AN EXAMINATION OF STUDENTS' REASONING ABOUT TRIGONOMETRIC FUNCTIONS WITH REPRESENTATIONS

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The Problem To Address

- Trigonometric functions have been identified as one of the more difficult topics for first-year college students and secondary students to learn (Byers, 2010; Thompson, 2008).
- But trigonometric functions have not been well represented in the research literature; far more attention has been paid to the teaching and learning of non-trigonometric functions, such as linear functions.
- Students often rely on drawing a right triangle, reciting memorized trigonometry values for common angles, and the use of calculators, which can limit their understanding of trig functions to a small set of properties and limited domain (Brown, 2005; Byers, 2010).

Research Questions

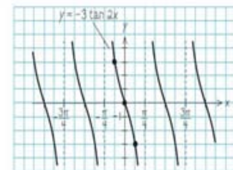
- How do college students work with and translate among multiple representations of trigonometric functions when performing mathematical tasks?
- In what ways do college students reason about trigonometric functions when working within a particular type of representation?

Literature Review – Multiple Representations

- Different types of representations provide students with different ways of thinking about functions (CCSSM, 2010; Even, 1993; Kaput, 1987; Kieran, 2007; NCTM, 1989, 2000).

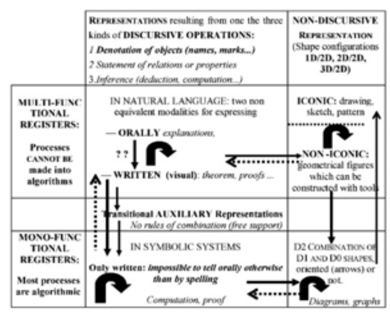
e.g., $y = -3tan2x$

x	$f(x)$
$-\frac{\pi}{4}$	undefined
0	0
$\frac{\pi}{4}$	undefined



- However, students often struggle with translating between different types of representations of functions (Galbraith & Haines, 2000; Goldenberg, 1988; Hitt, 1998; Selden & Selden, 1992).

Cognitive Approach Framework (Duval, 2006)



Mathematical Reasoning Framework (Lithner, 2008)

Reasoning types	Sub-type of reasoning	Characteristics of each type of reasoning
Imitative Reasoning	Memorized reasoning	Only recalls a complete answer in detail
		Only writes down without having considered preceding parts such as an identical copy of a textbook proof
	Algorithmic reasoning	Only recalls a solution algorithm without understanding but not whole answer detailed
		A careless mistake beyond computation error interferes gaining an answer
Creative Reasoning	Local Creative reasoning	A forgotten mathematical fact or concept is recreated without depending upon memorization
		The fact and concept is used only a few local part of solutions
	Global Creative reasoning	A forgotten mathematical fact or concept is recreated without depending upon memorization
		The fact and concept is mainly used based upon conceptual understanding

Methodology

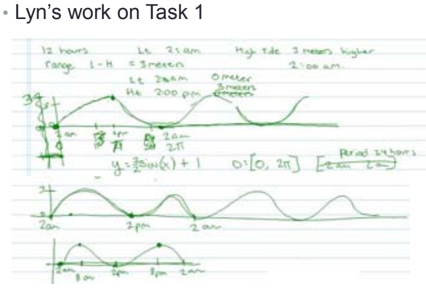
- Qualitative embedded multi-case study
  - An embedded multiple case study approach can be used when there is more than one sub-unit of analysis (Yin, 2003). An embedded design is used to study various units within an identifiable case.
  - In this study, the tasks serve as the cases. Each case/task was purposefully designed to begin in a different one of Duval's representation registers (natural language (N), drawings (D), symbolic systems (S), and graphs and mathematical diagrams (G)).
  - Analysis of six participants' work is embedded as sub-units within each of these cases (Yin, 2003).

