The Calculus Of Corvettes

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Jackie Murawska
Saint Xavier University
Chicago, Illinois
murawska@sxu.edu

Keith Nabb
Moraine Valley Community College
Palos Hills, Illinois
nabb@morainevalley.edu

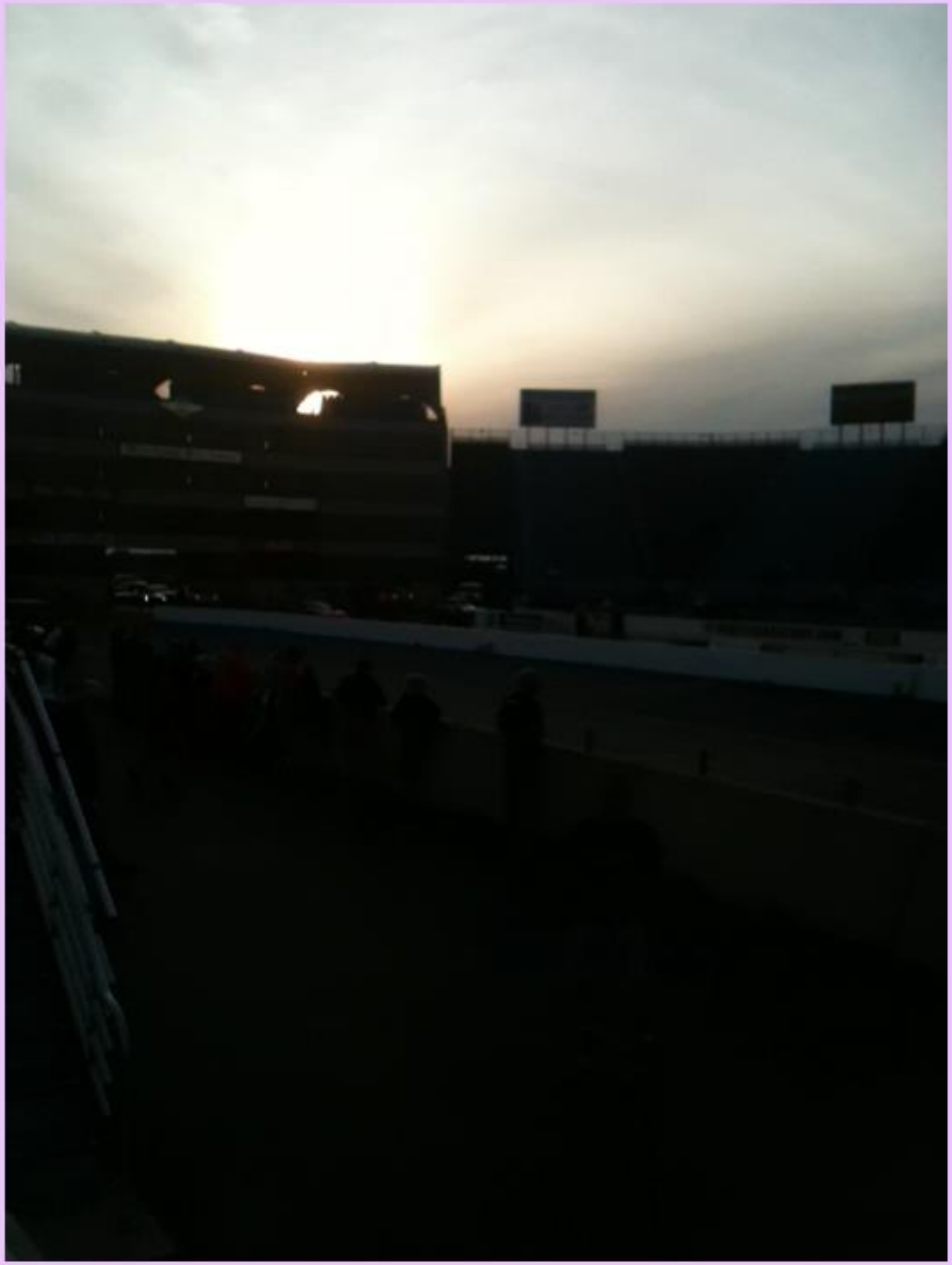
The Corvette Problem

1998 Corvette

Race Day

In June 2013, Al Lewis raced his Corvette on the quarter mile drag strip in Joliet

Race Day



Time Slip

“How many seconds did it take me to reach a speed of 60 mph?”

