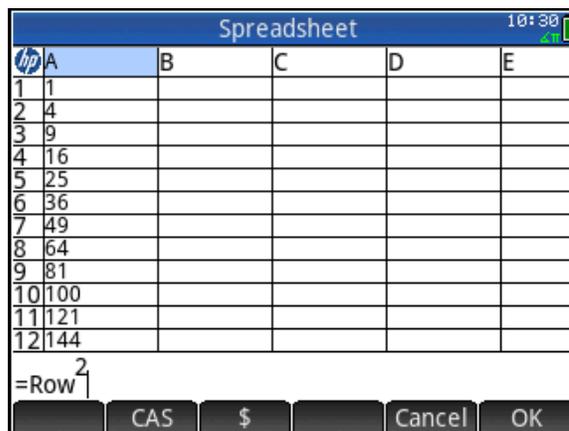
The constant 27 in the formula is not special. This constant could have been any integer of the form $5k+2$.

So now we have an infinite supply of curves with no integer solutions at all! This activity is also a good way to introduce a little number theory into your Algebra classes!

We did not offer a formal proof, but some of your scholars might pursue a more formal explanation of why numbers of the form x^2 cannot also be of the form $5k+2$.

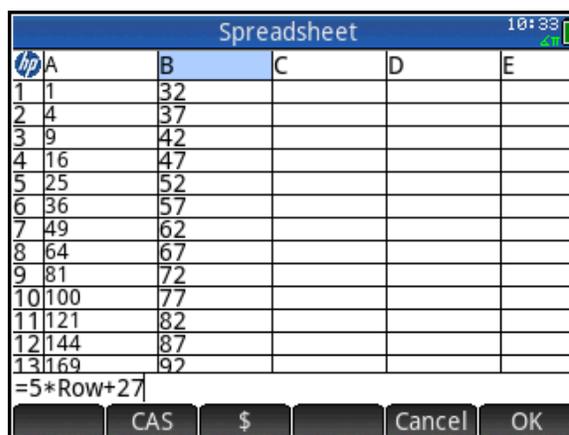
Hint: In order to end in the digits in the set above, the square would have to be in one of the following forms: $5t$, $5t+1$, or $5t-1$. In order to end in 2 or 7, the square would have to be in the form $5t+2$ or $5t-2$.

Challenge: Find another integer n such that $20 < n < 30$, for which the equation $y^2 = 5x + n$ has no integer solution. There are two solutions!



	A	B	C	D	E
1	1				
2	4				
3	9				
4	16				
5	25				
6	36				
7	49				
8	64				
9	81				
10	100				
11	121				
12	144				

=Row²



	A	B	C	D	E
1	1	32			
2	4	37			
3	9	42			
4	16	47			
5	25	52			
6	36	57			
7	49	62			
8	64	67			
9	81	72			
10	100	77			
11	121	82			
12	144	87			
13	169	92			

=5*Row+27

Problem 2: Cyclic Quadrilaterals

The instructor will send you an HP Prime app called CyclicQuads for this next problem. A cyclic quadrilateral is a convex quadrilateral that can be inscribed in a circle.

1. Press **Apps** to open the App Library and select the **CyclicQuads** app.

The construction consists of a cyclic quadrilateral $DEGH$ inscribed in circle A . Tap and drag any of the points D , E , G , or H to change the quadrilateral. Which of the following can $DEGH$ never be?

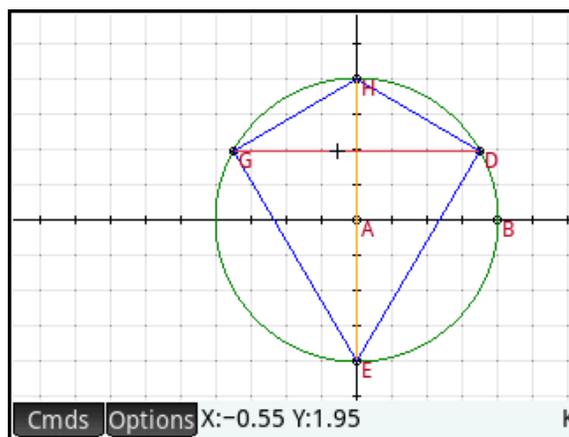
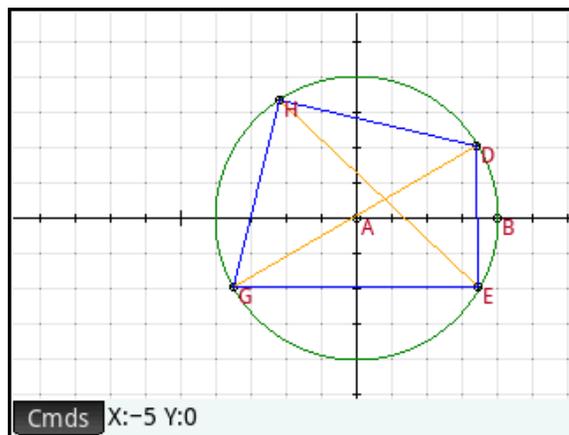
- Square
- Rhombus (non-square)
- Rectangle
- Parallelogram
- Isosceles trapezoid
- Kite

Just move the points of the quadrilateral around enough to convince yourself for each one.

