Using Area Models
To Teach Multiplying, Factoring
And Polynomial Division

http://tinyurl.com/nzz3wq9

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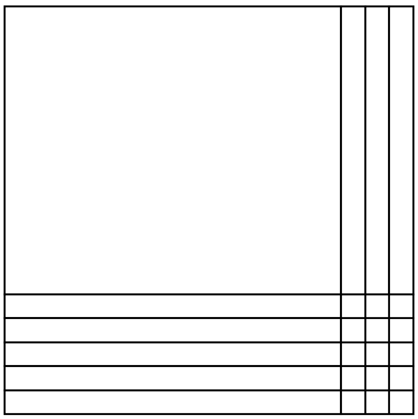
Modeling with Area

- Base Ten Blocks
- Generic Model for Multiplication
- Distributive Property
- Multiplying Polynomials
- Factoring
- Completing the Square
- Dividing Polynomials

Base Ten - Multiplication Area Model

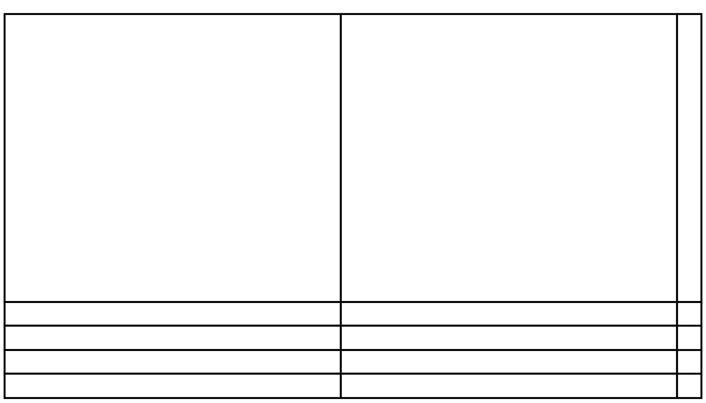
Example 1: Multiply 13 • 15

Patterns:



Example 2: Multiply 21 • 14

Patterns:



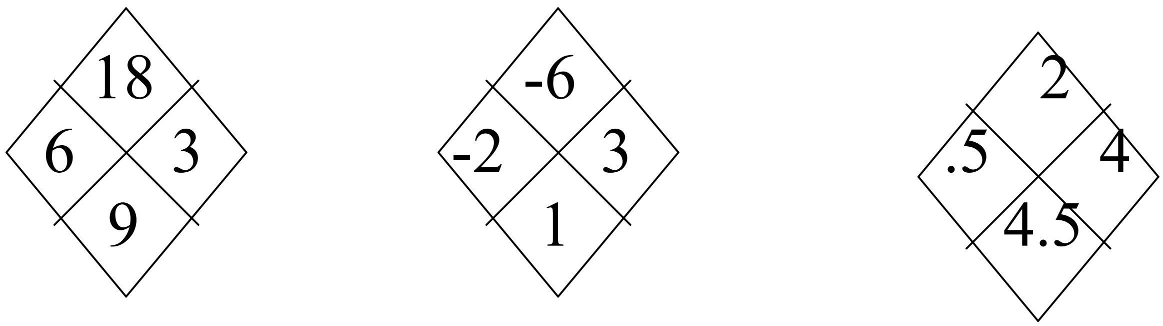
Base Ten - Generic Model

Example: 18 • 12

Example: 146 • 57

Diamond Problems

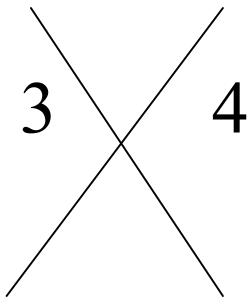
Can you find the Pattern?



When you think you know it, see if you can convince a neighbor.