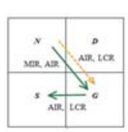
Case 1: a task starting N

Rainytown has low tides every 12 hours. Local fishermen note that one of the low tides occurs at 2am. The water level at high tide is 3 meters higher than it is at its lowest level. Create an equation to represent this situation.

Example of Student Work

- Lyn's work on Task 1





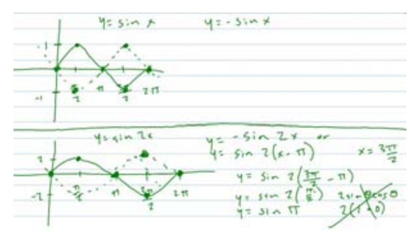
Case 2: a task starting S

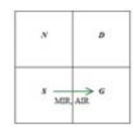
(a) If $y = sin\ x$ is changed into $y = sin(x - \pi)$, how does the graph change?

(b) If $y = sin\ 2x$ is changed into $y = sin\ 2(x - \pi)$, how does the graph change?

Example of Student Work

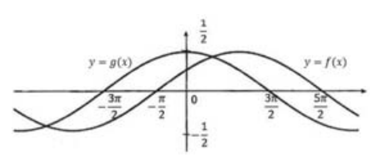
- Doug's work on Task 2





Case 3:a task starting G

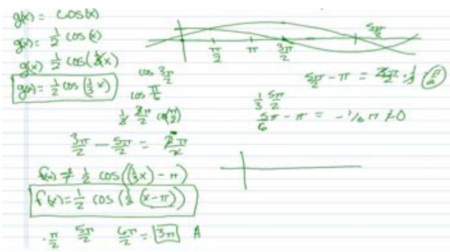
A graph of a trigonometric function horizontally moves $y = g(x)$ to $y = f(x)$. See the below.

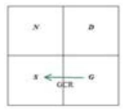


Determine possible equations for the graphs of $y = g(x)$ and $y = f(x)$ shown.

Example of Student Work

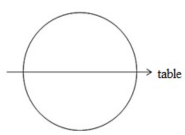
- Spiro's work on Task 3





Case 4: a task starting D

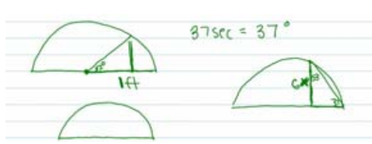
A large circular saw blade with a 1-foot radius is mounted so that exactly half of it shows above the table. It is spinning slowly, at one degree per second. One tooth is initially 0 feet above the table, and rising. See below.

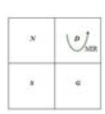


What is the height after 37 seconds? What is the height after t seconds?

Example of Student Work

- Moll's work on Task 4





Answering the research question 1

- *How do college students work with and translate among multiple representations of trigonometric functions when performing mathematical tasks?*
  - Without working with multiple representations, participants were not able to complete given tasks.
  - However, just using multiple representations did not indicate that the participants' had profound understandings of trigonometric functions or their properties. Without being able to unpack their understanding of the task in register G, none of the students would have been able to work on Task 1, for example.

Answering the research question 2

- *In what ways do college students reason about trigonometric functions when working within a particular type of representation?*
  - The multiple-functional registers N and D were used less often by the participants than the mono-functional registers S and G.
  - However, participants used mainly creative reasonings when employing the registers, N and D. It was likely to see registers S and G used together when registers N and D were employed.
  - Registers S and G were often used when imitative reasonings, although the use of register G did contribute to several examples of local and global creative reasonings.

Implications for teaching

- This study illustrated some ways in which students could misuse a formula, which can lead them to confuse the ideas of properties such as amplitude, shifts, and period in both the symbolic and graphical representations.
- It also showed how some students were able to creatively turn to another register to help them when they became stuck in the register in which they were working.
- Teachers and researchers could think of such a misuse and reflect back on the common instruction presented in most textbooks. They could explicitly encourage more translations among the four registers, helping students build more robust understandings of the properties of these important functions.