Welcome to
ROUTE 66 RACEWAY
Friday Night Test & Tune
June 14, 2013
Radio 88.5 FM
Event Hotlines 855-794-RACE

7:36 PM
14/JUN/2013

O'Reilly Auto Toyota

———— LEFT .... RIGHT

	LEFT	RIGHT
Car # ...	587	585
Class ...		
DIAL ...		
R/T ...	.264	.734
60' ...	1.626	2.063
330 ...	4.673	5.387
1/8 ...	7.250	8.025
MPH ...	94.99	94.23
1000 ...	9.498	10.312
E.T. ...	11.416	12.259
MPH ...	117.15	116.14

Left 1st 1.3131
Compulink AUTOSTART OFF

Rnd # TO 1155/1156

..... CompuLink StarTRAK 2012

Time Slip

Corvette

Car #	...	587	585
Class	...		
DIAL	...		
R/T	...	.264	.734
60'	...	1.626	2.063
330	...	4.673	5.387
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MPH	...	94.99	94.23
1000	...	9.498	10.312
E.T.	...	11.416	12.259
MPH	...	117.15	116.14

Student Worksheet

In June 2013, a 1998 Corvette raced down the quarter mile drag strip at the Route 66 Raceway in Joliet, Illinois. The following data were provided on a time slip after the race:

Distance	Time
0 feet	0.734 seconds
60 feet	2.063 seconds
330 feet	5.387 seconds
$\frac{1}{8}$ mile	8.025 seconds
1000 feet	10.312 seconds
$\frac{1}{4}$ mile	12.259 seconds

We also know that the car crossed the finish line at a speed of 116.14 mph. For sports cars, a common measure of performance is the number of seconds it takes the car to accelerate from 0 to 60 mph. The driver of this Corvette would like to know, according to these data, how many seconds it took him to reach a speed of 60 mph. Your task is to determine this time, and support your claim mathematically. Include an explanation in words and a graph, if necessary.

Let’s solve it!

Sample Student Work: Kenneth

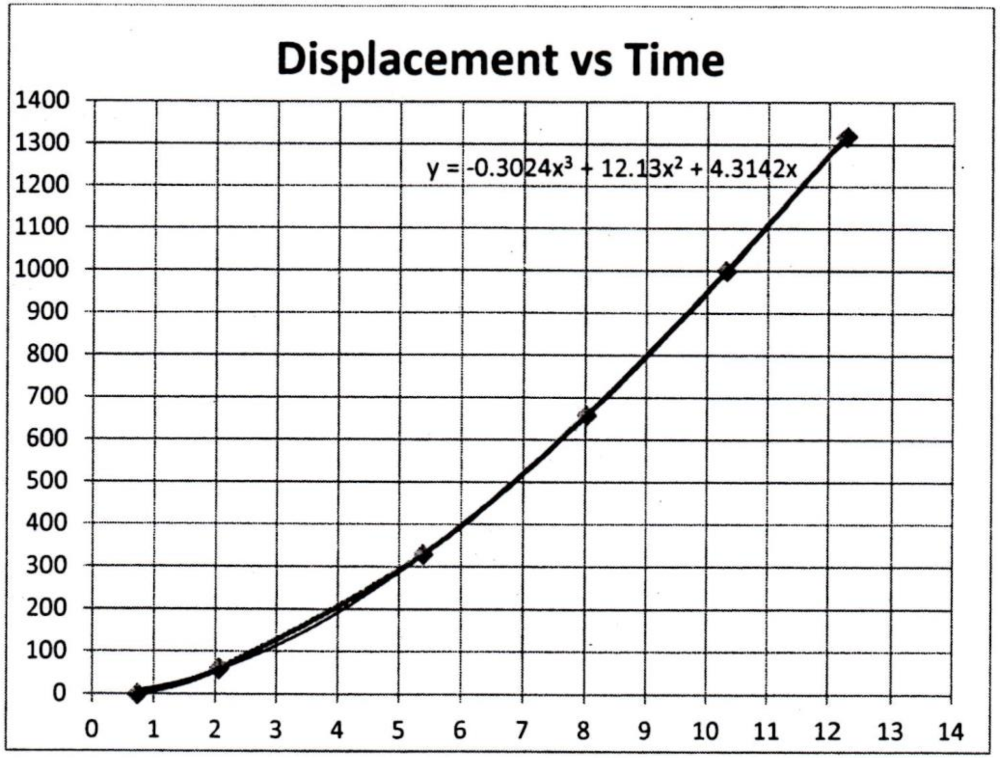
Kenneth's table showing the Corvette's acceleration is not constant.

$$a_n = \frac{v_n - v_{n-1}}{t_n - t_{n-1}}$$

Acceleration (ft/s²)	Time (s)
0	0.734
33.97045	2.063
10.85461	5.387
16.62901	8.025
10.30678	10.312
8.05806	12.259

Sample Student Work: Kenneth

Kenneth's regression curve, both graphically and algebraically.

