

**Focus on What Matters Most: Effective Teaching!**

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**Handout will be posted next week on Conference Site**

**High Quality Standards are Necessary for Effective Teaching and Learning, But Insufficient**

Standards do not describe or prescribe the essential conditions required to make sure mathematics works for all students.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

**Principles to Actions: Ensuring Mathematical Success for All**

The overarching message is that effective teaching is the non-negotiable core necessary to ensure that all students learn mathematics.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

**Guiding Principles for School Mathematics**

1. *Teaching and Learning*
2. *Access and Equity*
3. *Curriculum*
4. *Tools and Technology*
5. *Assessment*
6. *Professionalism*

Essential Elements of Effective Math Programs

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

**We Must Focus on Instruction**

Teaching has 6 to 10 times as much impact on achievement as all other factors combined ... Just three years of effective teaching accounts on average for an improvement of 35 to 50 percentile points.

Schmoker, M. (2006). *Results now: How we can achieve unprecedented improvements in teaching and learning*. Alexandria, VA: Association for Supervision and Curriculum Development.

**Teaching and Learning Principle**

*Teaching and Learning.* An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

**Eight Research-Informed Instructional Practices**

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning and problem solving.
- Use and connect mathematical **representations**.
- Facilitate meaningful mathematical **discourse**.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

**Eight Research-Informed Instructional Practices**

- Pose purposeful **questions**.
- Build **procedural fluency** from conceptual understanding.
- Support **productive struggle** in learning mathematics.
- **Elicit and use evidence** of student thinking.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

**Obstacles to Implementing Research-Informed Instructional Practices**

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms.

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### Discussion Question

With a shoulder partner:  
 What cultural beliefs about the teaching and learning of mathematics (that students and parents hold) do you believe stand as obstacles to consistent implementation of effective teaching and learning in mathematics classrooms?

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#### Beliefs about teaching and learning mathematics

Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.

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### Eight Research-Informed Instructional Practices

**Build procedural fluency from conceptual understanding.**

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

#### Build procedural fluency from conceptual understanding

Teacher and student actions

What are teachers doing?	What are students doing?
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems.
Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Striving to use procedures appropriately and efficiently.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Our Founding Fathers Did NOT Establish the "Standard Algorithms"

Standard algorithms were developed in India in the first centuries of the modern era, and further honed by traders and engineers in the Iraq-Persia region, in order to make pencil-and-paper calculation most efficient.

### Put Standard Algorithms in Historical Perspective

Standard algorithms sacrifice ease of understanding in favor of computational efficiency, and that made sense once. In today's world we have readily accessible machines to do calculations, so we can turn the educational focus on understanding the place-value system that lies beneath those algorithms.

### The Area Model Builds Understanding of the Traditional Algorithm

$$\begin{array}{r}
 43 \times 17 \\
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 301 \\
 + 430 \\
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 731
 \end{array}$$

### Don't confuse an "instructional strategy" with a "mathematical standard"

### Moving Forward: Consider How you Communicate with Parents

How, Why and When

We should emphasize **visual representations or models to build understanding** -- not "alternate" algorithms

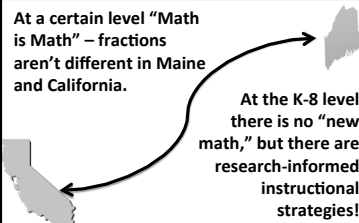
$$\begin{array}{r}
 43 \times 17 \\
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 301 \\
 + 430 \\
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 731
 \end{array}$$

**Which Makes More Sense?  
Which is More Grounded in Place Value?**

$\begin{array}{r} 43 \\ \times 17 \\ \hline 301 \\ + 430 \\ \hline 731 \end{array}$	$\begin{array}{r} 43 \\ \times 17 \\ \hline 21 \\ 280 \\ 30 \\ \hline 400 \\ 731 \end{array}$
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**We Need "Standards in Common"**

At a certain level "Math is Math" – fractions aren't different in Maine and California.



At the K-8 level there is no "new math," but there are research-informed instructional strategies!


