Developing and Assessing Addition Fact *Fluency*

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Our Session

- Overview of basic fact *fluency*
- Developing basic fact *fluency*
- Assessment strategies for basic fact *fluency*
- Results of a strategy-based approach to learning basic facts

What do we mean by fluency with basic facts?

CCSS-M Descriptions

Grade 1 (1.0A.C.6):

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.0A.B.2):

Fluently add and subtract within 20 using mental strategies (reference to **1.0A.C.6**). By end of Grade 2, know from memory all sums of two one-digit numbers.

CCSS-M Descriptions

Grade 1 (1.0A.C.6):

Add and subtract within 20, ***demonstrating fluency*** for addition and subtraction within 10. ***Use strategies*** such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums.

Grade 2 (2.0A.B.2):

Fluently add and subtract within 20 ***using mental strategies*** (reference to **1.0A.C.6**). By end of Grade 2, ***know from memory*** all sums of two one-digit numbers.

This would suggest that ***fluency is different*** from automatic retrieval. Research heavily supports this…

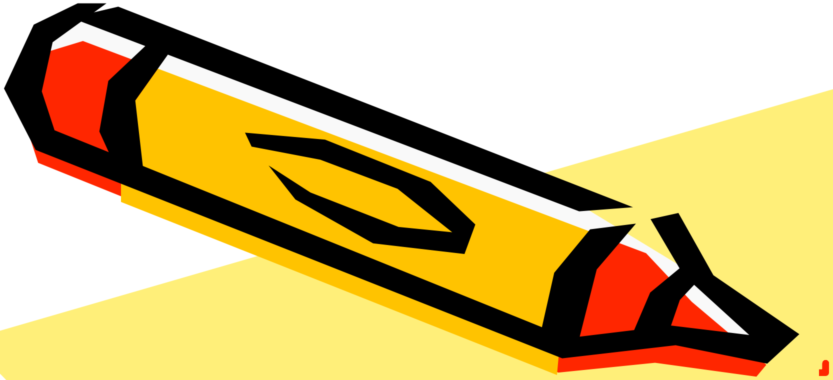
So, what does fluency *really* mean?

The Common Core State Standards for Mathematics (CCSS-M) describes procedural fluency as "skill in carrying out procedures **flexibly**, **accurately**, **efficiently** and **appropriately**"

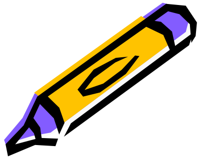
(CCSSO, 2010, p. 6).

Procedural Fluency

Knowing from Memory $\neq$ Memorization



Developing Addition Fact Fluency



Mastering Basic Facts

Phase 1: Counting

(counts with objects or mentally)

Phase 2: Deriving

(uses reasoning strategies based on known facts)

Phase 3: Mastery

(efficient production of answers)

Adapted from Baroody, 2006

Mastering Basic Facts

Phase 1 Example: Solving 5 + 7 by counting on from 5.

Phase 2 Example: Solving 5 + 7 by starting from 5 + 5 = 10 and then adding 2 more to get 12.

Phase 3 Example: Answering 5 + 7 = 12 with little/no hesitation.

Mastering Basic Facts

Some programs attempt to push children from Phase 1 directly to Phase 3 through drill and rote memorization.

- What aspects of fluency are/are not developed when this happens?
- What are we communicating to children about what it *means to do mathematics* when we do this?

Mastering Basic Facts

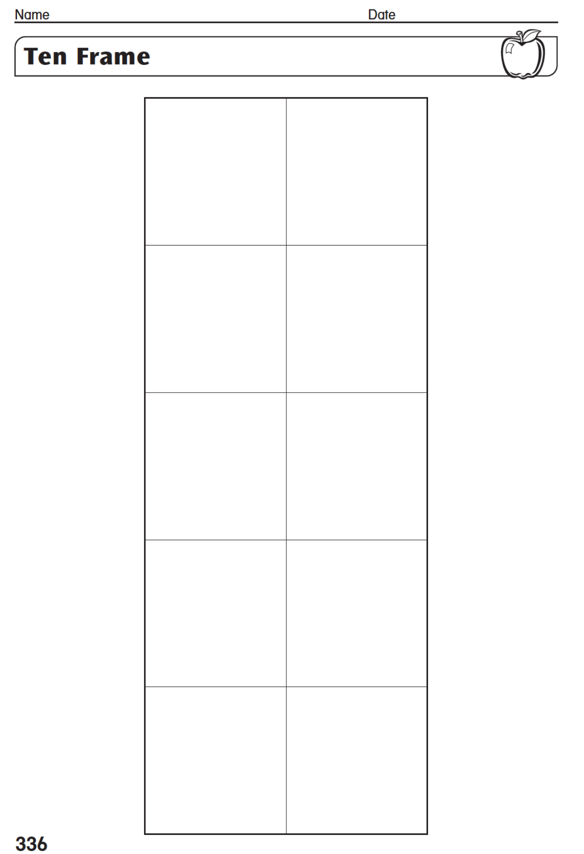
In contrast, to develop true *fluency*, children need adequate time to make sense of each operation and develop strategies in Phases 1 and 2. Through repeated, meaningful practice, children then naturally progress to Phase 3. This transition occurs with different groups of facts at different times throughout the year.

