

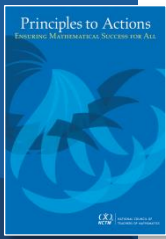
# Productive Strategies for Engaging Students in Productive Struggle

**Paul Kelley**  
**Diane Briars**

**NCTM Regional Meeting**  
**Minneapolis, MN**  
**November 13, 2015**



NATIONAL COUNCIL OF  
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# Outcomes for Session

- Explore the expectations for students, teacher actions, and classroom-based indicators of success for **productive** struggle.
- Engage in two strategies for supporting students in **productive** struggle when engaged in rigorous mathematics tasks.
- Consider how these strategies support students' engagement in **productive** struggle.



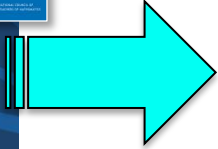
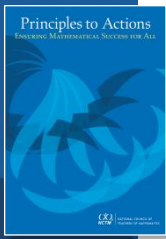
# **What Is Productive Struggle?**

## **Tasks Worth The Struggle**

## **Setting Up Productive Struggle**

## **Responding To Productive Struggle**





# **What Is Productive Struggle?**

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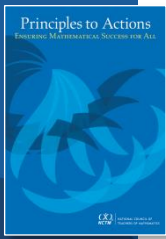
# Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



# Key Features of Minnesota Academic Standards

- All students should learn important mathematical concepts, skills, and relationships **with understanding**.
- The standards and benchmarks . . . describe a connected body of mathematical knowledge **that is acquired through the processes of problem solving, reasoning and proof, communication, connections, and representation.**



# *Effective* Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations.**
4. Facilitate meaningful mathematical **discourse.**
5. Pose purposeful **questions.**
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.



# Support Productive Struggle in Learning Mathematics

*Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*

*Principles to Actions, NCTM, 2014, p 48*



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# What is “Productive Struggle”?

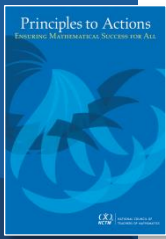
## Unproductive Struggle

- Leads to frustration
- Makes learning goals feel hazy and out of reach
- Feels fruitless
- Leaves students feeling abandoned and on their own
- Creates a sense of inadequacy

## Productive Struggle

- Leads to understanding
- Makes learning goals feel attainable and effort seem worthwhile
- Yields results
- Leads students to feelings of empowerment and efficacy
- Creates a sense of hope

Jackson and Lambert (2012)

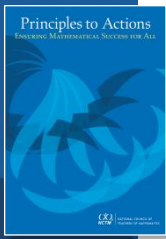


# Embracing Productive Struggle: Redefining Student and Teacher Success

- Examine the table that relates:
  - Expectations for students,
  - Teacher actions to support students, and
  - Classroom-based indicators of success

# Redefining Student and Teacher Success

Expectations for students	Teacher actions to support students	Classroom-based indicators of success
Most tasks that promote reasoning and problem solving take time to solve, and frustration may occur, but perseverance in the face of initial difficulty is important.	Use tasks that promote reasoning and problem solving; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students are engaged in the tasks and do not give up. The teacher supports students when they are “stuck” but does so in a way that keeps the thinking and reasoning at a high level.
Correct solutions are important, but so is being able to explain and discuss how one thought about and solved particular tasks.	Ask students to explain and justify how they solved a task. Value the quality of the explanation as much as the final solution.	Students explain how they solved a task and provide mathematical justifications for their reasoning.
Everyone has a responsibility and an obligation to make sense of mathematics by asking questions of peers and the teacher when he or she does not understand.	Give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.	Students question and critique the reasoning of their peers and reflect on their own understanding.
Diagrams, sketches, and hands-on materials are important tools to use in making sense of tasks.	Give students access to tools that will support their thinking processes.	Students are able to use tools to solve tasks that they cannot solve without them.
Communicating about one’s thinking during a task makes it possible for others to help that person make progress on the task.	Ask students to explain their thinking and pose questions that are based on students’ reasoning, rather than on the way that the teacher is thinking about the task.	Students explain their thinking about a task to their peers and the teacher. The teacher asks probing questions based on the students’ thinking.

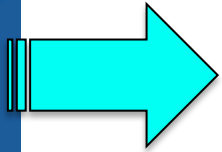


# Embracing Productive Struggle: Redefining Student and Teacher Success

- Examine the table that relates:
  - Expectations for students,
  - Teacher actions to support students, and
  - Classroom-based indicators of success
- What are the implications for our work?



