Cooperative Activities for Calculus

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Resources for Activities:

AP Calculus Teacher Community and Electronic Discussion Group:
https://apcommunity.collegeboard.org/

Designated Deriver: https://designatedderiver.wikispaces.com

AP Calculus Wiki: https://apcalcwiki.wikispaces.com/

Facebook Group -- AP Calc Teachers AB/BC:
https://www.facebook.com/groups/757156494357029/

Stu Schwartz' Website: http://www.mastermathmentor.com/

Sean Bird's Website: http://cchsindy.org/bird/Calculus.htm

Lin McMullin's Blog: http://teachingcalculus.com/

Whiteboarding Resources: http://www.mctmmathbits.org/?p=416

Slope Field Card Sort:
A) Created by Nancy Stephenson, St. Thomas High School, Houston, TX:
http://apcentral.collegeboard.com/apc/public/repository/ap08_calculus_slopefields_cardmatch.pdf

B) Created by Debbie Preston, Keystone School, San Antonio, TX:
https://apcalcwiki.wikispaces.com/file/view/L)+Binder2+Graphing+PDFS.pdf

Differential Equations Group Activities Lesson Plan:
These activities will take place over several class days. Have the students return to the same groups with the same differential equation each time.

Day 1: Slope Field Introduction
1. Model creating a slope field with the differential equation: $y' = -2x$. Use large graph paper and make a grid [-3, 3] x [-3, 3]. Skip at least 2 lines between lattice points. Prompt students to look for patterns in the calculations. (Option: Sketch a solution curve through (2, 1).)
2. Assign student groups DE's to draw their own slope fields on large graph paper. Differentiate instruction by choosing the equation for each group or use random choices.
3. Do a gallery walk. Have students determine the shape of general solution cuves for each slope field as an exit slip.

Day 2: Euler's Method
1. Using the slope field for the DE: $y' = -2x$ (or another DE with a drawn slope field), and the point (-2, -1), draw 4-6 tangent line segments with x-increment of ½ or 1/3 to estimate y(0). At the end of each line segment, recalculate the appropriate slope.
2. Have student groups use their slope fields to create a similar path. Have them begin with any point that has x-value of -2 and take steps the width of one square on their graphs.
3. If time permits, have them repeat the activity (using a different colored marker) for steps of width 1 or steps with width ½ of a grid square.
4. Display the graphs in the classroom.
5. Introduce Euler's Method using their paths as a reference for each step.

Day 3: Separable DE's
1. Determine which of the slope field DE's can be solved using separation of variables and which DE's will need to be solved using technology.
2. Solve the DE's using the initial value chosen for Euler's Method.
3. Compute y(0) for your solution equation. Compare the result with your estimate from Euler's Method.

$y' = y^2 - x^2$ $y' = \frac{-x}{y}$

$y' = y + x^2$ $y' = \frac{1}{2}y$

$y' = y^2 - x$ $y' = xy$

$y' = 2x - 3y + 1$

$y' = y(3 - y)$ $y' = \frac{y}{x}$

$y' = y + x$ $y' = x^2 y$

Partner Volume Project

Determine the volume of your crepe paper decoration.

What you will need to do and turn in:

- Trace the outline of your decoration on graph paper.
- Measure key points along your outline to determine x- and y-coordinates in cm.
- Create data tables to represent points on each curve of your outline.
- Compute equation(s) to match your curve(s) well.
- Write integral(s) to represent the volume of your decoration.
- Calculate the volume using your calculator.
- Does your answer seem reasonable? Explain why or why not.

Partner/Group Real Object Activity:

Include Name of Object

What you will need to do:

- Be sure your object will work.
- Make a table to find radii.
- Make a graph to show your data and the curve.
- Find equation(s) to match your curve well.
- Write integral(s) for volume.
- Calculate the volume.
- Use displacement (Thursday or Friday) to find volume.
- Calculate the error.

What you will turn in:

- Name of object
- Clear data table.
- Clear graph.
- Equations that model the curves.
- Correct integrals.
- Volume by integrals.
- Volume by displacement.
- Error calculation.

Due Wednesday.
20 points.

AP Calculus Group Project – Solid of Revolution Lesson Plan

Objective: Students will use an object from home as a model solid of revolution. Students will compute the volume of their object using calculus and then compare their result with the volume determined by submerging the object in water.

Advanced Preparation: Once the concept of a solid of revolution has been introduced, students should be told to begin looking for an appropriate object. Remind them every day until you do the project. Arrange to use a lab room for 1-2 periods for this project—the lab tables and access to beakers or graduated cylinders is helpful. (Switch with Chemistry for the day.) Review the regression features on their calculators and how to determine the curve of best fit.

Lab Day: Gather needed supplies. 2 large beakers and/or graduated cylinders. Measuring tools: rulers, calipers, string. Graph paper. Duct tape (to seal over holes in objects).

Procedure: Students should form pairs OR small groups (3-4) for this project.