Meaningful Activities for Learning Basic Addition Facts

- Solve Number Stories
- Use Quick Looks with dot patterns and ten frames
- Discuss/write about strategy use
- Play basic facts games with a focus on reasoning strategies

Alternative Approaches to Ten Frames

- What are different ways children could arrange 8 counters on a ten frame?
- What different ideas about 8 would each way elicit?
- *


Name Date

Ten Frame

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Phase 1 → Phase 2

Quick Looks

PreK:

- ✓ number recognition
- ✓ representation

Kindergarten:

- ✓ decomposing and recomposing numbers
- ✓ subitizing

Grade K- 2:

- ✓ recognizing and developing strategies for basic +/– combinations

Developing Fact Strategies through Quick Looks

- What mathematical concepts and skills can Quick Looks encourage?
- Why is the Quick Look format important?
- How does the arrangement of dots/counters impact the development of mathematical ideas?

Developing Fact Strategies through Quick Looks

- What mathematical ideas did children make use of when describing how they found the totals?
- What challenges do you anticipate children having with developing fact strategies via Quick Looks?

* ** ***

Reasoning Strategies for Addition Facts K-2 Trajectory

Foundational Fact Strategies

- Sums within 5 K
- +/– 1 or 2 K
- Doubles K 1

  (2 + 2, 6 + 6, etc.)
- Combinations of Ten

  (3 + 7, 8 + 2) K 1

Derived Fact Strategies

- Near Doubles 1 2

  (6 + 7, 8 + 7)
- Making Ten 1 2

  (8 + 3, 9 + 5)

Everyday Mathematics®

The University of Chicago School Mathematics Project

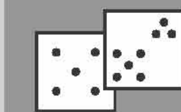
QUICK LOOK ACTIVITY BOOK

McGraw Hill Education

Decomposing Dot Patterns

Object Practice recognizing images composed of familiar images

Materials Quick Look Cards 22, 23, 25, 30, 31



Dot Patterns

Options for Differentiation

GRADES	K	1	2	3
Enrichment	•			
Core Program		•		
Reteaching			•	

Supplements Grade 1 Lesson 2-3.

As children progress through the year, their exposure to dot patterns through Quick Looks and dice games encourages the development of their abilities to subitize, or instantly recognize, small quantities of dots. In this activity, children will use that ability to decompose complicated dot patterns into smaller, recognizable components.

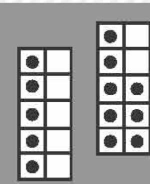
Show Quick Look Cards 25, 23, 22, 30, and 31 for 2-3 seconds each. Be sure to give a second look for each card. After each card, invite children to share both the number they saw and how they saw it.

Highlight strategies that involve decomposing the image into smaller, familiar images and then either counting on or using more advanced addition fact strategies as opposed to counting all the dots. For example, children might recognize Quick Look Cards such as 23 and 25 as doubles, and might either count on 4 more from 5 for Card 30, or apply a Near-Doubles Strategy, such as $5 + 5 = 10$, $10 - 1 = 9$. Encourage children to try these more efficient strategies on subsequent images.

Ten Frame Doubles

Object Recognize images of small doubles combinations

Materials Quick Look Cards 51, 54, 56, 58, 61, 67, 70



Ten Frames

Options for Differentiation

GRADES	K	1	2	3
Enrichment				
Core Program	•	•		
Reteaching			•	

Supplements Kindergarten Lesson 9-10 and Grade 1 Lesson 4-7 and Supports Grade 2 Lesson 2-3

Doubles addition facts, such as 2 + 2 = 4 or 7 + 7 = 14, are important facts for children to learn before the end of Grade 1, as they become key "helper facts," or known facts that can be used for deriving challenging related facts, such as 7 + 8. This activity provides children with practice recognizing small doubles combinations.