# What Is Productive Struggle?



## Tasks Worth The Struggle

## Setting Up Productive Struggle

## Responding To Productive Struggle



# Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

***By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.***

**Hiebert & Grouws, 2007, pp. 387-388**

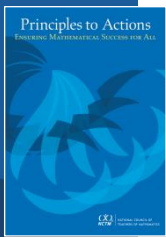


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# Tasks Worth the Struggle

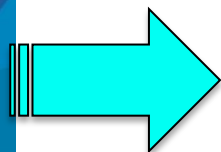
- Form a trio
- Study the tasks at your table.
- Each one, lead one:
  - What is the important mathematics in the task?
  - What misconceptions might students have about that mathematics?
  - What do you want students to struggle with when engaged in the task?





# **What Is Productive Struggle?**

## **Tasks Worth The Struggle**

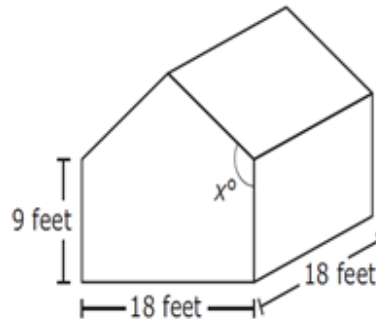


## **Setting Up Productive Struggle**

## **Responding To Productive Struggle**

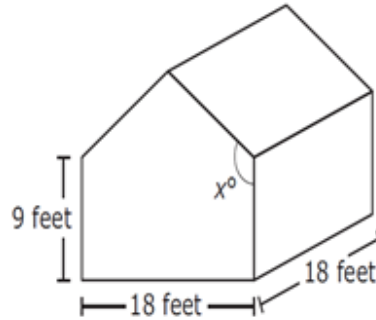


# Launching a Task: Design of a Shed



The base of the shed will be a square measuring 18 feet by 18 feet. The height of the rectangular sides will be 9 feet. The measure of the angle made by the roof with the side of the shed can vary and is labeled as  $x^\circ$ . Different roof angles create different surface areas of the roof. The surface area of the roof will determine the number of roofing shingles needed in constructing the shed. To meet drainage requirements, the roof angle must be at least  $117^\circ$ .

# Launching A Task: The Three Reads Protocol

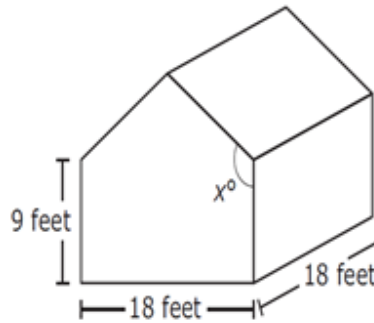


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## Read #1.

As I read the problem, think about “What is this situation about?” Turn and talk to a neighbor.

# The Three Reads Protocol

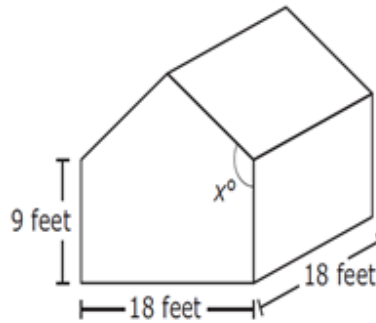


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## Read #2.

A student reads the problem. As s/he reads, listen for quantities in the situation. Turn and talk to a neighbor.

# The Three Reads Protocol



The base of the shed will be a square measuring 18 feet by 18 feet. The height of the rectangular sides will be 9 feet. The measure of the angle made by the roof with the side of the shed can vary and is labeled as  $x^\circ$ . Different roof angles create different surface areas of the roof. The surface area of the roof will determine the number of roofing shingles needed in constructing the shed. To meet drainage requirements, the roof angle must be at least  $117^\circ$ .

## Read #3.

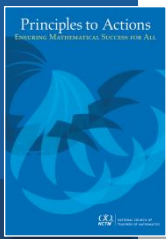
Another student reads the problem aloud. As s/he reads, think about what questions you could ask about the situation. Turn and talk to a neighbor.



# Designing a Shed

*The builder of the shed is considering using an angle that measures  $125^\circ$ . Determine the surface area of the roof if the  $125^\circ$  angle is used.*

- Share your thinking about how you might approach solving this task.
- Debrief
  - How did the Three Reads Protocol help you engage in productive struggle in making sense of this task?
  - How does engaging in productive struggle yourself make you think differently about your students/your classrooms?



# The Three Reads Protocol

1. The teacher reads the problem. Students turn and talk to the prompt: “What is this situation about?”
2. A student reads the problem. Students are guided to listen for quantities during this reading.
3. Another student reads the problem aloud again. Students are guided to generate questions about the situation.

Consider:

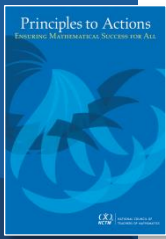
- Why aren’t the quantities identified until step 2?
- Why withhold the question until after step 3?