Use the Pattern you discovered to complete the Diamond Problems.

a)



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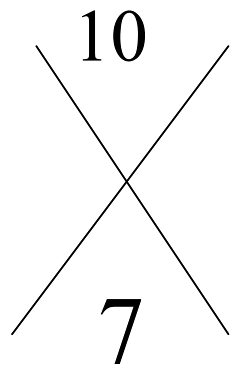

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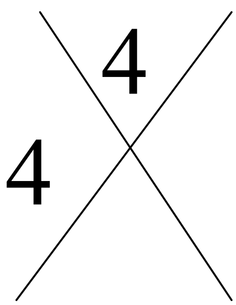
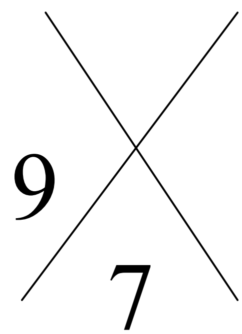
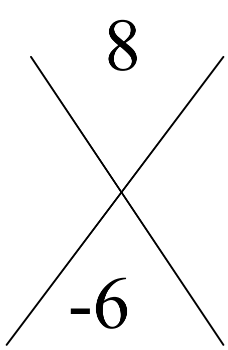


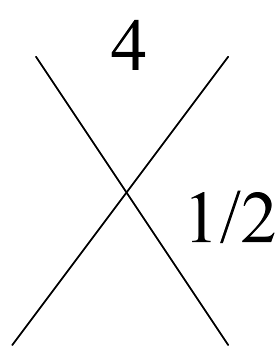
d)



e)



f)