Show Quick Look Cards 51, 56, 67, 58, 70, 54, and 61 for 2-3 seconds each. Be sure to give a second look for each card. After each card, invite children to share both the number they saw and how they saw it.

Cards are sequenced to help children recognize doubles, and so it will be important to highlight this idea once children have recognized it. For example, children might describe solving Quick Look Card 56 by remembering that Card 51 showed a stack of 2 dots and that now they see 2 stacks of dots to make 4 dots total. As children explain how they related the images, it may help to have both cards available to them. To further reinforce these ideas, you may wish to record number sentences to match the doubles they describe, such as 2 + 2 = 4 for Card 56.

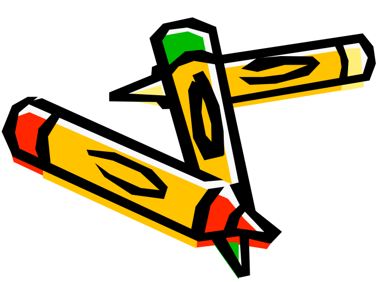
Phase 2 → Phase 3

Meaningful Practice

"Practice that follows substantial initial experiences that support understanding and emphasize 'thinking strategies' has been shown to improve student achievement with single-digit calculations." (NRC, 2001).

Key addition fact games to try in your classroom

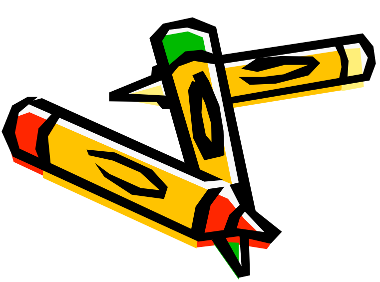
- *Fishing for 10* (Foundational)
- *Salute!* (General Practice)



Games as Meaningful Practice

As you explore each game, consider the following:

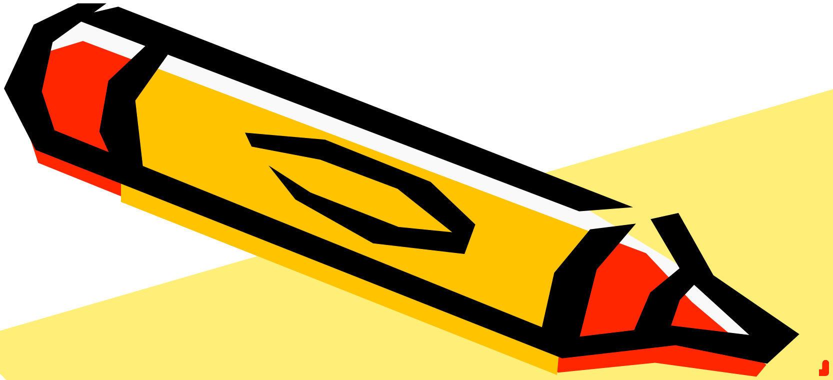
- What is the mathematical content of each game?
- What makes these games examples of “meaningful practice”?
- How might you differentiate these games?
- What questions do you have about these games?



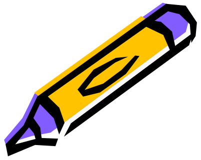
Meaningful Practice

Games:

- Are engaging
- Provide opportunities for strategy discussion and assessment
- Should be sequenced developmentally (for example, playing combinations of ten games before exploring making ten strategies)
- Can be targeted practice or general practice
- Lend to differentiation



Assessing Basic Fact Fluency



The Common Core State Standards for Mathematics (CCSS-M) describes <u>procedural fluency</u> as "skill in carrying out procedures **flexibly**, **accurately**, **efficiently** and **appropriately**"

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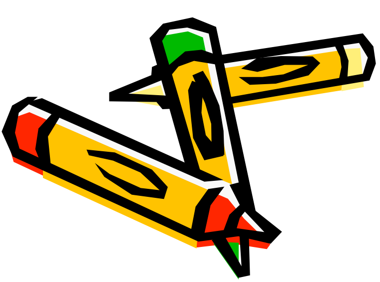


Assessing Basic Fact Fluency

What can we learn from this assessment related to:

- ✓ Flexibility
- ✓ Accuracy
- ✓ Efficiency
- ✓ Appropriate Strategy Use

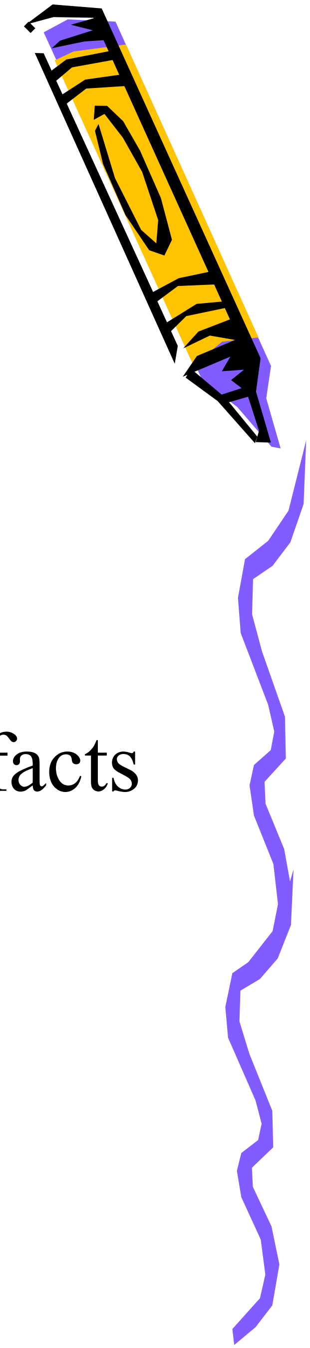
Aspects of Fluency

- ❑ Flexibility
- ❑ Accuracy
- ❑ Efficiency
- ❑ Appropriate Strategy Use

Timed Tests

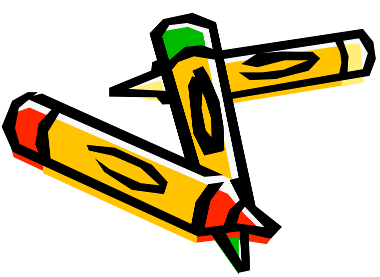
(1) 19 + 1 =	(10) 2 + 4 =	(19) 10 + 2 =	(28) 1 + 3 =
(2) 9 + 4 =	(11) 9 + 1 =	(20) 5 + 3 =	(29) 5 + 4 =
(3) 5 + 4 =	(12) 15 + 1 =	(21) 14 + 3 =	(30) 2 + 2 =
(4) 8 + 4 =	(13) 16 + 1 =	(22) 5 + 3 =	(31) 11 + 1 =
(5) 14 + 4 =	(14) 16 + 1 =	(23) 7 + 2 =	(32) 14 + 1 =
(6) 15 + 3 =	(15) 8 + 4 =	(24) 5 + 1 =	(33) 7 + 4 =
(7) 14 + 4 =	(16) 15 + 2 =	(25) 9 + 2 =	(34) 6 + 4 =
(8) 15 + 1 =	(17) 9 + 1 =	(26) 2 + 1 =	(35) 4 + 2 =
(9) 14 + 2 =	(18) 15 + 4 =	(27) 12 + 1 =	(36) 10 + 1 =

Timed Testing: Issues



The issues with timed testing include:

1) Limitations as an assessment tool
2) Can impede progress when mastering facts
3) Psychological effects



Timed Testing: Issues

1) Limitations with respect to the four components of fluency.

A child finishes a 20-fact timed test in 60 seconds.

- Did the child spend 3 seconds on each fact?

  Or…

- Did the child spend 1 second on 16 facts and 10 seconds each on 4 of the facts?

Timed Testing: Issues

2) Can impede progress in mastering facts

A study of nearly 300 first graders found that children who were *more frequently* exposed to timed testing demonstrated *lower* progress towards knowing facts from memory than their counterparts.

Henry & Brown, 2008

Timed Testing: Issues

2) Can impede progress in mastering facts

A study of 2^{nd} and 4^{th} graders showed that children in experimental classrooms with a focus on strategy development vastly outperformed those in the control classrooms, even on traditional timed assessments.

Thornton, 1978

Timed Testing: Issues

3) Can have negative psychological effects

- The stress that children experience with timed testing is not experienced when they complete the same tasks in untimed conditions.
- "Evidence strongly suggests that timed tests cause the early onset of math anxiety for students across the achievement range."

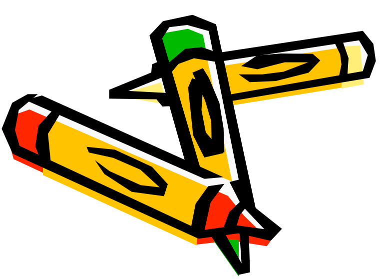
Boaler, 2014

Timed Testing: Issues

3) Can have negative psychological effects

Anxiety over timed testing is often not related to achievement. Even high-achieving children share concerns such as “I feel nervous. I know my facts, but this just scares me.”