# Implications for Our Work

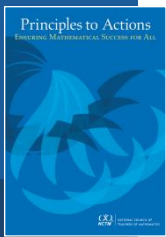
- How does the “Three Reads” protocol support productive struggle?
- How might the protocol support your English learners?
- How might the protocol support **all** students in the classrooms where you work?



# Three Reads with Your Tasks

- With a partner at your table, select one of the tasks you previewed.
- Engage in a discussion re Three Reads:
  - For the first read, what aspects of the problem would we want students to identify?
  - For the second read, what are the quantities (numbers and units) that students will need to identify?
  - For the third read, what questions might students generate?

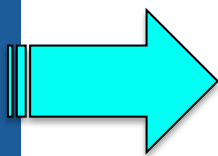




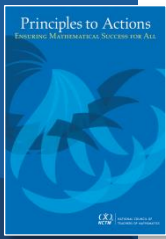
# **What Is Productive Struggle?**

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## **Responding To Productive Struggle**



# Shopping Trip Task

- Take a few minutes to consider the task below.

*Joseph went to the mall with his friends to spend the money that he had received for his birthday. When he got home, he had \$24 remaining. He had spent  $\frac{3}{5}$  of his birthday money at the mall on video games and food.*

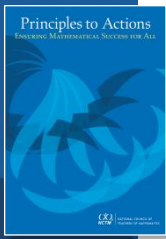
- *How much money did he spend?*
- *How much money had he received for his birthday?*



# Responding to Students When They Struggle

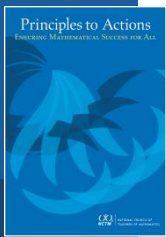
Read the vignette

- How did each teacher respond to her students' struggle?
- How do these responses support or hinder productive struggle?
- What messages are students in each classroom getting about productive struggle?



# Responding to Productive Struggle with Your Tasks

- With a partner at your table, select one of the tasks you previewed.
- Consider how you might respond to productive struggle:
  - Where do you anticipate students will struggle?
  - What actions might you take to ensure that the struggle is productive AND that you don't take over the struggle for your students?



# Planning with the Student in Mind

- Anticipate solutions, thoughts, and responses that students might develop as they struggle with the problem.
- Generate questions that could be asked to promote student thinking during the lesson, and consider the kinds of guidance that could be given to students who showed one or another types of misconception in their thinking.
- Determine how to end the lesson so as to advance students' understanding

Stigler & Hiebert, 1997

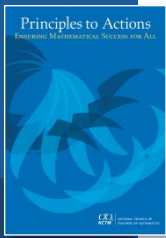


**How will you support  
productive struggle in  
your own setting?**

**Quick Share**



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# RESOURCES

- Illuminations: <https://illuminations.nctm.org>
- *Putting Essential Understanding into Practice* series
  - Addition and Subtraction, Pre K-2;
  - Multiplication and Division, 3-5;
  - Ratio and Proportions, 6-8;
  - Functions, 9-12; Statistics, 9-12
- Journals:
  - *Teaching Children Mathematics*
  - *Mathematics Teaching in the Middle School*
  - *Mathematics Teacher*
- Student Explorations in Mathematics:  
<https://www.nctm.org/sem/>

These grade-band specific professional learning modules are focused on the Effective Teaching Practices and Guiding Principles from *Principles to Actions: Ensuring Mathematical Success for All*.

Presentation, presenter notes, and required materials are provided in each module to support professional learning with teachers through analyzing artifacts of teaching (e.g., mathematical tasks, narrative and video cases, student work samples, vignettes) and abstracting from the specific examples general ideas about how to effectively support student learning.

The Teaching and Learning Modules were developed in collaboration with the [Institute for Learning \(IFL\)](#) at the University of Pittsburgh.

Learning modules are available exclusively to NCTM members. Limited open examples are provided for each grade level and are denoted on modules marked **(EXAMPLE)**.

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[www.nctm.org/referafriend](http://www.nctm.org/referafriend)

## Effective Teaching Practices

[The Case of Mr. Harris and the Band Concert](#)

**Overview of Effective Teaching Practices** – Elementary **(EXAMPLE)**

During this session you will discuss NCTM's publication, *Principles to Actions: Ensuring Mathematical Success for All*, read and analyze a short case of a teacher, and discuss the eight teaching practices and relate them to the case.

[The Case of Millie Brooks and the Half of a Whole Task](#)





# NCTM Interactive Institutes

[www.nctm.org](http://www.nctm.org)

## Effective Teaching with *Principles to Actions*:



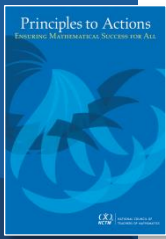
Implementing College- and Career-Readiness Standards

NCTM INTERACTIVE INSTITUTES

**Grades PK-5, 6-8, High School,  
and Administrators  
February 5-6, 2016  
Dallas**



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# Thank You!

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