Developing Essential Understandings of Addition & Subtraction

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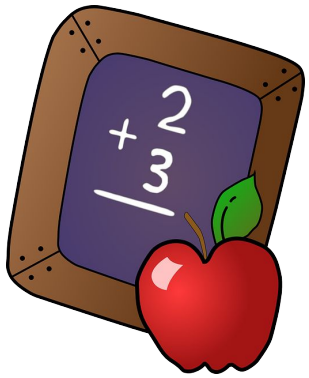

Addition and subtraction are essential to understand *deeply* because they are foundational to many other mathematical concepts.

Karp, Karen, Jennifer Bay-Williams, Rose Mary. Zbiek, and Janet Caldwell. "Introduction." *Developing Essential Understanding of Addition and Subtraction ForTeaching Mathematics in Pre-K-Grade 2*. Reston: National Council of Teachers of Mathematics, 2011. 2. Print. A Ser. for Teaching Mathematics.

What does this mean for K-2?

Activities in **kindergarten** should center on joining and separating sets

Build on sequential counting

- ★ Using counters:
  Addition:Count all; Count on from the first number; Count on from the larger number
  Subtraction: count back, count down, count up from
- ★ Using number cubes and dominoes
- ★ Using five-frame and ten-frame

In **grade 1**, instruction should focus on developing students' understanding of addition and subtraction as well as related facts and strategies associated with these operations.

In **grade 2**, the instructional focus should shift to helping students develop quick recall of addition and related subtraction facts, as well as **fluency** with multi-digit addition and subtraction

Karp, Karen, Jennifer Bay-Williams, Rose Mary. Zbiek, and Janet Caldwell. "Introduction." *Developing Essential Understanding of Addition and Subtraction ForTeaching Mathematics in Pre-K-Grade 2*. Reston: National Council of Teachers of Mathematics, 2011. 7-8. Print. A Ser. for Teaching Mathematics.

Addition and Subtraction Situations by Grade Level

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
Put Together/ Take Apart	**Total Unknown**	**Both Addends Unknown**[1]	**Addend Unknown**[2]
	A red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
Compare	**Difference Unknown**	**Bigger Unknown**	**Smaller Unknown**
	"How many more?" version. Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy?	**"More" version suggests operation.** Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have?	**"Fewer" version suggests operation.** Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have?
	"How many fewer?" version Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	**"Fewer" version suggests wrong operation.** Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	**"More" version suggests wrong operation.** Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Pink shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Blue shaded problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2.

Adapted from CCSS, p. 88, which is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity, National Research Council, 2009, pp. 32–33. Charts adapted from "Progressions for CCSS in Math, K, Counting and Cardinality; K-5 Operations and Algebraic Thinking"

[1] This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with blanks on the left help children understand that = does not always mean "makes" or "results in" but always means "is the same number as." Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all the decompositions of a number and reflecting on the patterns involved.

[2] Either addend can be unknown; both variations should be included.

The National Research Council concluded that attaining computational fluency - the **efficient**, **appropriate**, and **flexible** application of single-digit and multi-digit calculation skills - is an essential aspect of mathematical proficiency.

Adapted from *Adding It Up: Helping Children Learn Mathematics* (Kilpatric, Swafford, and Findell 2001)

FLUENCY

Developing Fluency

- **Fast and accurate** recall and use of **basic facts**.
- **Automaticity** is the ability to recall answers with both speed and accuracy at an unconscious level.
- Students develop quick fact recall by using their own or common **strategies** to promote their retention.

Games

Bay-Williams, Jennifer. "Enriching Addition and Subtraction Fact Mastery through Games." *Teaching Children Mathematics* 21.4 (2014): 238-47. Print.

Addition Facts Table: 0–9

	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

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2			4	5	6	7	8	9	10	11
3				6	7	8	9	10	11	12
4					8	9	10	11	12	13
5						10	11	12	13	14
6							12	13	14	15
7								14	15	16
8									16	17
9										18

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2			4	5	6	7	8	9	10	11
3				6	7	8	9	10	11	12
4					8	9	10	11	12	13
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3				6	7	8	9	10	11	12
4					8	9	10	11	12	13
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6							12	13	14	15
7								14	15	16
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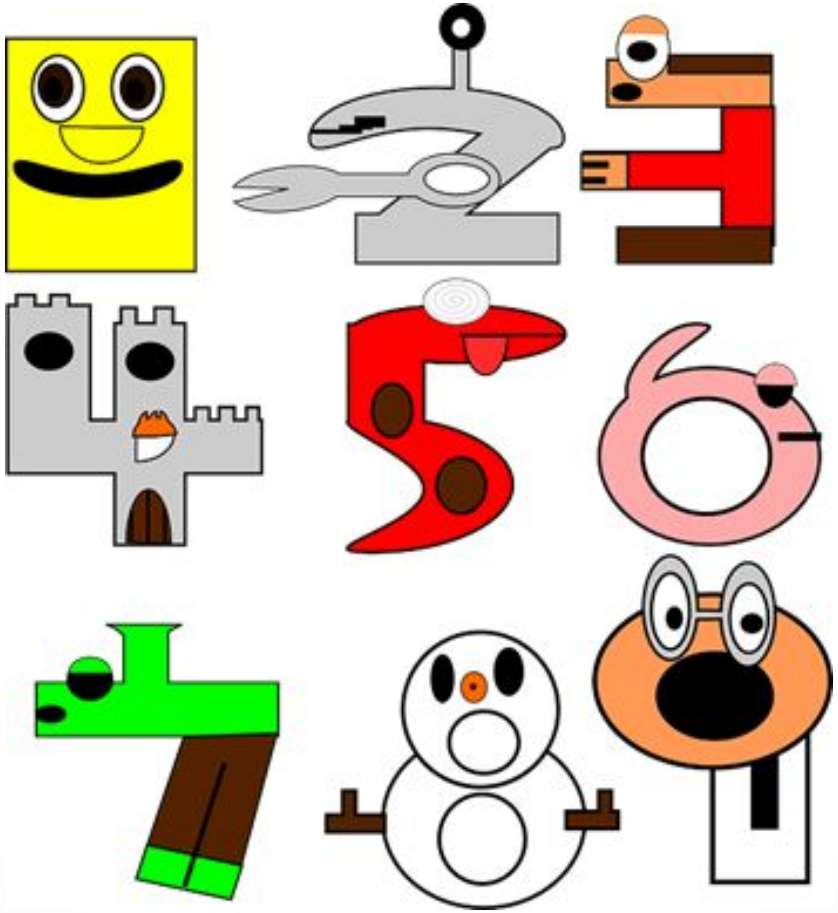
9										18