Boaler, 2012



Timed Testing: Issues

3) Can have negative psychological effects

Children experience math anxiety as early as first grade and this anxiety is not correlated with reading achievement. This suggests that the children’s anxiety is specific to mathematics, not general academic work.

Ramirez et al. 2013

Timed Testing: Issues

3) Can have negative psychological effects

Children who tended to use more sophisticated mathematical strategies experienced the most negative impact on achievement due to math anxiety. Thus, it appears that some of our best mathematical thinkers are often those most negatively impacted by timed testing.

Ramirez et al. 2013

Timed Tests: Alternatives

- ✓ Observation
- ✓ Interviews
- ✓ Writing prompts
- ✓ Strategy quizzes
- ✓ Self-assessment

Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

Journal Writing

If your friend didn't know the answer to 4 + 5, how would you tell him to figure it out?

Aspects of Fluency

- Flexibility
- Accuracy
- Efficiency
- Appropriate Strategy Use

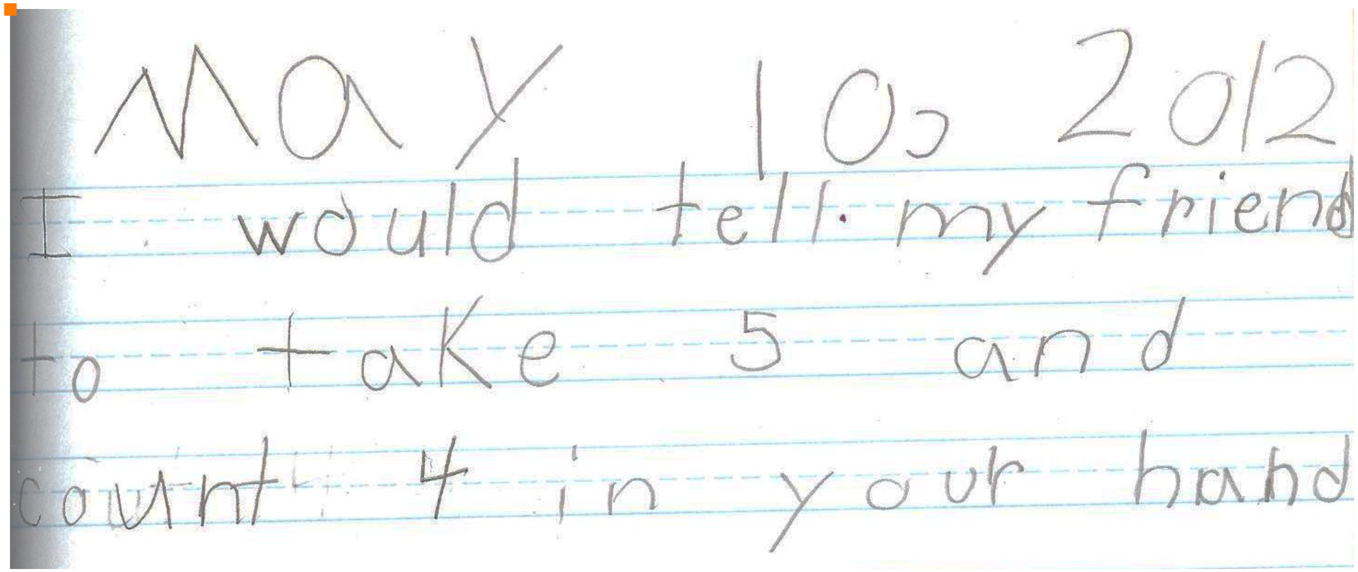
Journal Writing

Review the four student responses:

What might you infer about each child's level of fluency?