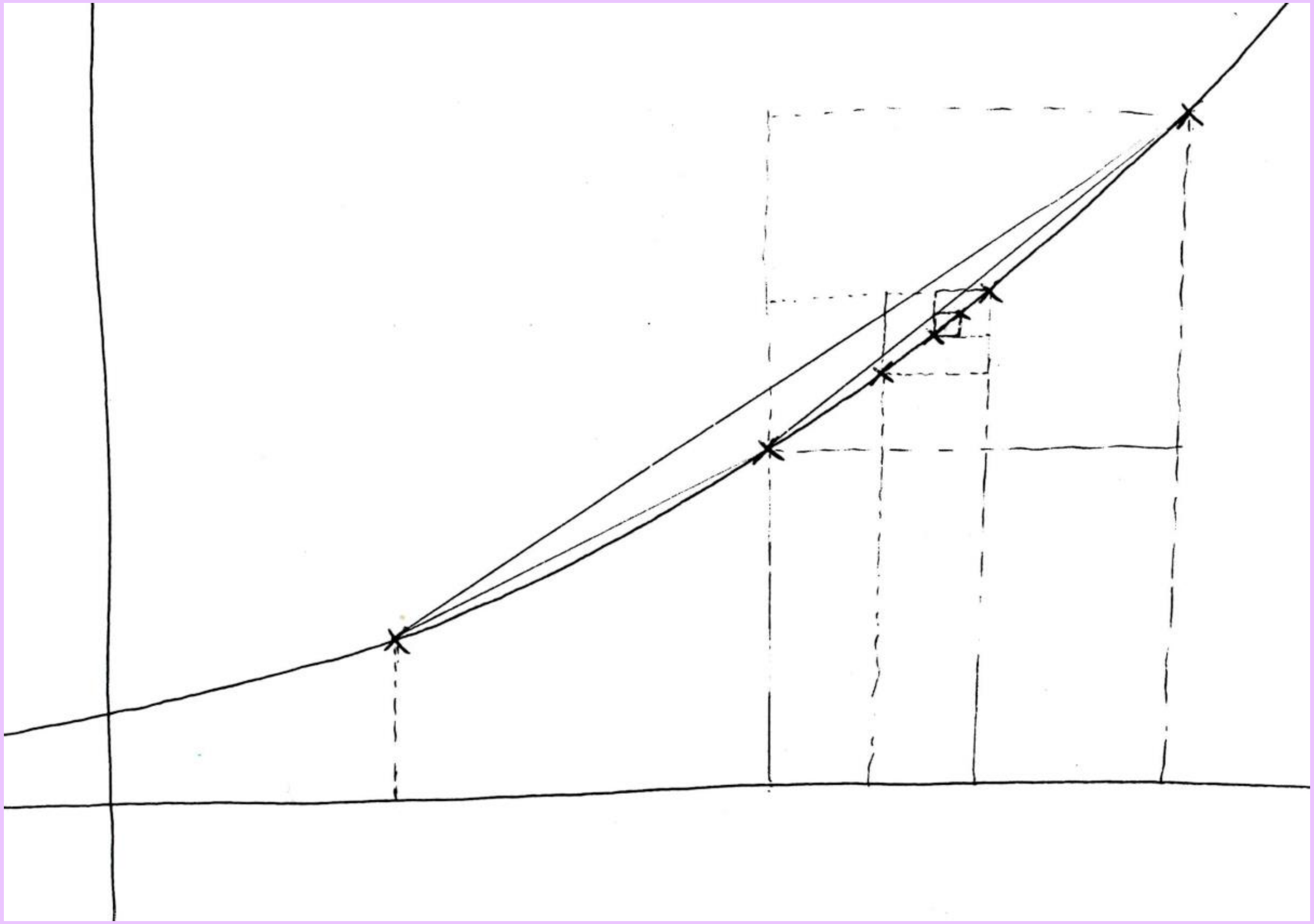


We have fitted a 3rd order polynomial from the given data to model the position of the car with respect to time. This is:

$$s = -0.3024t^3 + 12.13t^2 + 4.3142t$$

Sample Student Work: Jeremy

Jeremy’s graphical explanation when asked about his specific strategy.