Here is an important message ....

There is no such thing as "new math" or "common core math."

But there are "new" instructional strategies – aimed at supporting students in knowing "why" in addition to "how."

If the past, or "traditional instruction," were so wonderful, why do so many adults proudly proclaim that they "cannot do math"?

Few people make a similar claim with respect to reading.



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Status: Approved  
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Card Number: XXXXXXXXXXXX 9/XXX/XXXX  
Card Name: [REDACTED]  
Swipe/Manual: Swipe  
AMOUNT: 114.55  
TIP: 12.00  
TOTAL: 125.00  
SUGGESTED TIP CALCULATOR  
15% = 20.62  
20% = 22.91  
25% = 28.64

**Eight Research-Informed Instructional Practices**

**Implement tasks that promote reasoning and problem solving.**

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and that allow for multiple entry points and varied solution strategies.

NCTM. (2014). Principles to Actions: Ensuring Mathematical Success for All. Reston, VA: NCTM.

**Choice of Mathematical Tasks is Critical**

"If the tasks students encounter require memorizing facts or practicing procedural computations (i.e., tasks with low-level cognitive demands), students are likely to become facile with facts and computational skills. If tasks require students to think, reason, and make sense of mathematical ideas (i.e., tasks with high-level cognitive demands, or cognitively challenging tasks), students are likely to be constructing a rich understanding of mathematics" (p. 82).

Boston, M. (2012). Assessing instructional quality in mathematics. *The Elementary School Journal*, 113, 76-104.

Task A: Smartphone Plans	Task B: Solving systems of equations
<p>You are trying to decide which of two smartphone plans would be better. Plan A charges a basic fee of \$30 per month and 10 cents per text message. Plan B charges a basic fee of \$50 per month and 5 cents per text message. How many text messages would you need to send per month for plan B to be the better option? Explain your decision.</p> <p><small>(Adapted from Illustrative Mathematics Illustrations: <a href="http://www.illustrativemathematics.org/Illustrations/469">www.illustrativemathematics.org/Illustrations/469</a>)</small></p>	<p>Solve each of the following systems:</p> $\begin{array}{l} -4x - 2y = -12 \\ 4x + 8y = -24 \\ \hline x - y = 11 \\ 2x + y = 19 \\ \hline 8x + y = -1 \\ -3x + y = -5 \\ \hline 5x + y = 9 \\ 10x - 7y = -18 \end{array}$

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**Cognitively-Demanding Tasks are Atypical**

Typical classroom mathematics teaching in the United States tends not to use challenging tasks, nor to promote students' thinking about and engagement with mathematical ideas, and thus fails to help students develop understanding of the mathematics they are learning.

Silver, E. (2016). Examining what teachers do when they display their best practice: Teaching mathematics for understanding. *Journal of Mathematics Education at Teachers College*, 1(1), 1-6.

**Selecting a Quality Task is not Enough**

Selecting a task with high cognitive demand does not ensure that students will be provided opportunities to engage in rigorous mathematical activity.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle grades mathematics instruction. *Journal for Research in Mathematics Education*, 44(4), 646-682.

Beliefs about teaching and learning mathematics	
Unproductive beliefs	Productive beliefs
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Eight Research-Informed Instructional Practices

**Support productive struggle in learning mathematics.**

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

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### Support Productive Struggle in Learning Mathematics

Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to 'rescue' students by breaking down the task and guiding students step-by-step through the difficulties.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Support Productive Struggle in Learning Mathematics

Struggle does not mean needless frustration or extreme levels of challenge. It means students expend some effort to make sense of mathematics. Unproductive struggle is a situation in which students make no progress towards sense making.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. Charlotte, NC: Information Age Publishing.

Warschauer, H. K. (2011). *The Role of Productive Struggle in Teaching and Learning Middle School Mathematics*. Doctoral Dissertation, University of Texas at Austin.

### Productive Struggle Can Lead to a "Growth Mindset"

Teachers must acknowledge and value students for their perseverance and effort in reasoning and sense making in mathematics in order to develop in students a "growth mindset."