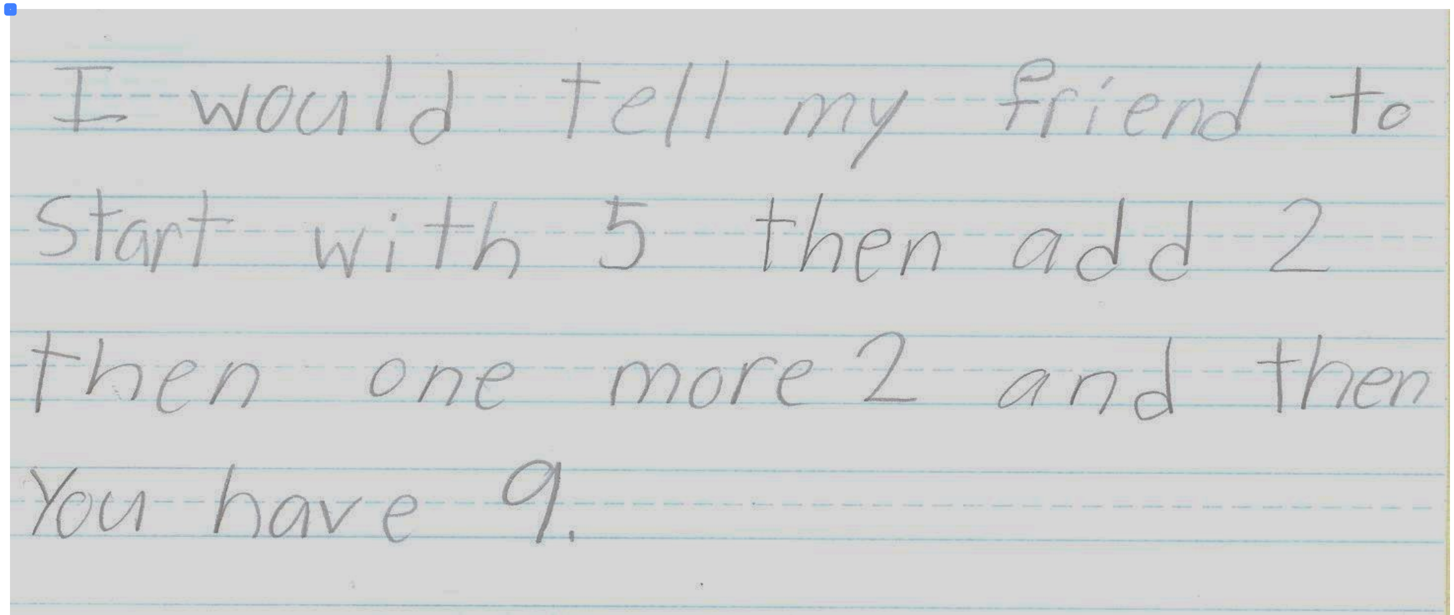
Timed Tests: Alternatives

Journal Writing Samples



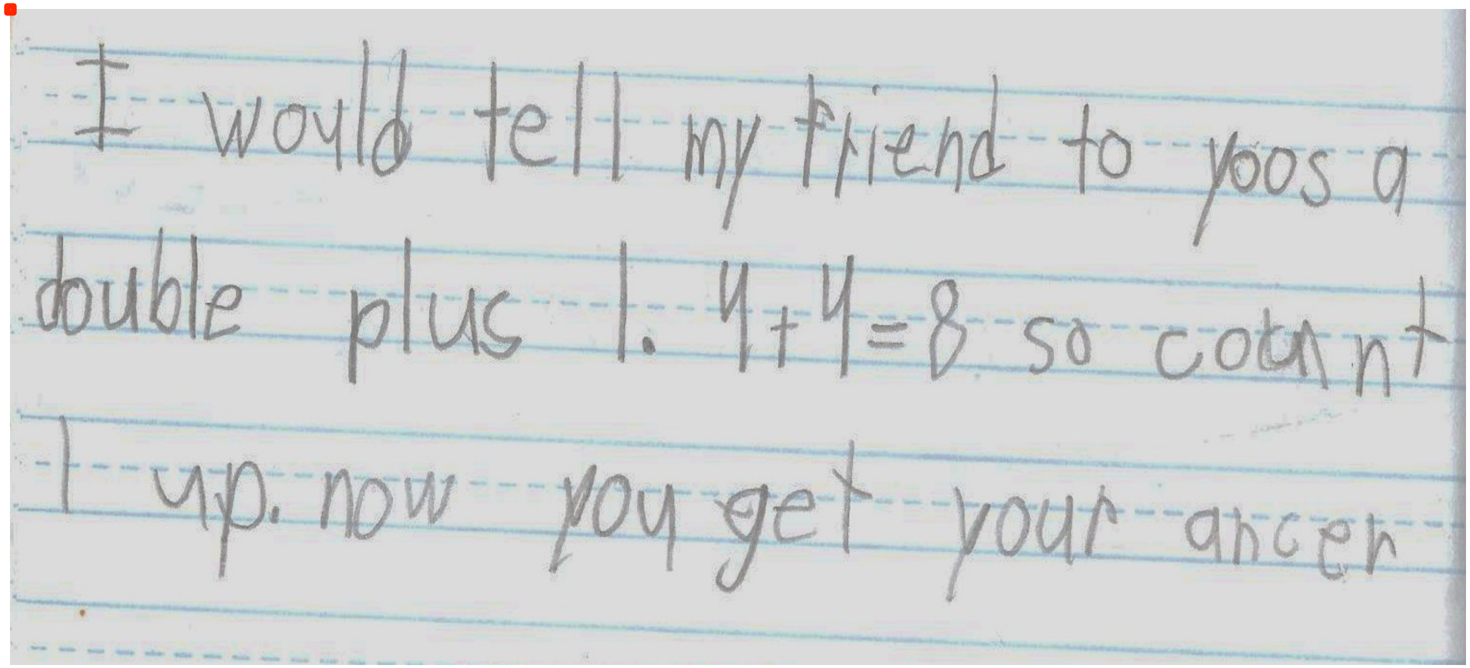
Timed Tests: Alternatives