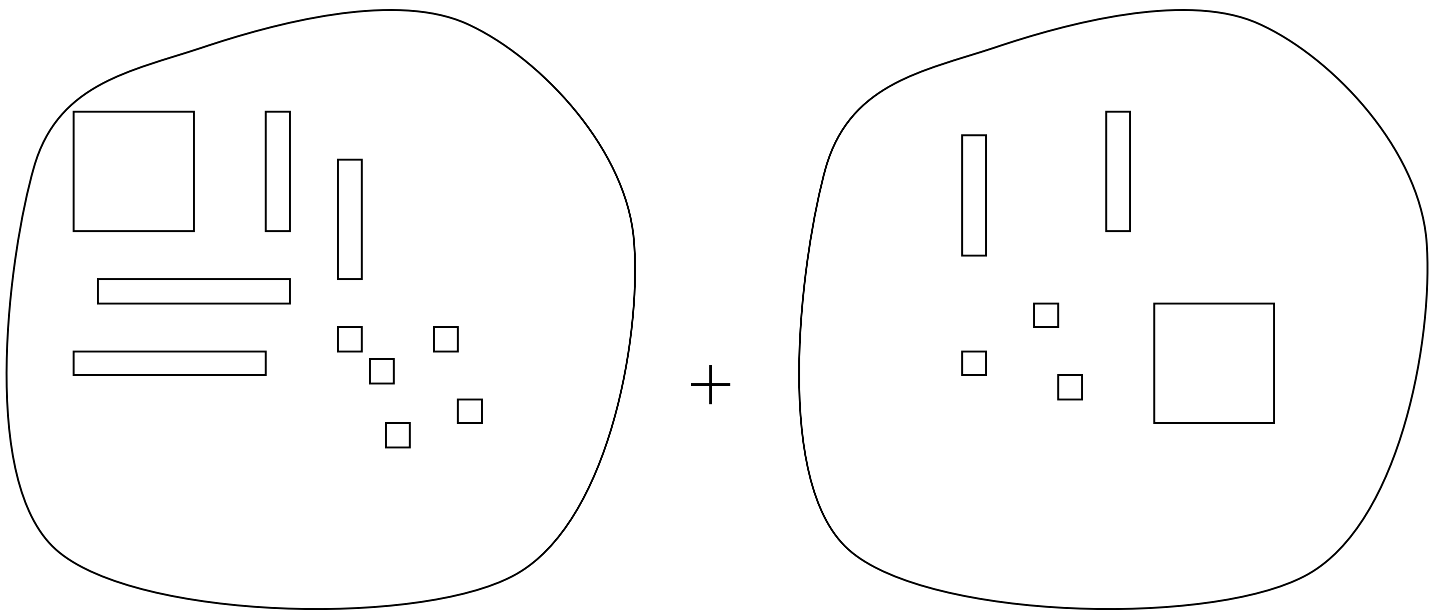
Meet the Algebra Tiles

Maya has 4 large squares, 6 rectangles and five small squares. Logan borrows 3 large squares, 4 rectangles and 2 small squares. What does Maya have left?

(   ) – (   ) = (   )

Combine the Like Terms

Iconic



( ) + ( ) = ( )

Symbolic

$$(5x^2 + 6x + 3) + (2x^2 + 4x + 7) = \underline{\hspace{6cm}}$$

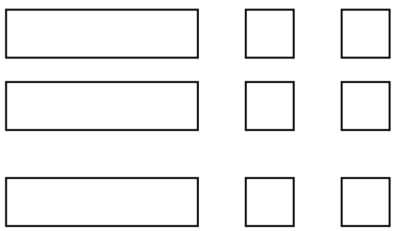
Use Algebra Tiles to show that

$1x + 2x \neq 2x^2$

$2x - x \neq 2$

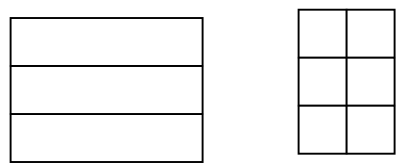
Grouping Algebra Tiles

Figure A



$3x + 6$

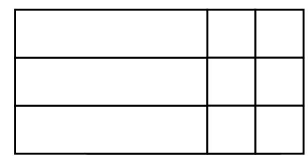
Figure B



$3(x) + 3(2)$

3 rows of x
3 rows of 2

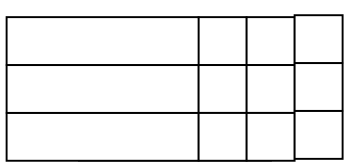
Figure C

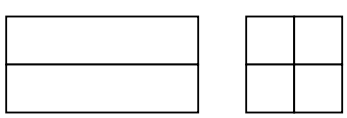


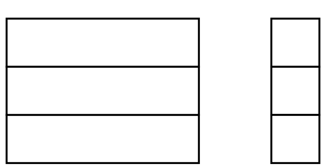
$3(x + 2)$

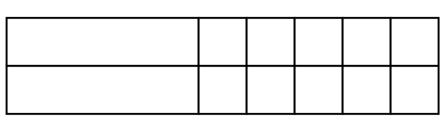
3 rows of
(x+ 2)

Match the following Algebra Tile groupings.

a) 

b) 

c) 

d) 

1. $2(x) + 2(2)$
2. $3(x + 3)$
3. $2\ (x + 5)$
4. $3(x) + 3(1)$

Distributive Property

Use your Algebra Tiles to represent the following. Write your answer as a number sentence.

Example 1:

$3(x + 5)$ “3 groups of x plus 5”

Length • Width = Area

$3\ (x + 5)\ =$ ___________

Example 2:

$2(3x + 1)$ “2 groups of 3x plus 1”

Length • Width = Area

$2\ (3x + 1) =$ _________

Rectangles: Length • Width = Area

What is the area of a rectangle with dimensions $(x + 3)$ by $(x + 2)$? Build it.

Recall the Base 10 Block Patterns? Do they apply here?

Build It.......

Write the <u>area as a product</u> and <u>as a sum.</u>

1) A rectangle with dimension $(2x+1)$ by $(x+3)$.

2) A rectangle with dimensions $(x+4)$ by $(x+3)$.

3) A rectangle with dimensions $(2x+3)$ by $(x+1)$.

