

Reasoning about Fractions: Using Number Lines to Understand Fraction Comparison

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Session Overview

We will discuss:

- Relevant CCSS Standards and other recommendations about fraction reasoning and fraction comparison.
- Models, activities, and online resources to help students understand and reason about comparing fractions on the number line.

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Some of the CCSS “Big Ideas (Clusters) in Grades 3 – 5: Number and Operations—Fractions

1. Develop **understanding** of fractions as numbers (gr. 3)
2. Extend understanding of fraction **equivalence** and **ordering** (gr. 4)
3. Use **equivalent fractions** as a strategy to add and subtract fractions. (gr. 5)

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More about CCSS

- Greater emphasis on using the **number line model** to represent and act on fractions.

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Grade Three CCSS

- Understand a fraction as a number on the number line; represent fractions on a number line diagram. (3.NF.A.2)

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Grade Three CCSS (cont.)

- Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and **partitioning** it into **b equal parts**. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.
(3.NF.A.2.A)

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Grade Three CCSS (cont.)

- Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (3.NF.A.2.B)

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Grade Three CCSS (cont.)

- Explain equivalence of fractions in special cases, and **compare fractions by reasoning about their size.** (3.NF.A.3)

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Grade Three CCSS (cont.)

- Compare two fractions with the **same numerator** or the **same denominator** by **reasoning about their size.**
- Recognize that comparisons are valid only when the two fractions refer to the **same whole.**

(continued)

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Grade Three CCSS (cont.)

- Record the results of comparisons with the symbols $>$, $=$, or $<$, and **justify the conclusions**, e.g., by using a **visual fraction model.** (3.NF.A.3.D)

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Grade FOUR CCSS (cont.)

- Compare two fractions with different numerators and different denominators, e.g., by creating **common denominators or numerators**, or by **comparing to a benchmark fraction** such as $1/2$. (continued)

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Grade Four CCSS (cont.)

- Recognize that comparisons are valid only when the two fractions refer to the same whole.
- Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. (4.NF.A.2)

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Improving Fractions Instruction

Help students recognize that **fractions are numbers** and that they expand the number system beyond whole numbers. Use **number lines** as a central representational tool in teaching this and other fraction concepts from the early grades onward.

Developing Effective Fractions Instruction for Kindergarten through Eighth Grade: A Practice Guide (Siegler, Carpenter, Fennell, Geary, Lewis, Okamoto, Thompson, & Wray, 2010).

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Considerations

- Most children need to **use concrete models over extended periods of time** to develop **mental images** needed to think conceptually about fractions
- Students who don't have mental images for fractions often resort to whole number strategies.

(Post et al., 1985; Cramer et al., 1997)

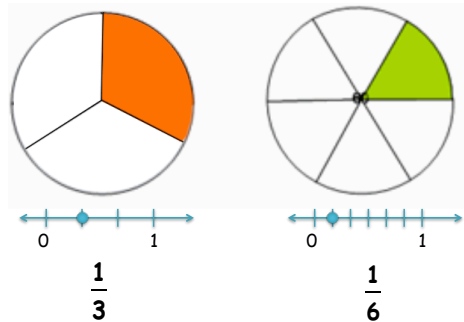
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Types of Models for Fractions

- Area/region
 - Fraction circles, pattern blocks, paper folding, geoboards, fraction bars, fraction strips/kits
- Set/discrete
 - Chips, counters, painted beans
- **Linear**
 - **Number lines, rulers**

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Comparing Fractions with a Model



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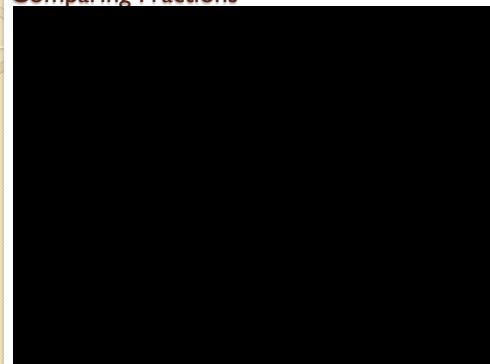
One Fifth-Grader's Understanding of Comparing Fractions

Circle the larger number or write "=" if they are equal in the pairs below:

- | | |
|--------------------------------|---------------------------------|
| 1. $\frac{1}{6}$ $\frac{1}{3}$ | 4. $\frac{1}{7}$ $\frac{2}{7}$ |
| 2. $\frac{4}{3}$ | 5. $\frac{3}{10}$ $\frac{1}{2}$ |
| 3. $\frac{3}{6}$ $\frac{1}{2}$ | 6. $\frac{1}{2}$ $\frac{4}{6}$ |

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One Fifth-Grader's Understanding of Comparing Fractions



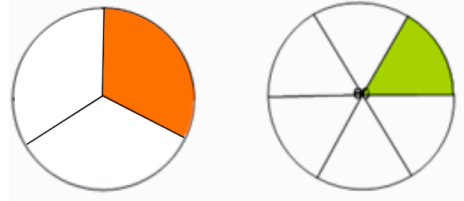
Comparing 1/6 and 1/3:

- According to Ally, “1/3 is bigger, because if you **change the digit** down from 3, if it was 1/1 it would be equal to 1 and one’s a whole number so it’s bigger”.
- What does she understand and what is she struggling to understand about comparing fractions?

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Think about the Language of Comparison

- Should we use “Bigger” or “Greater”? (or “Smaller” or “Less than”?)



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Ordering Fractions

$$\frac{5}{8}, \frac{3}{8}, \frac{6}{8}$$

Fractions with the same denominator have the **same-sized pieces**, so the numerators tell which fraction has more pieces (and is greater).

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Ordering Fractions

$$\frac{4}{8}, \frac{4}{5}, \frac{4}{6}$$

Fractions with the same numerator have the **same number of pieces**, and the denominators tell us which pieces are larger (and which fraction is greater).

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Ordering Fractions

$$\frac{3}{4}, \frac{2}{5}, \frac{1}{2}$$

Fractions **close to a benchmark** (such as $\frac{1}{2}$ or 1) can be compared by finding their distance from the benchmark.

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Fractions Equivalent to One-half

$$\frac{2}{5} = \frac{1}{2}$$

The denominator is **twice the value of the numerator**, so it's equal to $\frac{1}{2}$

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Ordering Fractions

$$\frac{7}{8}, \frac{3}{4}, \frac{2}{3}$$

Fractions **close to one** can be compared by finding their distance from one, for example, by focusing on the amount that's missing from the whole.

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Ordering Fractions

$$\frac{99}{100}, \frac{6}{7}, \frac{15}{16}$$

Fractions close to one can be compared by finding their distance from one, for example, by focusing on the amount that's missing from the whole.

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Ordering Fractions on a Number Line: The "Clothesline" Activity

- Task:
 - **Order** fraction tents using a clothesline to represent a number line and
 - mathematically **justify** the reasons for your ordering.
- Materials: fraction tents and clothesline (string, yarn, etc.)

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In A Third Grade Classroom



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"Clothesline" Fractions Activity

$$\frac{1}{2}, \frac{3}{4}, 1$$

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"Clothesline" Fractions Activity

$$1\frac{2}{3}, \frac{7}{4}$$

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"Clothesline" Fractions Activity

$$\frac{1}{3}, \frac{3}{4}, \frac{5}{8}$$

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"Clothesline" Fractions Activity

$$\frac{3}{5}, \frac{4}{9}, \frac{3}{4}$$

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"Clothesline" Fractions Activity

$$\frac{1}{8}, \frac{7}{8}, \frac{11}{12}$$

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"Clothesline" Fractions Activity

$$\frac{1}{4}, \frac{3}{13}, \frac{6}{27}$$

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In A Third Grade Classroom



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Free Online Fraction Resources

[ConceptuaMath](http://www.conceptuamath.com)
www.conceptuamath.com

Resources → Tool Library →
"Try the Tools"

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Strengthen Students' Fraction Reasoning by Helping Them:

- Develop understanding of fractions as numbers.
- Understand fraction concepts, order, and equivalence,
- Use **number lines** as a central representational tool (but not as the first model students use for fractions) in teaching fraction concepts from the early grades onward.
- Make “**Why?**”, “**How do you know?**”, “**Can you explain?**” classroom mantras.

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A Few iPad Activities for Fractions

- Number Line
- Motion Math HD
- Math Tappers: Estimate Fractions

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