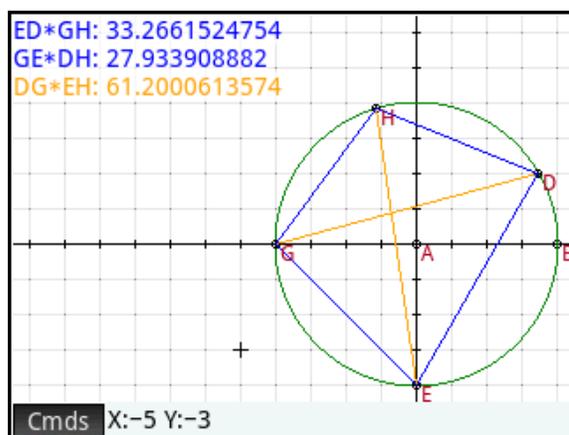
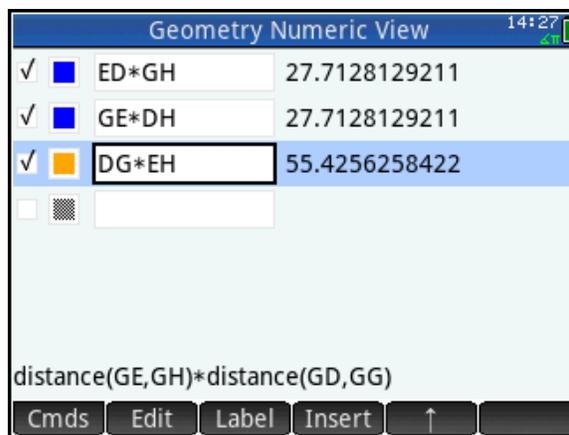
2. Make $DEGH$ into a kite, similar to that shown to the right. Tap segment DG and press **Enter** to select it. Now use **▲** and **▼** to move the diagonal vertically. What do you notice about the angles of this kite? Press **Esc** to deselect segment DG . Make a conjecture to prove later:

Theorem: If a cyclic quadrilateral is a kite,

then _____?



3. Press  to open Numeric view.
Here we have defined the following products:
 - a. The product of the opposite side lengths ED and GH
 - b. The product of the opposite side lengths GE and DH
 - c. The product of the lengths of the diagonals DG and EH
4. Tap the checkbox next to each one (once to select it and again to check it). Once checked, these measurements show up in Plot view.
5. Press  to return to Plot view and again move the points of the cyclic quadrilateral about the circle. What do you notice about these products? Make a conjecture:



The _____

of a cyclic quadrilateral is equal to the _____.

This conjecture is known as Ptolemy's Theorem. If DEGH is a rectangle, to what theorem does it become equivalent?

There is also a famous formula for the area of a cyclic quadrilateral. This formula looks a bit like Heron's formula for the area of a triangle. It is called Brahmagupta's Formula.

Problem 3

Suppose you have a set of linear equations in Standard Form: $Ax + By = C$. Suppose further that the coefficients of each equation (A , B , and C) form an arithmetic sequence. What surprising property do they all have in common?

Note: I first saw this one at an NCTM presentation by Dan Kennedy.

1. Press  to open the App Library and select the Advanced Graphing app.
2. Create several linear equations, each of whose coefficients form an arithmetic sequence. The figure to the right shows a varied set of 7 such equations.

I like to challenge students to come up with their own outlandish examples and use them!

3. Press  to see the graphs.

It is surprising the first time one sees that these lines are all concurrent at the point whose coordinates are $(-1, 2)$! Students will be curious about the mathematics that shows why this must be true!

How do we re-write the equation $AX+BY=C$ so that it reflects the fact that A , B , and C form an arithmetic sequence?

Start with AX . Then B must be the sum of A and some value. Let's write $AX + (A+B)Y$. So the constant C must be B greater than this, or $A+2B$. So now we have $AX + (A+B)Y = A+2B$.

By inspection, $Y=2$ in order to have $2B$ on both sides. That leaves us:

$$AX+2(A+B)=A+2B \text{ or } AX + 2A + 2B = A + 2B$$

Now we have $2A$ on the left but only $1A$ on the right. So $A = -1$ is the only way to make the equation true. So any equation with this property contains the point $(-1, 2)$.

In these exercises, we have used 21st Century technology (the HP Prime graphing calculator and the HP Wireless Network) to explore interesting problems from Algebra and Geometry. These problems could form the basis of student projects.

Come see us in Booth #1209 to learn more!