Journal Writing Samples

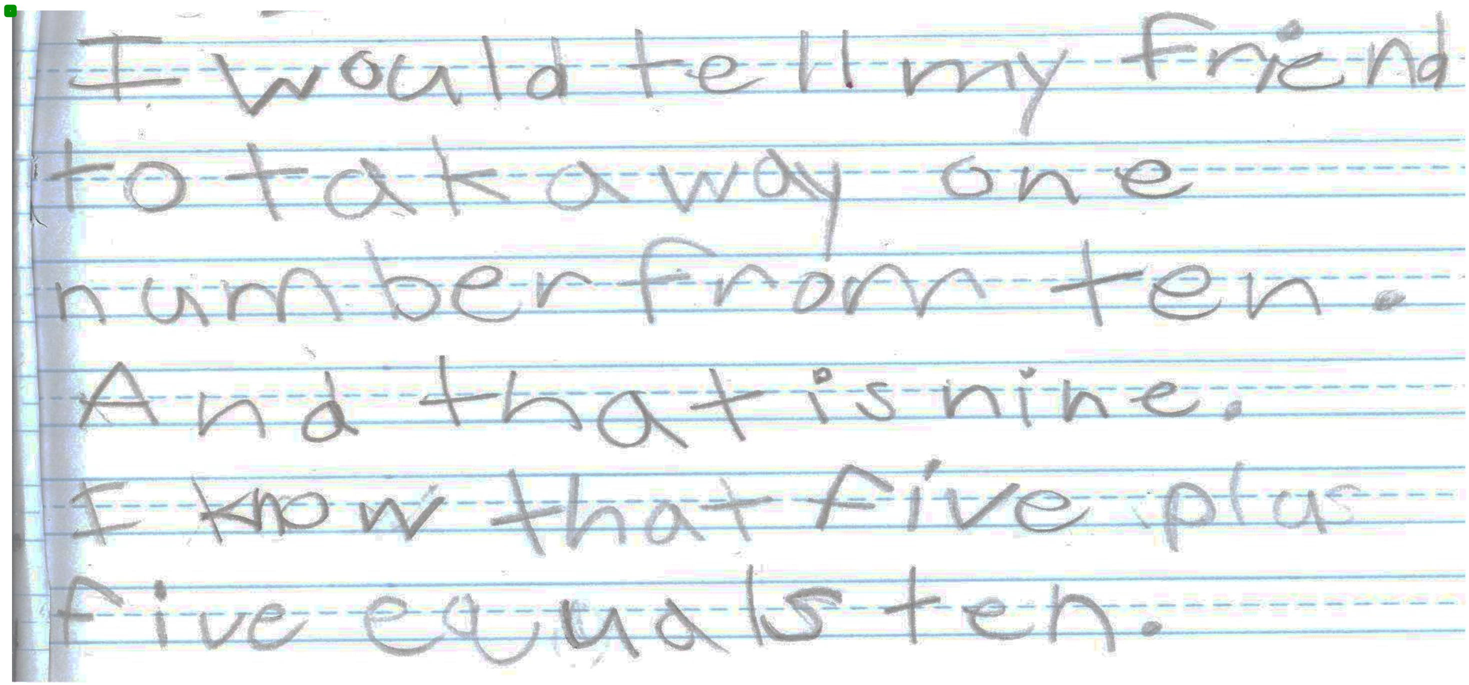


I would tell my friend to Start with 5 then add 2 then one more 2 and then You have 9.