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Avoiding Productive Struggle Decline

There are three common teaching moves that generally lead to teachers taking over a student's thinking.

1. Interrupting the child's strategy
2. Manipulating the tools
3. Asking a series of closed questions

These moves can result in leading a student to an answer without engaging him/her in the reasoning about mathematical ideas.

Jacobs, V. R., Martin, H. A., Ambrose, R. C., & Philipp, R. A. (2014). Warning signs! *Teaching Children Mathematics*, 21(2), 107-113.

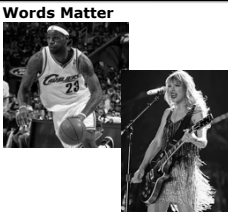
**If your students are going home at the end of the day less fired than you are, the division of labor in your classroom requires some attention.**

William, D. (2011). *Embedded formative assessment*. Bloomington, IN: Solution Tree Press.

### Moving Forward: Watch How You Communicate with Parents!

**Words Matter**

With parents we should talk about **expending effort**, not productive struggle.



NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Eight Research-Informed Instructional Practices

**Facilitate meaningful mathematical discourse.**

Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Facilitate Meaningful Mathematical Discourse

Students who learn to articulate and justify their own mathematical ideas, reason through their own and others' mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics ...

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary schools*. Portsmouth, NH: Heinemann.

### Facilitate meaningful mathematical discourse

Teacher and student actions

What are teachers doing?	What are students doing?
<p>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</p> <p>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</p> <p>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</p> <p>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</p>	<p>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</p> <p>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</p> <p>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others.</p> <p>Identifying how different approaches to solving a task are the same and how they are different.</p>

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Five Practices to Promote Productive Math Discussions

1. **Anticipating** likely student responses to challenging mathematical tasks.
2. **Monitoring** students' actual responses to the tasks (while students work on the tasks in pairs or small groups).
3. **Selecting** particular students to present their mathematical work during whole-class discussion.
4. **Sequencing** the student responses that will be displayed in a specific order.
5. **Connecting** different students' responses and connecting the responses to key mathematical ideas.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

### Eight Research-Informed Instructional Practices

**Pose purposeful questions.**

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Effective Teachers are Effective Questioners

**"Effective mathematics teachers ... pose more questions with higher cognitive demand and ask more follow-up questions, and their students ask more questions as well. Effective teachers orchestrate productive discussions through purposefully prepared questions" (p. 22).**

McNeil, (2010). *What we know about mathematics teaching and learning*, third edition. Bloomington, IN: Solution Tree Press.

### We Need More Challenging Questions

**If the questions are not causing students to struggle and think, they are probably not worth asking ... mistakes are evidence that the questions asked are tough enough to make students smarter.**

William, D. (2014). The right questions the right way. *Educational Leadership*, 71, 16-19.

### Make "Why?" "How do you know?" and "Can you explain?" Classroom Mantras

**Creating an environment where all students in the class get to hear explanations or justifications and where multiple explanations are valued makes it safer for students to take intellectual risks.**

Leinwand, S. (2009). *Accessible mathematics: 10 instructional shifts that raise student achievement*. Portsmouth, NH: Heinemann.

### Five Essential Elements of Effective Mathematics Programs

Effective teaching and learning, while the non-negotiable core of successful mathematics programs, are part of a system of essential elements of excellent mathematics programs.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Five Essential Elements of Effective Mathematics Programs

- Access and Equity
- Curriculum
- Tools and Technology
- Assessment
- Professionalism**

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Guiding Principles for School Mathematics: Professionalism

**Professionalism.** In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Professionalism Obstacle

In too many schools, professional isolation severely undermines attempts to significantly increase professional collaboration ... A danger in isolation is that it can lead to teachers developing inconsistencies in their practice.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.

### Overcoming the Obstacle: Professional Learning Communities

Teachers have a professional responsibility to participate in group decision making to improve the art and practice of teaching. One of the most powerful forums for teacher improvement is involvement in a professional learning community.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

### But What Happens in Your PLCs?

A Chinese teacher sees a lesson as a performance and puts in many hours of preparation to cover the standard forty-five minute period ...