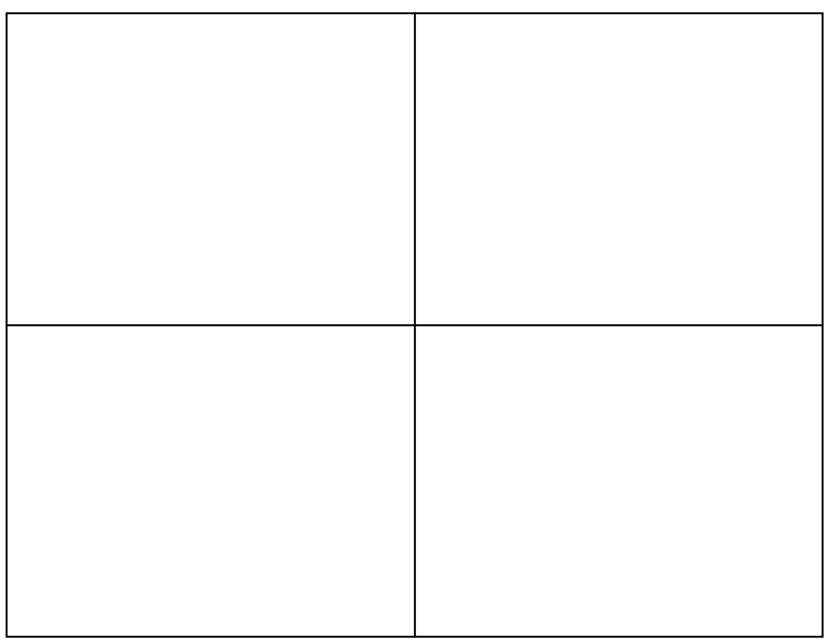
4) A rectangle with dimensions $(x+4)$ by $(x+5)$.

Generic Rectangles

Multiplying Polynomials

$(2x + 7)(x + 4)$

$(2x - 3)(x - 1)$



Note: Do not introduce negative or large numbers while working with Tiles or Pictures.

More Generic Rectangles

$$(x+3)(x^2+4x+5)$$

$$(2x-3)(4x^2-7x+8)$$

Challenge: Using the following Algebra Tiles, build a rectangle

$2x^2$ 7x 3

What are the dimensions of your rectangle?

Write the area as a product and a sum.

Area as a product = Area as a sum.

Factoring……Use your Algebra Tiles

1. What are the dimensions of a rectangle with the area of $3x^2 + 7x + 2$

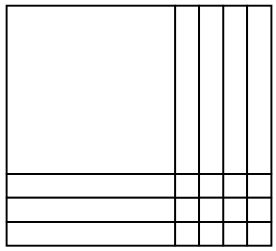
________________ = ________________

Area as a product = Area as a sum.

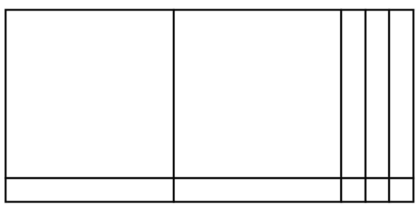
2. What are the dimensions of a rectangle with the area of $4x^2 + 5x + 1$ **= Area as a sum.**

Factoring – Write an <u>algebraic equation</u> for the area of each rectangle. **Area as a Product=Area as a Sum.**

a)



b)



c)