Timed Tests: Alternatives
Journal Writing Samples


I would tell my friend to yoos a double plus 1. 4+4=8 so count 1 up. now you get your ancer

Timed Tests: Alternatives
Journal Writing Samples



I would tell my friend to take away one number from ten. And that is nine. I know that five plus five equals ten.

Tests: Alternatives

Addition Fact Fluency Quiz

Solve these problems and tell how you solved it.

4 + 5 = ______ Check one: ____ I used this strategy: _____________
____ I just knew.

10 + 6 = ______ Check one: ____ I used this strategy:_____________
____ I just knew.

6 + 2 = ______ Check one: ____ I used this strategy: _____________
____ I just knew.

Timed Tests: Alternatives

Interviews: "Riley" at the end of Grade 1

At first, Riley incorrectly answers that 8 + 7 = 17.

- Which components of fluency were you able to assess through the follow-up questions?
- How does this assessment experience compare to traditional facts assessments?

Timed Tests: Alternatives

Interviews: “Riley” at the end of Grade 2

Riley solves 5 + 9 in one second.

- Which components of fluency were you able to assess through the follow-up questions?

- How does this assessment experience compare to traditional facts assessments?

Interviews

Flexibility	Accuracy
Solve 6 + 7 using one strategy. Now try solving it using a different strategy.	What is the answer to 7 + 8? How do you know it is correct (how might you check it)?
Efficiency	**Appropriate Strategy Selection**
For which facts did you **just know**? For which facts did you **use a strategy**?	Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.

Feedback to Parents

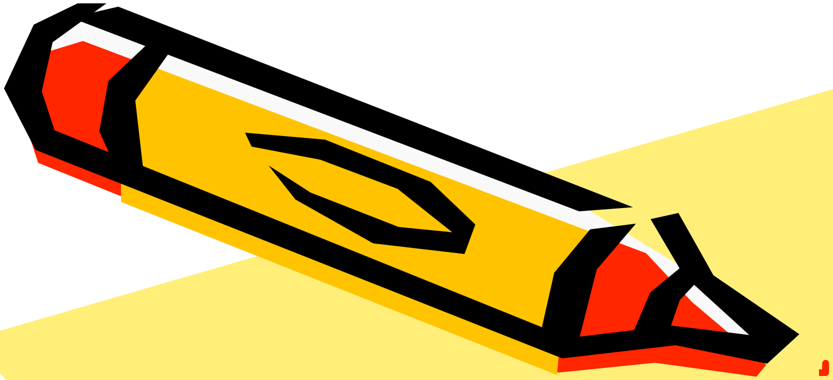
Although it is important for your child to practice **all** addition and subtraction facts this summer to maintain fluency, the end-of-year addition facts assessment showed your child,

________________________________, will benefit from focusing in particular on the following addition facts:

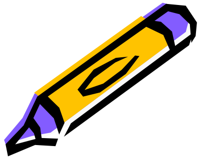
Assessing Basic Fact *Fluency*

Have you had it with timed tests, which present a number of concerns and limitations? Try a variety of alternative assessments from this sampling that allows teachers to accurately and appropriately measure children's fact fluency.

By Gina Kling and Jennifer M. Bay-Williams



What Works with Addition Fact Fluency



Promising Results

Kling (2013) followed 30 children from 2 different schools, 4 different classrooms, in Kalamazoo, MI.; 21 had no exposure to timed testing or drill in the classroom in either 1st or 2nd grade. By the end of 2nd grade those 21 children demonstrated:

- automaticity with addition facts (solved within 3 seconds) 95% of the time.
- Strategy use (e.g., making ten) so quickly that it was impossible to distinguish between strategy use and “knowing from memory.”

… and RETENTION

18 of the children were interviewed once more in the first week of 3rd grade *prior to any fact strategy review*.

- Children demonstrated automaticity 91% of the time.
- Were accurate and used strategies (not counting) 99.99% of the time.

Conclusions

Basic facts instruction, practice, and assessment must truly encompass all four components of fluency:

- **Flexibility**
- **Accuracy**
- **Efficiency**
- **Appropriate strategy use**

Traditional approaches to teaching and assessing basic facts do not support these goals. But when *fluency* is the focus, children can achieve meaningful mastery of basic facts.

Assessing Your Fact [teaching] Fluency

Whip Around

At your table, share your answer to one of these prompts:

- An activity I will use is…
- An assessment strategy I will use is…
- Something surprising I heard is…
- I still have questions about…

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