Sample Student Work: Jeremy

Snapshots of Jeremy's calculations.

16)	4.00	209.6767	.125	10.6425	85.14
17)	4.125	220.5065	.125	10.8298	86.6384
18)	4.25	231.5236	.125	11.0171	88.1368
19)	4.375	242.7279	.125	11.2043	89.6344
20)	4.5	254.1195	.125	11.3916	
21)	4.625	265.6984	.125	11.5789	
22)	4.75	277.4645	.125	11.7661	

4)	4.1625	223.7920	.0125	1.097	87.76
5)	4.175	224.8907	.0125	1.0989	87.912
6)	4.1875	225.9917	.0125	1.1008	88.064
7)	4.2	227.0943	.0125	1.1026	88.208
8)	4.2125	228.1988	.0125	1.1055	

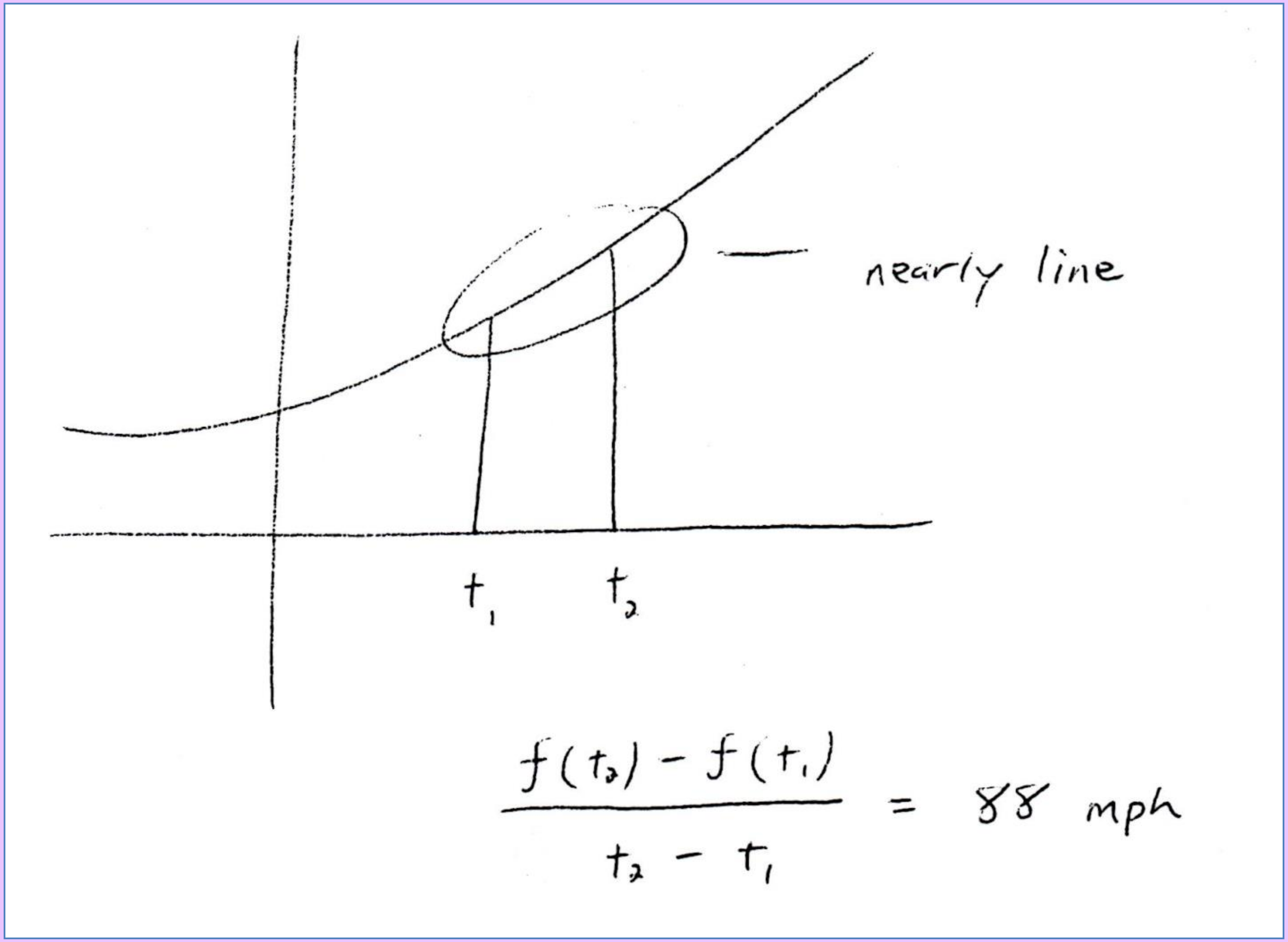
Table 3

	Time	Distance	Δt	ΔD	Avg. velocity
1)	4.175	224.8909			
2)	4.18125	225.4410	.00625	.5501	88.016
3)	4.1875	225.9917	.00625	.5507	88.112