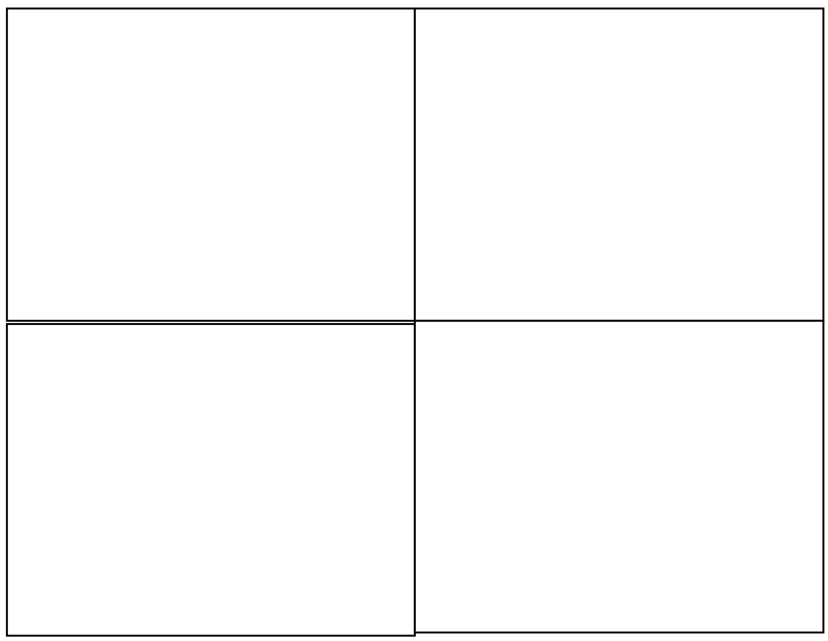
x^2	$5x$
$3x$	15

d)

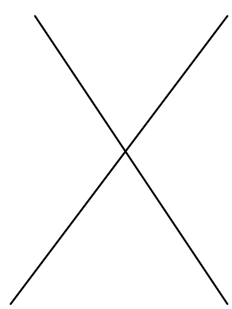
x^2	$6x$
$3x$	18

Factoring Using Diamond Problems

Start with $x^2 + 8x + 12$ and draw a generic rectangle. Use the patterns we discovered and fill in the parts we know.



a) Use this information to write and solve a Diamond Problem.



b) Complete the rectangle and write your equation.

Try some more…..

$$x^2 + 13x + 12$$

$$x^2 + 10x - 24$$

Special Cases

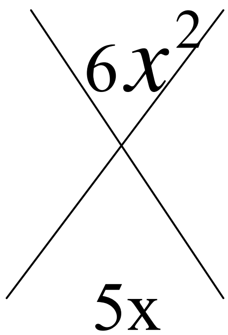
1) $x^2 + 14x + 49$

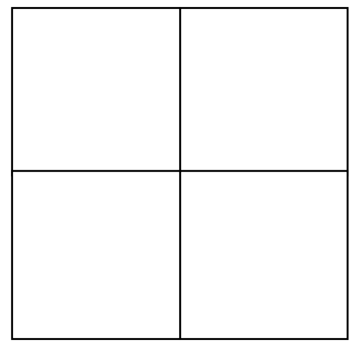
2) $x^2 - 36$

3) $25x^2 - 64y^2$

Using Diamond Problems to factor when the co-efficient $\neq 1$.

$$2x^2 + 5x + 3$$





Step 1: Multiply the coefficient of x^2 by the constant to find the product.

Step 2: Use the generic rectangle to fill in the inside pattern.

Step 3: Find the rectangle's dimensions.
(Pull out the G.C.F. Since $2x$ is the greatest common factor of $2x^2 + 2x$ we know where the $2x$ should be placed)

Try Some More……

$$4x^2 + 5x + 1$$

$$7x^2 + 4x - 3$$

Completing the Square

Make a square using the tiles given below.

1) $x^2 + 4x + 5$

2) $x^2 + 6x + 2$

3) $x^2 + 3x + 1$

Dividing Polynomials

$$\left(2x^3+2x^2-4x+24\right) \div (x + 3)$$

$$\left(4x^3+23x^2+14x-5\right) \div (x + 5)$$

Polynomials (with remainders)

$$\left(3x^3+4x^2+2x+15\right)\div(x+2)$$

$$\left(5x^3-12x-13\right)\div(x-2)$$