Cheng, K. (2011). Shanghai: How a big city in a developing country leaped to the head of the class. In *Surpassing Shanghai: An agenda for American education built on the world's leading systems*. Ed. M.S. Tucker. 21-50. Cambridge, MA: Harvard University Press.

### Lesson Planning is Cultural

The tendency to spend relatively little time developing lessons and to produce lesson outlines appears to be a cultural style specific to the U.S.

Ding, M., & Carlson, M. A. (2013). Elementary teachers' learning to construct high-quality mathematics lesson plans. *The Elementary School Journal*, 113(3), 359-385.

### You Should Collaboratively Plan One Lesson in Each Unit

The lack of time to devote this careful planning and reflection to all lessons cannot be used as an excuse to never collaboratively learn, plan, and reflect on the effectiveness of key lessons.



Kanold, T., & Larson, M. R. (2012). *Common Core Mathematics in a PLC at Work: Leader's Guide*. Bloomington, IN: Solution Tree Press; Reston, VA: NCTM.

### Why Focus on Lesson Planning?

... the co-planning of lessons is the task that has one of the highest likelihoods of making a marked positive difference on student learning.

Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. New York: Routledge, Taylor & Francis Group.

### Collaboration Can Improve Learning

Students whose teachers regularly collaborate and participate in professional communities show more growth in mathematics achievement than those whose teachers are more isolated ... in addition, teachers' participation in such communities leads to smaller learning gaps between diverse racial and socioeconomic groups.

Moller, S., Mickelson, R. A., Sterns, E., Banerjee, N., & Bottia, M. C. (2013). Collective pedagogical teacher culture and mathematics achievement: Differences by race, ethnicity, and socioeconomic status. *Sociology of Education*, 86, 174-194.

### It Can All Seem Overwhelming and Change Often Feels Sisyphean!





**It Can All Seem Overwhelming and Change is Hard!**

The most likely reason for the stability of teaching practices over time is that teaching is a cultural activity and cultural activities, by their very nature, are highly resistant to change.

Stigler, J. W., & Thompson, B. J. (2009). Thoughts on creating, accumulating, and utilizing shareable knowledge to improve teaching. *The Elementary School Journal*, 109(5), 442-457.

**Some Practices are a Cultural Trap**

Cultural routines evolve over time to enable adaptation to the environment. However, sometimes the environment changes, and yet, the cultural routine persists, even if it is now highly maladaptive. It may be that mathematics instruction is an example of a cultural trap.

Stigler, J. W., & Thompson, B. J. (2009). Thoughts on creating, accumulating, and utilizing shareable knowledge to improve teaching. *The Elementary School Journal*, 109(5), 442-457.

We live in the educational shadow of the 18<sup>th</sup> century.

Nicholas Pike's 1788 *Arithmetic*

Cultural Teaching Script: State a Rule, Provide an Example, Practice the Rule

**Why is it so hard to reform mathematics education?**

The traditional math "teaching script" is VERY embedded in our culture ... The inertia of the past is incredibly strong.

**We Know What to Do – We Just Need to Do It!**

It is critical that schools learn the lesson that "best practice" in effective organizations is rarely new practice ... the most effective actions are well-known practices, with the extra dimension that they are reinforced and carried out reliably.

Schmoker, M. (2011). *Focus: Elevating the essentials to radically improve student learning*. Alexandria, VA: ASCD.

**Moving Forward: Support Research-Informed Instructional Practices**

We expect physicians to use research-informed treatments. Society should expect and support the same of teachers!

**Moving Forward: Support and Implement Research-Informed Instructional Practices**

The six guiding principles constitute the foundation of high-quality mathematics education.

**We Know What Best Practice is for School Mathematics**

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism

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**Principles to *Actions***

What actions will you take when you leave this conference to improve teaching and learning in your classroom, your school, and your district?

Extend the experience to others!!!!