→ 88 ft/s achieved between 4.175 & 4.18125 seconds

Sample Student Work: Dan

Dan’s graph showing local linearity.



Sample Student Work: Dan

Dan’s work on a computer algebra system.

```
> f:=5.99*t^2+37.95*t-38:
> t1:=5: t2:=6:
> f1:=eval(f,t=t1): f2:=eval(f,t=t2):
> (f2-f1)/(t2-t1);
                                                  103.84
> f:=5.99*t^2+37.95*t-38:
> t1:=4.5: t2:=5:
> f1:=eval(f,t=t1): f2:=eval(f,t=t2):
> (f2-f1)/(t2-t1);
                                                  94.85500000
> f:=5.99*t^2+37.95*t-38:
> t1:=4: t2:=4.5:
> f1:=eval(f,t=t1): f2:=eval(f,t=t2):
> (f2-f1)/(t2-t1);
                                                  88.86500000
> f:=5.99*t^2+37.95*t-38:
> t1:=4.2: t2:=4.3:
> f1:=eval(f,t=t1): f2:=eval(f,t=t2):
> (f2-f1)/(t2-t1);
                                                  88.86500000
> f:=5.99*t^2+37.95*t-38:
> t1:=4.1: t2:=4.2:
> f1:=eval(f,t=t1): f2:=eval(f,t=t2):
> (f2-f1)/(t2-t1);
                                                  87.66700000
> f:=5.99*t^2+37.95*t-38:
> t1:=4.17: t2:=4.18:
> f1:=eval(f,t=t1): f2:=eval(f,t=t2):
> (f2-f1)/(t2-t1);
                                                  87.96650000
```

What do we see in these solutions?

- More than D = RT
- Local Linearity
- Limits
- Connections to other areas of Calculus
  - Physics
  - Geometry
  - Iterative Schemes

Hallmarks of a Good Task

- The *problem solver* must decide what mathematics to bring in
- The task uses real-life (often messy!) data
- The task requires mathematical modeling

Learning Trajectories

- The problem is authentic, has a low entry point for engagement, yet challenges the problem solver in a multitude of ways.
- Natural components of this problem are average and instantaneous rates of change, data analysis and statistics, technology use, and mathematical modeling.

Corvettes, Curve Fitting, and Calculus

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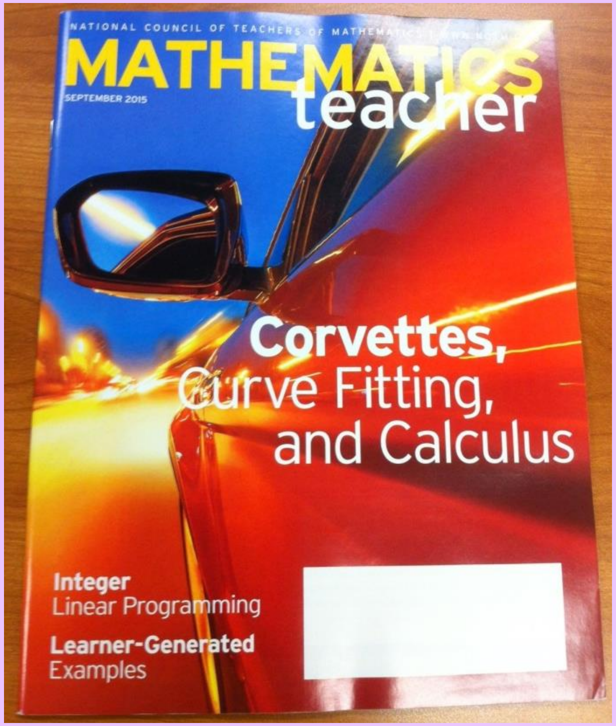

Jackie Murawska
Saint Xavier University
Chicago, Illinois
murawska@sxu.edu
twitter: @murawskamath

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