Student Assignment Sheet:

Find an object of revolution at home and bring it in for this project. Objects should be about the size of your fist and be submersible. The objects cannot be cylinders, cones or spheres.

You need to choose at least 2 objects from your group to complete this project. Measure everything in centimeters!

1. Sketch a graph of the function(s) that could form a region to rotate and create your object. Use the measuring tools in class to find actual lengths on your object and plot points carefully. Be precise!
2. Calculate the appropriate equation(s) for your function(s). A piecewise-defined equation will work best for most objects.
3. Write the integral(s) that determine the volume of your object. Use your calculator to evaluate the integral(s). Round your answer to the nearest cubic centimeter.
4. Submerge your object and determine its actual volume. Compare the actual volume with the result from your calculations. Calculate your percentage of error. Are there any factors that would lead to large error amounts?
5. Repeat for another object.

Closing: Remind the students to bring their work back tomorrow if they are not done. If finished, they should turn in their work at the end of class.

AP Calculus Group Review Project

Your task is to re-teach a major concept of this course as part of our preparation for the AP Test. You will need to work in a group of 2 or 3 to produce a 5-10 minute presentation. Each group will sign up for a presentation topic and date. You will also be responsible to participate in all other groups' presentations.

Each Project Must Include:

- **Presentation:** Remember to have any needed copies or materials for each member of class.
- **Assessment:** One blank master copy and one copy with complete worked out solutions.

Presentation Choices:

- **Song:** Write new lyrics to the song (or songs) of your choice that teach how to do your topic. Your group will need to sing your song and teach it to the class. Everyone will need copies of the lyrics.
- **Skit:** Act out your topic. Your dialogue will need to teach your topic and your actions will need to clarify your work. You may do this as a video or live action drama. You will need to turn in a written script. If you produce a video, you will need to turn in a copy of your tape/DVD.
- **Cartoon:** Create a comic that teaches your topic. Use poster boards, a comic book (copies for everyone), or powerpoint format. Be colorful, detailed, and creative. Neatness and style counts!
- **Short Story:** Write a short story that teaches your topic. Develop characters and a plot. Your story needs to be at least 2 pages long. Use complete sentences and proper grammar. Your group will need to lead the reading of the story. Everyone will need copies or a large-print format will need to be projected.
- **Game:** Create a game to review your topic. Design the game for either whole class or small group play. Directions need to be clear and written. Bring in enough materials for everyone to play.
- **Create your own:** Have another idea for a project? Let's talk and see if it will work.

Assessment:

Be the "Evil AP Test Writer": Write 2 multiple choice questions and 1 free response question on your topic. Be creative—**don't repeat problems that we have already done in class**! Your questions need to completely cover the topic and be at an AP-appropriate level of difficulty.

Write the Answer Key: Write clear, complete solutions.

Your Grade:

Your group will be graded on the completeness of your presentation, the clarity of your presentation, the quality of your assessment questions, your creativity, over-all project quality, and your individual participation in the group project. This is a 40-point assignment.

Each Group Will Turn In:

A copy of the presentation, assessment questions, and a grading key must be turned in by each group.

You will need to work on your own time to complete this project. Don't wait until the last minute!

AP Calculus Review Project

*Your Mission, should you choose to not FAIL, is to study for your AP Exam. You need to choose three topics (from either the AB or BC exam board) and complete the project requirements check list for **EACH** topic. Your topics must form a Tic-Tac-Toe! ☺*

For Each Topic:

Consult an Expert

- Look up your topic in another Calculus resource (textbook in class, review book, your Mom's Calculus book, Ask Mr Calculus.com). Choose a different source for each topic.
- Write a ½ page summary of what they said about the topic.
- Include a source citation.
- Finish 2 of these statements (3-5 sentences each). Be specific.
  - The example I liked best was _____ because …
  - Now I understand …
  - I am still confused about …
  - Wow! I can see how this is connected to …

WORK A PROBLEM SET

- Choose another Calculus resource book (It can be the same as above or you can switch to another.)
- Work a "typical" length homework assignment on this topic.
- Include a source citation with page numbers.

You're the Teacher (Choose a different option for each topic.)

- Write the "Notes" for this topic. Be sure to include theory and examples.
- Write the "Quiz" for this topic. Be sure to include an answer key.
- You have a friend at another school who is just starting to study this topic. Write a letter to help explain it all. Be creative!
- Write a full-proof (or fool-proof) calculator program to help you do problems from this topic. Turn in a copy of your program.

AB Topics

Optimization	Separable Differential Equations	Related Rates
Implicit Derivatives	Volumes Of Solids Of Revolution	Max ☹ & Minnie ☺
Slope Fields	**Position Velocity Acceleration**	**Riemann Sums**

BC Topics

Volumes By Cross Sections	Parametric Arc Length	Power Series: Interval Of Convergence
Partial Fractions	Separable Differential Equations	Related Rates
Position Velocity Acceleration Vectors	Maclaurin & Taylor Series	Integration By Parts