Common Addition Strategies

- **Doubles**… all the facts that have two addends that are the same quantity
- **Commutative Property**… Ex: realization that 2 + 5 and 5 + 2 result in the same sum
- **Additive identity**… adding 0 to a number
- **1 or 2 more than**… increasing a given number by 1 or 2
- **Near doubles**… **derive** facts from known doubles
- **Combinations that make 10**… addition facts for sums of 10
- **Using a Make Ten Strategy**… addition two one digit numbers with a sum above ten

Number Talks



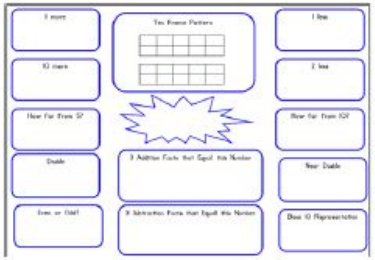
Common Subtraction Strategies

- **Think addition** (fact families)… natural and unconscious when prompted to look at the relationship of the total
- **Down over 10**… using 10 as a benchmark
- **Take from 10**… whole number combinations that sum to 10
- **Compare**… (Part-whole)

Basic Fact Practice

Number Relationship Mat

Materials: laminated mats, dry erase markers



1. Work with a friend. Each of you needs to collect a mat and dry erase marker.
2. Choose a number between 6 and 20 that you will both write inside the star on your mat.
3. Fill in all the sections on your mats.
4. Compare your mats. What is the same about your mats? What is different?

1 more

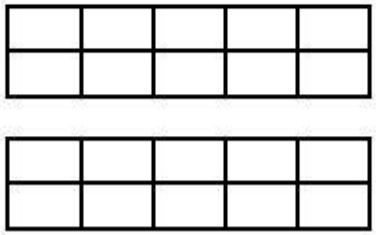
10 more

How far from 5?

Double

Even or Odd?

Ten Frame Pattern



3 Addition Facts that Equal this Number

3 Subtraction Facts that Equal this Number

1 less

2 less

How far from 10?

Near Double

Base 10 Blocks

http://www.k-5mathteachingresources.com/

Development for basic fact mastery

	Addition	Subtraction
COUNTING	Direct modeling Counting objects and fingers	Counting objects
	Counting abstractly	Counting fingers
REASONING	Add 0; add 1 or 2; Commutative Property	Subtract 0; subtract 1 or 2
	Foundational facts	Inverse/complement of foundational facts
	Derived facts	Inverse/complement of derived facts
RETREIVAL	From long-term memory	From long-term memory

Required fluency

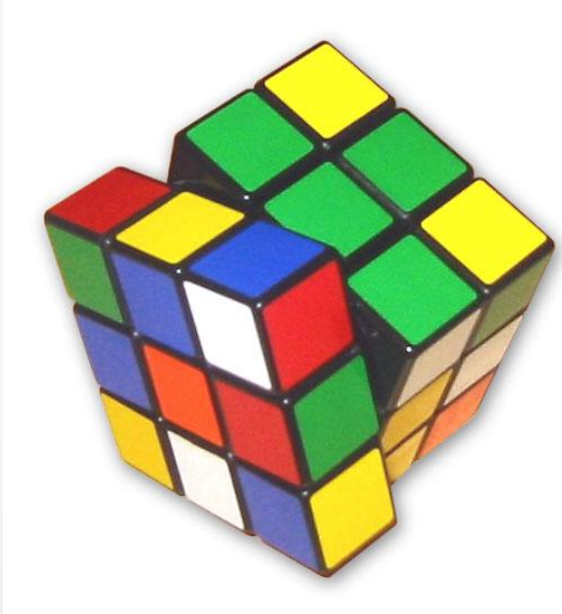
Grade	Standard	Fluency
K	K.OA.5.	Add and subtract **within 5**
1	1.OA.6.	Add and subtract **within 10**
2	2.OA.2.	Add and subtract **within 20** using mental strategies (know from memory all sums of two one-digit numbers)
2	2.NBT.5.	Add and subtract **within 100**
3	3.NBT.3	Add and subtract **within 1000**
4	4.NBT.4	Add and subtract **multi-digit whole numbers** using the standard algorithm

https://grade4commoncoremath.wikispaces.hcpss.org/

GWAEA Mathematical Understanding

tinyurl.com/mathematicalunderstanding

Solving with Multiple Methods



In order to fully implement Common Core State Standards, it is necessary to adjust some traditional teaching practices. One way to do this is to de-emphasize the answer and focus on the use of multiple strategies.

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